Property of $1/2^{-}$ baryon states

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Outline

- Symmetric quark model and some selection rules
- A revisit to the mixing of S-wave states and implication of their structures
  - Strong coupling
  - S-wave contributions in $\pi^- p \to \eta n$ and $K^- p \to \Sigma^0 \pi^0$
  - EM coupling
- Summary

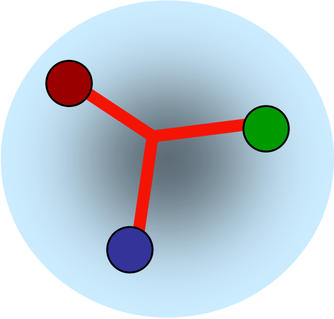
1. Baryons in SU(6)⊗O(3) symmetric quark model

Basic assumptions:

i) Chiral symmetry spantaneous breaking leads to the presence of massive constituent quarks as effective degrees of freedom inside hadrons;

ii) Hadrons can be viewed as quark systems in which the gluon fields generate effective potentials that depend on the spins and positions of the massive quarks.

Color	$SU(3)$	$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}$	$=\mathbf{10}_s+\mathbf{8}_\rho+\mathbf{8}_\lambda+\boxed{\mathbf{1}_a}$
Spin	$SU(2)$	$\mathbf{2}\otimes\mathbf{2}\otimes\mathbf{2}$	$=\mathbf{4}_s+\mathbf{2}_\rho+\mathbf{2}_\lambda,$
Flavor	$SU(3)$	$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}$	$=\mathbf{10}_s+\mathbf{8}_\rho+\mathbf{8}_\lambda+\mathbf{1}_a,$
Spin-flavor	$SU(6)$	$\mathbf{6}\otimes\mathbf{6}\otimes\mathbf{6}$	$=\mathbf{56}_s+\mathbf{70}_\rho+\mathbf{70}_\lambda+\mathbf{20}_a,$
Spatial	$\mathrm{O}(3)$	L^P	s, ρ, λ, a



Baryon wavefunction as representation of 3-dimension permutation group:

$$\phi_c|\mathrm{SU}(6)\otimes\mathrm{O}(3)\rangle=\phi_c|\mathrm{N}_6,\ {}^{2S+1}\mathrm{N}_3,\ N,L,J\rangle$$

symmetric

PDG2004: 22 nucleon resonances (uud, udd)

Particle	$L_{2I \cdot 2J}$	Overall status	Status as seen in —						
			$N\pi$	$N\eta$	ΛK	ΣK	$\Delta\pi$	$N\rho$	$N\gamma$
$N(939)$	P_{11}	****							
$N(1440)$	P_{11}	****	****	*			***	*	***
$N(1520)$	D_{13}	****	****	*			****	****	****
$N(1535)$	S_{11}	****	****	****			*	**	***
$N(1650)$	S_{11}	****	****	*	***	**	***	**	***
$N(1675)$	D_{15}	****	****	*	*		****	*	****
$N(1680)$	F_{15}	****	****				****	****	****
$N(1700)$	D_{13}	***	***	*	**	*	**	*	**
$N(1710)$	P_{11}	***	***	**	**	*	**	*	***
$N(1720)$	P_{13}	****	****	*	**	*	*	**	**
$N(1900)$	P_{13}	**	**					*	
$N(1990)$	F_{17}	**	**	*	*	*			*
$N(2000)$	F_{15}	**	**	*	*	*	*	**	
$N(2080)$	D_{13}	**	**	*	*				*
$N(2090)$	S_{11}	*	*						
$N(2100)$	P_{11}	*	*	*					
$N(2190)$	G_{17}	****	****	*	*	*		*	*
$N(2200)$	D_{15}	**	**	*	*				
$N(2220)$	H_{19}	****	****	*					
$N(2250)$	G_{19}	****	****	*					
$N(2600)$	$I_{1\,11}$	***	***						
$N(2700)$	$K_{1\,13}$	**	**						

(**) not well-established

18 Lambda resonances (uds)

Particle	$L_{I\cdot 2J}$	Overall status	Status as seen in —			
			$N\overline{K}$	$\Lambda\pi$	$\Sigma\pi$	Other channels
$\Lambda(1116)$	P_{01}	****		F		$N\pi$(weakly)
$\Lambda(1405)$	S_{01}	****	****	o	****	
$\Lambda(1520)$	D_{03}	****	****	r	****	$\Lambda\pi\pi, \Lambda\gamma$
$\Lambda(1600)$	P_{01}	***	***	b	**	
$\Lambda(1670)$	S_{01}	****	****	i	****	$\Lambda\eta$
$\Lambda(1690)$	D_{03}	****	****	d	****	$\Lambda\pi\pi, \Sigma\pi\pi$
$\Lambda(1800)$	S_{01}	***	***	d	**	$N\overline{K}^*, \Sigma(1385)\pi$
$\Lambda(1810)$	P_{01}	***	***	e	**	$N\overline{K}^*$
$\Lambda(1820)$	F_{05}	****	****	n	****	$\Sigma(1385)\pi$
$\Lambda(1830)$	D_{05}	****	***	F	****	$\Sigma(1385)\pi$
$\Lambda(1890)$	P_{03}	****	****	o	**	$N\overline{K}^*, \Sigma(1385)\pi$
$\Lambda(2000)$		*		r	*	$\Lambda\omega, N\overline{K}^*$
$\Lambda(2020)$	F_{07}	*	*	b	*	
$\Lambda(2100)$	G_{07}	****	****	i	***	$\Lambda\omega, N\overline{K}^*$
$\Lambda(2110)$	F_{05}	***	**	d	*	$\Lambda\omega, N\overline{K}^*$
$\Lambda(2325)$	D_{03}	*	*	d		$\Lambda\omega$
$\Lambda(2350)$		***	***	e	*	
$\Lambda(2585)$		**	**	n		

Some selection rules in the symmetric quark model

The first orbital excitation states:

$|70,\ ^2 8,\ 1,1,J\rangle$ • $S_{11}(1535)$ (****), $D_{13}(1520)$ (****);

$|70,\ ^4 8,\ 1,1,J\rangle$ • $S_{11}(1650)$ (****), $D_{13}(1700)$ (***), $\boxed{D_{15}(1675)\ (****)}$;

• <u>Moorhouse selection rule</u> (Moorhouse, PRL16, 771 (1966))

$$\gamma + p(|\mathbf{56}, {}^2\mathbf{8}; 0, 0, 1/2\rangle) \not\leftrightarrow \quad N^*(|\mathbf{70}, {}^4\mathbf{8}\rangle)$$

$$\gamma + n(|\mathbf{56}, {}^2\mathbf{8}; 0, 0, 1/2\rangle) \leftrightarrow N^*(|\mathbf{70}, {}^4\mathbf{8}\rangle)$$

EM transition of groundstate 0^+ ($1/2^+$) → first excited state 1^- ($1/2^-$, $3/2^-$, $5/2^-$)

• Λ selection rule (Zhao & Close, PRD74, 094014(2006)) in strong decays

$$N^*(|\mathbf{70}, {}^4\mathbf{8}\rangle) \not\leftrightarrow \quad K(K^*) + \Lambda$$

N* (p,n) $$|70,\ {}^4 8, N, L, J\rangle = \sum_{L_z+S_z=J_z} \langle LL_z, \frac{3}{2}S_z|JJ_z\rangle \frac{1}{\sqrt{2}}[\phi^\rho \chi^s_{S_z} \psi^\rho_{NLL_z} + \phi^\lambda \chi^s_{S_z} \psi^\lambda_{NLL_z}].$$

Λ $$|56,\ {}^2 8, 0, 0, \frac{1}{2}\rangle = \frac{1}{\sqrt{2}}[\phi^\rho \chi^\rho_{S_z} + \phi^\lambda \chi^\lambda_{S_z}]\psi^s_{000},$$

$$\langle \phi^\lambda_\Lambda | \hat{I}_3 | \phi^\lambda_p \rangle = 0$$

• Faiman-Hendry selection rule (Faiman & Hendry, PR173, 1720 (1968)).

$$\Lambda^*(|\mathbf{70}, {}^4\mathbf{8}\rangle) \not\leftrightarrow \quad N(|\mathbf{56}, {}^2\mathbf{8}; 0, 0, 1/2\rangle) + \bar{K}$$

Isgur, Karl, & Koniuk, PRL41, 1269(1978)

2. Spin-dependent potential from one-gluon-exchange (OgE) and SU(6)⊗O(3) symmetry breaking

$$H_{hyper} = \frac{2\alpha_s}{3m_i m_j}\left[\frac{8\pi}{3}\mathbf{S}_i \cdot \mathbf{S}_j \delta^3(\mathbf{r}_{ij}) + \frac{1}{r_{ij}^3}\left(\frac{3(\mathbf{S}_i \cdot \mathbf{r}_{ij})(\mathbf{S}_j \cdot \mathbf{r}_{ij})}{r_{ij}^2} - \mathbf{S}_i \cdot \mathbf{S}_j\right)\right]$$

Introduces mass splittings and configuration mixings in the SU(6) multiplets.

Nucleon:

$$|N\rangle = 0.90|\mathbf{56}, {}^2\mathbf{8}; 0, 0, 1/2\rangle - 0.34|\mathbf{56}, {}^2\mathbf{8}; 2, 0, 1/2\rangle \boxed{-0.27|\mathbf{70}, {}^2\mathbf{8}; 2, 0, 1/2\rangle} - 0.06|\mathbf{70}, {}^4\mathbf{8}; 2, 2, 1/2\rangle$$

$$|N\rangle \xrightarrow{-0.27|\mathbf{70}, {}^2\mathbf{8}; 2, 0, 1/2\rangle} |70, \ {}^4 8, N, L, J\rangle$$

⇒ **Moorhouse selection rule must be violated !**

Selection rule violations

TABLE I. Violations of some SU(6) rules.

Quantity	SU(6) (Relative values)	This calculation (Relative values)	Experiment (Various units)	PDG2008
$A_{3/2}^{n}(D_{15}\to n\gamma)$	$-\alpha$	$-\alpha$	-60 ± 33 [a]	-58 ± 13
$A_{1/2}^{n}(D_{15}\to n\gamma)$	-0.71α	-0.71α	-33 ± 25 [a]	-43 ± 12
$A_{3/2}^{p}(D_{15}\to p\gamma)$	0	$+0.31\alpha$	$+20\pm13$ [a]	$+15\pm9$
$A_{1/2}^{p}(D_{15}\to p\gamma)$	0	$+0.22\alpha$	$+19\pm14$ [a]	$+19\pm8$
$A(D_{15}\to\bar{K}N)$	β	β	$+0.41\pm0.03$ [b]	
$A(D_{05}\to\bar{K}N)$	0	-0.28β	-0.09 ± 0.04 [c]	
$\langle\sum e_i r_i^2\rangle_p$	γ	γ	$+0.82\pm0.02$ [d]	
$\langle\sum e_i r_i^2\rangle_n$	0	-0.16γ	-0.12 ± 0.01 [e]	

Isgur, Karl, & Koniuk, PRL41, 1269(1978)

PDG2008

$N(1535) \rightarrow p\gamma$, helicity-1/2 amplitude $A_{1/2}$

VALUE ($\mathrm{GeV}^{-1/2}$)	*DOCUMENT ID*
$+0.090 \pm 0.030$ OUR ESTIMATE	

$N(1535) \rightarrow n\gamma$, helicity-1/2 amplitude $A_{1/2}$

VALUE ($\mathrm{GeV}^{-1/2}$)	*DOCUMENT ID*
-0.046 ± 0.027 OUR ESTIMATE	

$N(1650) \rightarrow p\gamma$, helicity-1/2 amplitude $A_{1/2}$

VALUE ($\mathrm{GeV}^{-1/2}$)	*DOCUMENT ID*
$+0.053 \pm 0.016$ OUR ESTIMATE	

$N(1650) \rightarrow n\gamma$, helicity-1/2 amplitude $A_{1/2}$

VALUE ($\mathrm{GeV}^{-1/2}$)	*DOCUMENT ID*
-0.015 ± 0.021 OUR ESTIMATE	

Both S11(1535) and S11(1650) are not pure SU(6) states.

Λ selection rule violation

Hyperfine interaction in Λ(uds) is relatively suppressed compared with that in nucleon due to the heavier s-quark

$$H_{hyper} = \frac{2\alpha_s}{3m_i m_j}\left[\frac{8\pi}{3}\mathbf{S}_i\cdot\mathbf{S}_j\delta^3(\mathbf{r}_{ij}) + \frac{1}{r_{ij}^3}\left(\frac{3(\mathbf{S}_i\cdot\mathbf{r}_{ij})(\mathbf{S}_j\cdot\mathbf{r}_{ij})}{r_{ij}^2} - \mathbf{S}_i\cdot\mathbf{S}_j\right)\right]$$

In the constitute regime, the Λ wavefunction will be dominated by $|56, {}^2 8>$. One expects relatively smaller configuration mixings from $|70, {}^2 8>$. Hence

$$\boxed{N^*(|\mathbf{70}, {}^4\mathbf{8}\rangle) \not\leftrightarrow \quad K(K^*) + \Lambda}$$

Approximately hold !

D15(1675) $\xrightarrow{?}$ KΛ

F17(1990) $\xrightarrow{?}$ KΛ

PDG2008

$(\Gamma_i\Gamma_f)^{1/2}/\Gamma_{total}$ **in** $N\pi \to N(1675) \to \Lambda K$ $(\Gamma_1\Gamma_3)^{1/2}/\Gamma$

VALUE	DOCUMENT ID		TECN	COMMENT
±0.04 to ±0.08 OUR ESTIMATE				
−0.01	BELL	83	DPWA	$\pi^- p \to \Lambda K^0$
+0.036	[5] SAXON	80	DPWA	$\pi^- p \to \Lambda K^0$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
−0.034±0.006	DEVENISH	74B		Fixed-t dispersion rel.

If the Λ selection rule holds approximately, all states of [70, ^{4}8] will not contribute to Λ production channel. – Ideal place to disentangle baryon resonances.

A coherent study of $\gamma p \to K^+\Lambda$ and $\gamma n \to K^0\Lambda$ would be useful for distinguishing conventional QM states and states beyond 3q scenario.

2. A revisit to the S-wave state mixing

The mixing between pure $[70,\ ^2 8]$ and $[70,\ ^4 8]$ states is defined as

$$\begin{pmatrix} S_{11}(1535) \\ S_{11}(1650) \end{pmatrix} = \begin{pmatrix} \cos\theta_s & -\sin\theta_s \\ \sin\theta_s & \cos\theta_s \end{pmatrix} \begin{pmatrix} |[70,\ ^2 8]\rangle \\ |[70,\ ^4 8]\rangle \end{pmatrix}$$

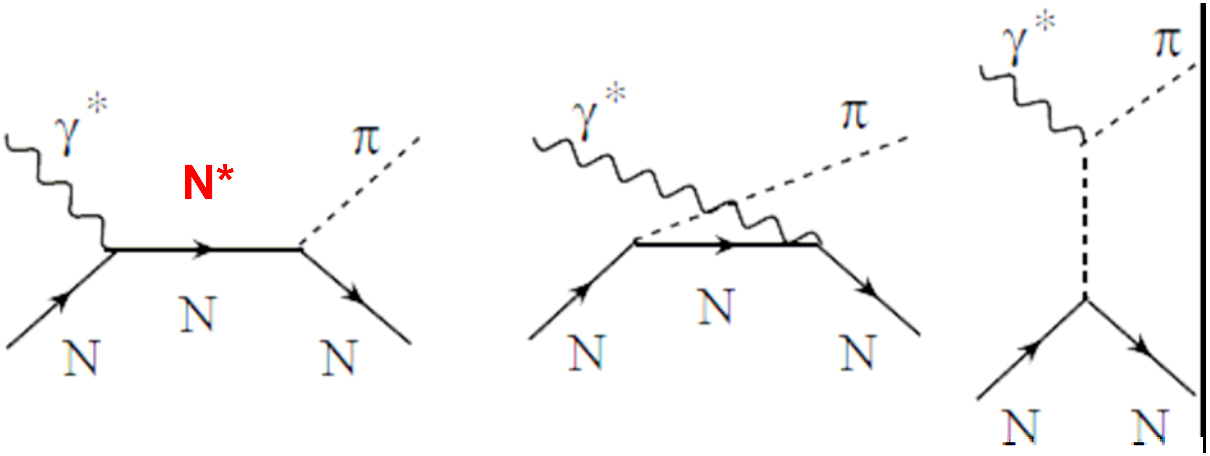
Questions:
Can we take the quark model seriously here?
What is the success and what is the failure?

A list to check:

1) **Mass spectrum**
2) **Couplings to the same final states, e.g. γN, πN, ηN, KΛ, KΣ, …**
3) **Excitations in photo and electroproduction**
4) **Excitations in meson-baryon scatterings**
5) **… …**

arXiv: 0810.0997[nucl-th] by Aznauryan, Burkert and Lee.

It is important to have a correct definition of the common sign of amplitudes and relative sign between helicity amplitudes, i.e. A1/2, A3/2, and S1/2.

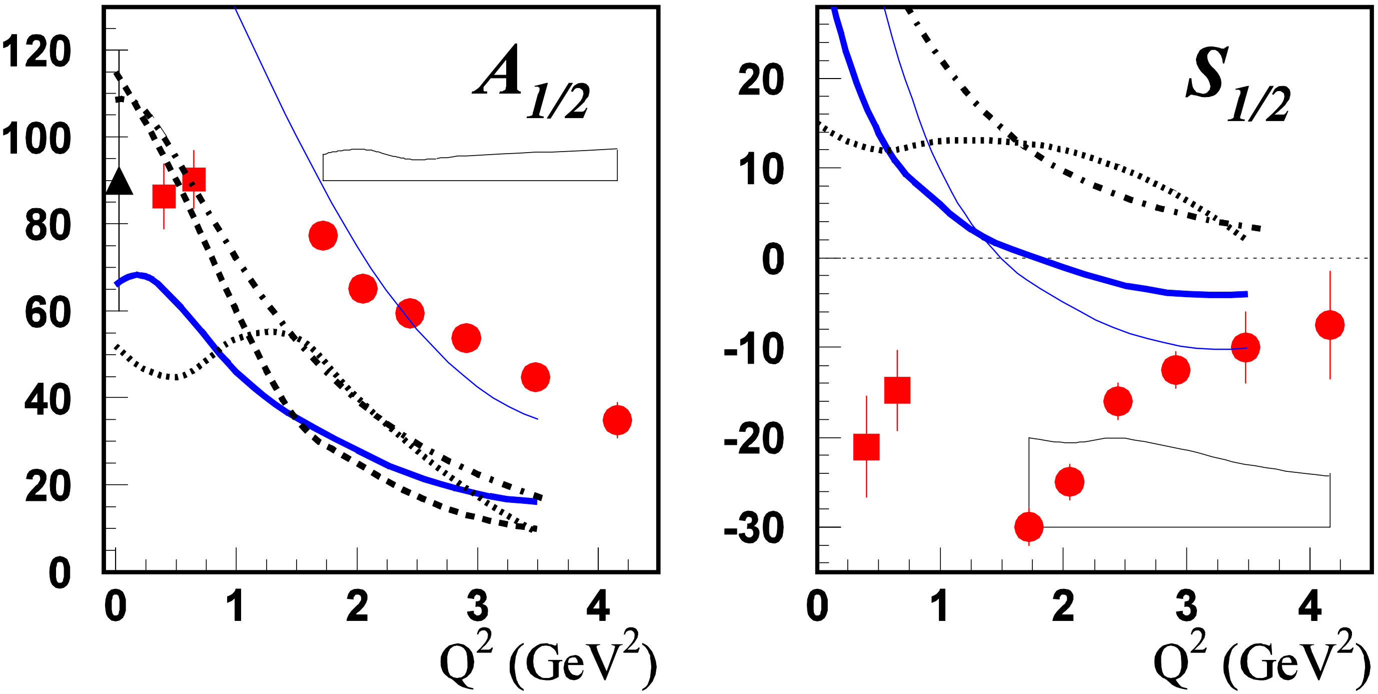


$\mathcal{A}_{1/2},\ \mathcal{A}_{3/2},\ \mathcal{S}_{1/2}$:

$$A_{\frac{1}{2},\frac{3}{2}} = \zeta \mathcal{A}_{\frac{1}{2},\frac{3}{2}}, \quad S_{\frac{1}{2}} = \zeta \mathcal{S}_{\frac{1}{2}}.$$

$$\boxed{\zeta = -sign(g^*/g)}$$

$\gamma^* p \rightarrow S_{11}$ (1535): 3q picture



Opposite sign of $S_{1/2}$!!!

Impossible to change in quark model !!!

Combined with the difficulties in the description of large width of **S_{11}(1535) → η N** and large **S_{11}(1535) → φN,ΛK** couplings, this shows that <u>3q picture for S_{11}(1535) should be complemented</u>

LF RQM:

Capstick, Keister, PR D51 (1995) 3598

Pace, Simula et.al., PR D51 (1995) 3598

From I. Aznauryan, Electromagnetic N-N Transition Form Factors Workshop, 2008*

- **What are the difficulties in the description of the couplings for S11(1535) and S11(1650) → ηN, πN, KΛ, and KΣ?**

- **What does the mixing tell about the S-wave resonance photo- and hadronic excitations?**

- **What are the mixing effects on the helicity amplitudes?**

(I) A chiral quark model for quark-meson interactions

In the chiral quark model, the low-energy quark-meson interactions are described by the effective Lagrangian

$$\mathcal{L} = \bar{\psi}[\gamma_\mu(i\partial^\mu + V^\mu + \gamma_5 A^\mu) - m]\psi + \cdots, \tag{1}$$

where V^μ and A^μ correspond to vector and axial currents, respectively. They are given by

$$\begin{aligned} V^\mu &= \frac{1}{2}(\xi\partial^\mu\xi^\dagger + \xi^\dagger\partial^\mu\xi), \\ A^\mu &= \frac{1}{2i}(\xi\partial^\mu\xi^\dagger - \xi^\dagger\partial^\mu\xi), \end{aligned} \tag{2}$$

with $\xi = \exp(i\phi_m/f_m)$, where f_m is the meson decay constant.

For the SU(3) case, the pseudoscalar-meson octet ϕ_m can be expressed as

$$\phi_m = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}, \quad (3)$$

and the quark field ψ is given by

$$\psi = \begin{pmatrix} \psi(u) \\ \psi(d) \\ \psi(s) \end{pmatrix}. \quad (4)$$

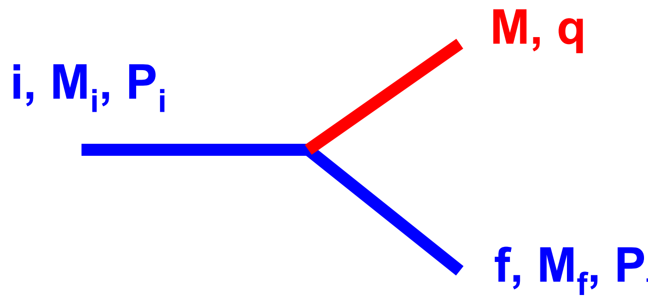
From the leading order of the Lagrangian [see Eq. (1)], we obtain the standard quark-meson pseudovector coupling at tree level

$$H_m = \sum_j \frac{1}{f_m} \bar{\psi}_j \gamma_\mu^j \gamma_5^j \psi_j \partial^\mu \phi_m, \quad (5)$$

where ψ_j represents the j-th quark field in the nucleon.

❑ Strong decay couplings

Nonrelativistic operator:



$$H_m^{nr} = \frac{1}{f_m} \sum_j \left\{ \frac{\omega_m}{E_f + M_f} \sigma_{\mathbf{j}} \cdot \mathbf{P_f} + \frac{\omega_m}{E_i + M_i} \sigma_{\mathbf{j}} \cdot \mathbf{P_i} - \sigma_{\mathbf{j}} \cdot \mathbf{q} + \frac{\omega_m}{2\mu_q} \sigma_{\mathbf{j}} \cdot \mathbf{p_j} \right\} \hat{I}_j e^{-i\mathbf{q}\cdot\mathbf{r_j}}$$

With the baryons described by the SU(6)⊗O(3):

$$\begin{aligned} H_m^{nr} &= 3 \times \left[-\left(\frac{\omega_m}{E_f + M_f} + 1 \right) \sigma_{3z} q + \frac{\omega_m}{2\mu_q} \sigma_{\mathbf{3}} \cdot \mathbf{p_3} \right] \hat{I}_3 e^{i\sqrt{\frac{2}{3}}\lambda_z q} \\ &\equiv (C_1 \sigma_{3z} q + C_2 \sigma_{\mathbf{3}} \cdot \mathbf{p_3}) \hat{I}_3 e^{i\sqrt{\frac{2}{3}}\lambda_z q} \\ &\equiv C_1 \hat{H}_1 + C_2 \hat{H}_2. \end{aligned}$$

$$\left\{ \begin{aligned} &\hat{H}_2 = \sigma_{3+} \hat{p}_{3-} + \sigma_{3-} \hat{p}_{3+} + \sigma_{3z} \hat{p}_{3z}, \\ &\sigma_{3\pm} \equiv \frac{1}{2}(\sigma_{3x} \pm i\sigma_{3y}), \\ &\hat{p}_{3\pm} \equiv \hat{p}_{3x} \pm i\hat{p}_{3y}. \end{aligned} \right.$$

A. $[70,^2 8] \to \mathrm{NM}$

$\hat{H}_1(\alpha), \hat{H}_2(\gamma - \sqrt{2}\beta)$	$S_{11}^+ \to \Lambda K^+$	$S_{11}^+ \to p\eta$	$S_{11}^+ \to n\pi^+$	$S_{11}^+ \to p\pi^0$	$S_{11}^+ \to \Sigma^+ K^0$
$\langle N, J_z = \frac{1}{2} \vert \hat{H}_1 \vert S_{11}^+, J_z = \frac{1}{2}\rangle$	$-\frac{1}{6}$	$-\frac{\cos\theta}{3\sqrt{3}}$	$-\frac{2\sqrt{2}}{9\sqrt{3}}$	$\frac{2}{9\sqrt{3}}$	$-\frac{1}{9\sqrt{6}}$
$\langle N, J_z = \frac{1}{2} \vert \hat{H}_2 \vert S_{11}^+, J_z = \frac{1}{2}\rangle$	$-\frac{1}{6}$	$-\frac{\cos\theta}{3\sqrt{3}}$	$-\frac{2\sqrt{2}}{9\sqrt{3}}$	$\frac{2}{9\sqrt{3}}$	$-\frac{1}{9\sqrt{6}}$

B. $[70,^4 8] \to \mathrm{NM}$

$\hat{H}_1(\alpha), \hat{H}_2(\gamma - \sqrt{2}\beta)$	$S_{11}^+ \to \Lambda K^+$	$S_{11}^+ \to p\eta$	$S_{11}^+ \to n\pi^+$	$S_{11}^+ \to p\pi^0$	$S_{11}^+ \to \Sigma^+ K^0$
$\langle N, J_z = \frac{1}{2} \vert \hat{H}_1 \vert S_{11}^+, J_z = \frac{1}{2}\rangle$	0	$\frac{\cos\theta}{3\sqrt{3}}$	$-\frac{\sqrt{2}}{9\sqrt{3}}$	$\frac{1}{9\sqrt{3}}$	$\frac{2\sqrt{2}}{9\sqrt{3}}$
$\langle N, J_z = \frac{1}{2} \vert \hat{H}_2 \vert S_{11}^+, J_z = \frac{1}{2}\rangle$	0	$\frac{\cos\theta}{3\sqrt{3}}$	$-\frac{\sqrt{2}}{9\sqrt{3}}$	$\frac{1}{9\sqrt{3}}$	$\frac{2\sqrt{2}}{9\sqrt{3}}$

$$
\left\{
\begin{aligned}
\alpha &\equiv \langle \psi_{000}^s | q e^{i\sqrt{\frac{2}{3}} q\lambda_z} | \psi_{110}^\lambda \rangle = i\frac{q^2}{\sqrt{3}\alpha_h} e^{-q^2/6\alpha_h^2}, \\
\beta &\equiv \langle \psi_{000}^s | e^{i\sqrt{\frac{2}{3}} q\lambda_z} \hat{p}_{3-} | \psi_{111}^\lambda \rangle = -\langle \psi_{000}^s | e^{i\sqrt{\frac{2}{3}} q\lambda_z} \hat{p}_{3+} | \psi_{11-1}^\lambda \rangle \\
&= -i\sqrt{\frac{2}{3}} \alpha_h e^{-q^2/6\alpha_h^2}, \\
\gamma &\equiv \langle \psi_{000}^s | e^{i\sqrt{\frac{2}{3}} q\lambda_z} \hat{p}_{3z} | \psi_{110}^\lambda \rangle = i\frac{\alpha_h}{\sqrt{3}} \left(1 + \frac{q^2}{3\alpha_h^2}\right) e^{-q^2/6\alpha_h^2},
\end{aligned}
\right.
$$

C. Configuration Mixing

$$\begin{pmatrix} S_{11}(1535) \\ S_{11}(1650) \end{pmatrix} = \begin{pmatrix} \cos\theta_s & -\sin\theta_s \\ \sin\theta_s & \cos\theta_s \end{pmatrix} \begin{pmatrix} |[70,\ {}^2 8]\rangle \\ |[70,\ {}^4 8]\rangle \end{pmatrix}$$

With the data from PDG2008:

$$\begin{aligned} Br(S_{11}(1535) \to N\pi) &= 35 \sim 55\% \\ Br(S_{11}(1650) \to N\pi) &= 60 \sim 95\% \end{aligned}$$

$$\begin{aligned} Br(S_{11}(1535) \to N\eta) &= 45 \sim 60\% \\ Br(S_{11}(1650) \to N\eta) &= 3 \sim 10\% \end{aligned}$$

$$\Rightarrow \quad \boxed{\theta_s \approx 24.6^\circ \sim 32.1^\circ.}$$

$$\begin{aligned} \mathrm{OPE:} &\quad \theta_s = 25.5^\circ, \\ \mathrm{OGE:} &\quad \theta_s = -32^\circ. \end{aligned}$$

D. Coupling Strength of $S_{11}(1535)$ to NM

$$\frac{g_{N^*NM}}{g_{N^*p\eta}} \equiv \frac{f_\eta}{f_M}\frac{|\mathcal{M}_{N^*\to NM}|}{|\mathcal{M}_{N^*\to p\eta}|}$$

	$S_{11}^+ p\eta$	$S_{11}^+ \Lambda K^+$	$S_{11}^+ n\pi^+$	$S_{11}^+ p\pi^0$	$S_{11}^+ \Sigma^+ K^0$
$\frac{g_{S_{11}NM}}{g_{S_{11}p\eta}}$	1	0.61 ± 0.02	0.89	0.89	0.25

$$R \equiv g_{N^*K\Lambda}/g_{N^*p\eta} = 1.3 \pm 0.3$$

is given by Liu and Zou with 5q contributions. See also Zou's talk.

- **The couplings for $S_{11}\Lambda K$ and $S_{11}\Sigma K$ are not necessarily zero even though the kinematics are not allowed.**
- **The relative signs for the g_{N^*NM}/g_{NNM} vertices are determined by the quark model which is useful for extracting the helicity amplitudes.**
- **The resonance relative signs can be examined in hadronic productions such as $\pi^- p \to \eta n$.**

❑ S-channel resonance excitations in hadronic productions

The process $\pi^- p \rightarrow \eta n$ can be expressed in term of the Mandelstam variables:

$$\mathcal{M} = \mathcal{M}_s + \mathcal{M}_u + \mathcal{M}_t .$$

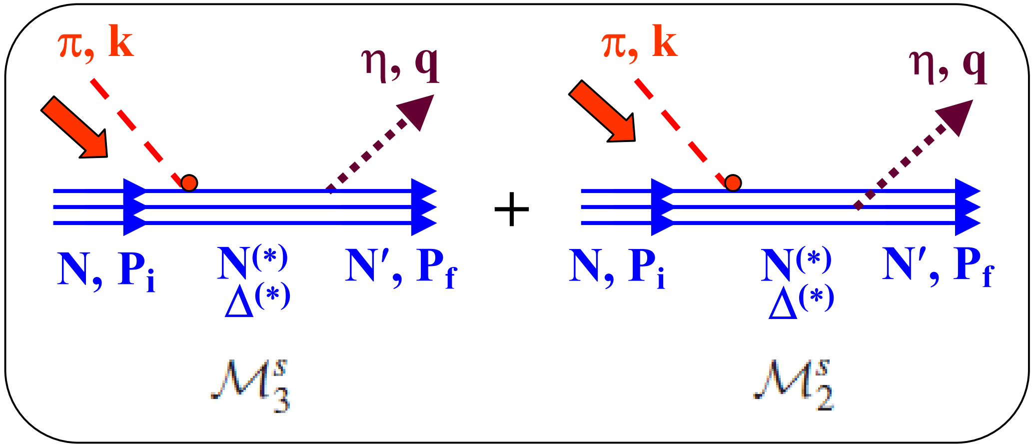
The s- and u-channel transitions are given by

$$\mathcal{M}_s = \sum_j \langle N_f | H_\eta | N_j \rangle \langle N_j | \frac{1}{E_i + \omega_\pi - E_j} H_\pi | N_i \rangle$$

$$\mathcal{M}_u = \sum_j \langle N_f | H_\pi \frac{1}{E_i - \omega_\eta - E_j} | N_j \rangle \langle N_j | H_\eta | N_i \rangle$$

Zhong, Zhao, He, and Saghai, PRC76, 065205 (2007)

s-channel



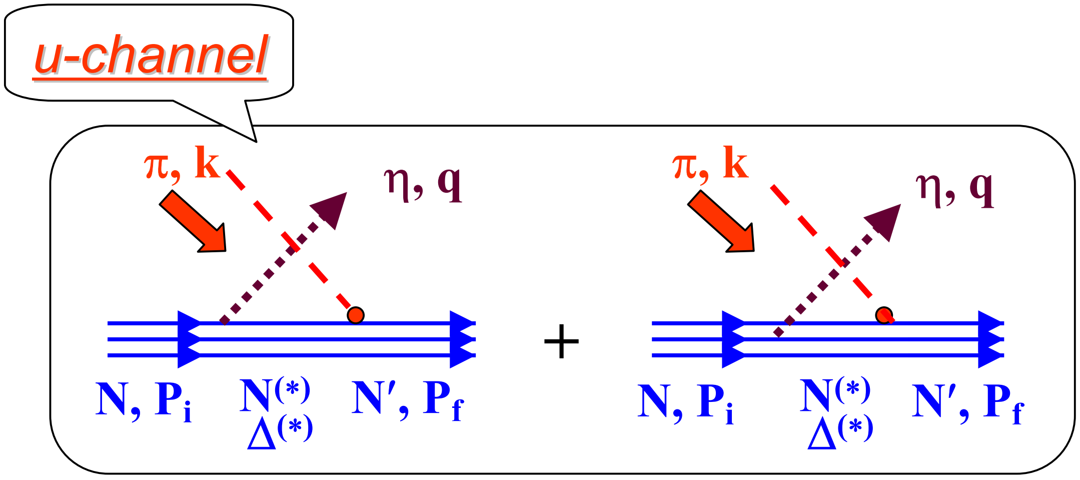
$$\mathcal{M}_s = \sum_j \langle N_f|H_\eta|N_j\rangle\langle N_j|\sum_n \frac{1}{\omega_\pi^{n+1}}(\hat{H} - E_i)^n H_\pi|N_i\rangle$$

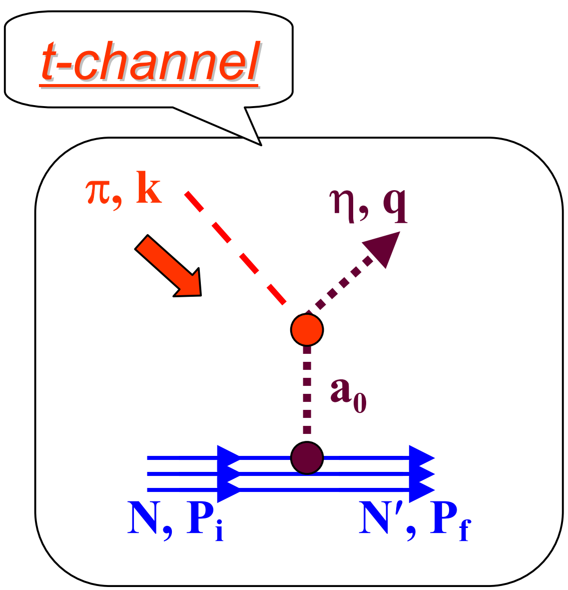
for any operator $\mathcal{O}$, one has

$$(\hat{H} - E_i)\mathcal{O}|N_i\rangle = [\hat{H}, \mathcal{O}]|N_i\rangle$$

Refs.

Zhao, Li, & Bennhold, PLB436, 42(1998); PRC58, 2393(1998);
Zhao, Didelez, Guidal, & Saghai, NPA660, 323(1999);
Zhao, PRC63, 025203(2001);
Zhao, Saghai, Al-Khalili, PLB509, 231(2001);
Zhao, Al-Khalili, & Bennhold, PRC64, 052201(R)(2001); PRC65, 032201(R) (2002);





$$\begin{cases} \mathcal{L}_{a_0\pi\eta} = g_{a_0\pi\eta} m_\pi \eta \vec{\pi} \vec{a}_0 \\ H_{a_0} = \sum_j g_{a_0 qq} m_\pi \bar{\psi}_j \psi_j \vec{a}_0 \end{cases}$$

$$\mathcal{M}_t = g_{a_0\pi\eta} m_\pi \langle N_f | H_{a_0} | N_i \rangle \frac{1}{t^2 - m_{a_0}^2}$$

◆ S-channel transition amplitude with quark level operators

Non-relativistic expansion:

$$H_\pi = \sum_j \frac{I_j}{g_A^\pi}\sigma_j \cdot \left[\mathbf{A}_\pi e^{i\mathbf{k}\cdot\mathbf{r}_j} + \frac{\omega_\pi}{2m_q}\{\mathbf{p}_j, e^{i\mathbf{k}\cdot\mathbf{r}_j}\}\right],$$

$$H_\eta = \sum_j \frac{I_j}{g_A^\eta}\sigma_j \cdot \left[\mathbf{A}_\eta e^{-i\mathbf{q}\cdot\mathbf{r}_j} + \frac{\omega_\eta}{2m_q}\{\mathbf{p}_j, e^{-i\mathbf{q}\cdot\mathbf{r}_j}\}\right],$$

with

$$\mathbf{A}_\pi = -\left(\frac{\omega_\pi}{E_i + M_i} + 1\right)\mathbf{k},$$

$$\mathbf{A}_\eta = -\left(\frac{\omega_\eta}{E_f + M_f} + 1\right)\mathbf{q}.$$

$$\mathcal{M}^s = \sum_n \left(\mathcal{M}_3^s + \mathcal{M}_2^s\right) e^{-(\mathbf{k}^2+\mathbf{q}^2)/6\alpha^2}$$

with

$$\mathcal{M}_3^s = \langle N_f | \frac{3I_3}{g_A^\pi} \left\{ \boldsymbol{\sigma}_3 \cdot \mathbf{A}_\eta \boldsymbol{\sigma}_3 \cdot \mathbf{A}_\pi \sum_{n=0} \frac{F_s(n)}{n!} \mathcal{X}^n \right.$$

$$\begin{aligned}
\mathcal{M}_2^s = \langle N_f | \frac{6I_1}{g_A^\pi} \Bigg\{ & \boldsymbol{\sigma}_1 \cdot \mathbf{A}_\eta \boldsymbol{\sigma}_3 \cdot \mathbf{A}_\pi \sum_{n=0} \frac{F_s(n)}{n!} \frac{\mathcal{X}^n}{(-2)^n} \\
& + \left[-\boldsymbol{\sigma}_1 \cdot \mathbf{A}_\eta \frac{\omega_\pi}{3m_q} \boldsymbol{\sigma}_3 \cdot \mathbf{q} - \frac{\omega_\eta}{3m_q} \boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_3 \cdot \mathbf{A}_\pi \right. \\
& \left. + \frac{\omega_\eta}{m_q} \frac{\omega_\pi}{m_q} \frac{\alpha^2}{3} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_3 \right] \sum_{n=1} \frac{F_s(n)}{(n-1)!} \frac{\mathcal{X}^{n-1}}{(-2)^n} \\
& + \frac{\omega_\eta}{3m_q} \frac{\omega_\pi}{3m_q} \boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_3 \cdot \mathbf{k} \sum_{n=2} \frac{F_s(n)}{(n-2)!} \frac{\mathcal{X}^{n-2}}{(-2)^n} \Bigg\} | N_i \rangle
\end{aligned}$$

where

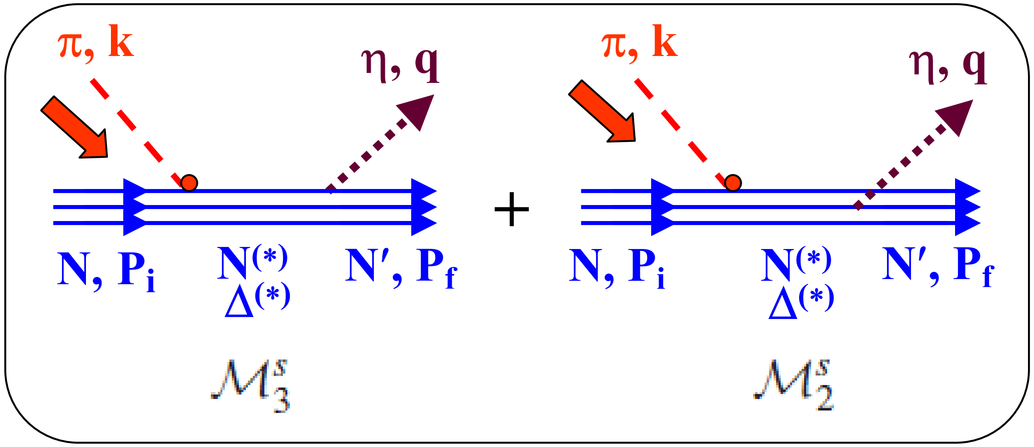
◆ quark level → hadron level

$$
\begin{aligned}
\mathcal{M}^s = \frac{1}{g_A^\pi}\Bigg\{ & \mathbf{A}_\eta \cdot \mathbf{A}_\pi \sum_{n=0} [g_{s1} + (-2)^{-n} g_{s2}] \frac{F_s(n)}{n!} \mathcal{X}^n \\
& + \left(-\frac{\omega_\pi}{3m_q} \mathbf{A}_\eta \cdot \mathbf{q} - \frac{\omega_\eta}{3m_q} \mathbf{A}_\pi \cdot \mathbf{k} + \frac{\omega_\eta}{m_q} \frac{\omega_\pi}{m_q} \frac{\alpha^2}{3} \right) \\
& \times \sum_{n=1} [g_{s1} + (-2)^{-n} g_{s2}] \frac{F_s(n)}{(n-1)!} \mathcal{X}^{n-1} \\
& + \frac{\omega_\eta \omega_\pi}{(3m_q)^2} \mathbf{k} \cdot \mathbf{q} \sum_{n=2} \frac{F_s(n)}{(n-2)!} [g_{s1} + (-2 \\
& + i\boldsymbol{\sigma} \cdot (\mathbf{A}_\eta \times \mathbf{A}_\pi) \sum_{n=0} [g_{v1} + (-2)^{-n} g_{v2}] \frac{F_s(n)}{\cdot} \mathcal{X}^n \\
& + \frac{\omega_\eta \omega_\pi}{(3m_q)^2} i\boldsymbol{\sigma} \cdot (\mathbf{q} \times \mathbf{k}) \\
& \times \sum_{n=2} [g_{v1} + (-2)^{-n} g_{v2}] \frac{F_s(n)}{(n-2)!} \mathcal{X}^{n-2} \Bigg\} e^{-(\mathrm{k}^2+\mathrm{q}^2)/6\alpha^2}
\end{aligned}
$$

$$F_s(n) = \frac{M_n}{P_i \cdot k - n M_n \omega_h}$$

$$\to F_s(R) = \frac{2M_R}{s - M_R^2 + i M_R \Gamma_R}$$

s-channel



- Compared with M^s_3, amplitude M^s_2 is relatively suppressed by a factor of $(-1/2)^n$ for each n.
- Higher excited states are relatively suppressed by $(k\cdot q/3\alpha^2)^n/n!$
- One can identify the quark motion correlations between the initial and final state baryon
- Similar treatment can be done for the u channel

◆ Separate out individual resonances

A. $n = 0$ shell resonances

For $n = 0$, only the nucleon pole term contributes to the transition amplitude. Its s-channel amplitude is

$$\mathcal{M}_N^s = \mathcal{O}_N \frac{2M_0}{s - M_0^2} e^{-(\mathbf{k}^2+\mathbf{q}^2)/6\alpha^2},$$

with

$$\mathcal{O}_N = [g_{s1} + g_{s2}]\,\mathbf{A}_\eta \cdot \mathbf{A}_\pi + [g_{v1} + g_{v2}]\, i\boldsymbol{\sigma} \cdot (\mathbf{A}_\eta \times \mathbf{A}_\pi),$$

where M_0 is the nucleon mass.

B. $n = 1$ shell resonances

For $n = 1$, only S and D waves contribute in the s channel. Note that the spin-independent amplitude for D waves is proportional to the Legendre function $P_2^0(\cos\theta)$ and the spin-dependent amplitude for D waves is in proportion to $\frac{\partial}{\partial\theta}P_2^0(\cos\theta)$. Moreover, the S-wave amplitude is independent of the scattering angle.

$$\mathcal{M}^s(S) = \mathcal{O}_S F_s(R) e^{-(\mathbf{k}^2+\mathbf{q}^2)/6\alpha^2},$$
$$\mathcal{M}^s(D) = \mathcal{O}_D F_s(R) e^{-(\mathbf{k}^2+\mathbf{q}^2)/6\alpha^2},$$

with

$$\mathcal{O}_S = \left(g_{s1} - \frac{1}{2}g_{s2}\right)\left(|\mathbf{A}_\eta||\mathbf{A}_\pi|\frac{|\mathbf{k}||\mathbf{q}|}{9\alpha^2} - \frac{\omega_\pi}{3m_q}\mathbf{A}'_\eta \cdot \mathbf{q} - \frac{\omega_\eta}{3m_q}\mathbf{A}_\pi \cdot \mathbf{k} + \frac{\omega_\eta}{m_q}\frac{\omega_\pi}{m_q}\frac{\alpha^2}{3}\right),$$

$$\mathcal{O}_D = \left(g_{s1} - \frac{1}{2}g_{s2}\right)|\mathbf{A}_\eta||\mathbf{A}_\pi|(3\cos^2\theta - 1)\frac{|\mathbf{k}||\mathbf{q}|}{9\alpha^2} + \left(g_{v1} - \frac{1}{2}g_{v2}\right) i\boldsymbol{\sigma}\cdot(\mathbf{A}_\eta \times \mathbf{A}_\pi)\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}.$$

$$\mathcal{M}^s(S) = [g_{S_{11}(1535)} + g_{S_{11}(1650)}]\mathcal{M}^s(S),$$
$$\mathcal{M}^s(D) = [g_{D_{13}(1520)} + g_{D_{13}(1700)} + g_{D_{15}(1675)}]\mathcal{M}^s(D)$$

In the SU(6) symmetry limit,

Factor	Value	Factor	Value	Factor	Value
g_{s1}	1	$g_{S_{11}(1535)}$	2	g_2	5/3
g_{s2}	2/3	$g_{S_{11}(1650)}$	-1	$g_{P_{11}(1710)}$	180/619
g_{v1}	5/3	$g_{D_{13}(1520)}$	2	$g_{P_{13}(1900)}$	18/619
g_{v2}	0	$g_{D_{13}(1700)}$	$-1/10$	$g_{P_{11}(2100)}$	$-16/619$
g_A^π	5/3	$g_{D_{15}(1675)}$	$-9/10$	$g_{F_{15}(1680)}$	5/3
g_A^η	1	$g_{P_{11}(1440)}$	225/619	$g_{F_{15}(2000)}$	$-2/21$
g_1	1	$g_{P_{13}(1720)}$	180/619	$g_{F_{17}(1990)}$	$-4/7$

Features of this approach

Advantage:

i) A complete set of NRCQM resonances is included with very few parameters as leading contributions.
ii) The same parameters for the production of SU(3) multiplets.
iii) The same framework for meson photoproduction turns out to be successful.

Disadvantage: Neither covariant nor unitary.

◆ Model parameters

Goldberger-Treiman relation:

$$g_{mNN} = \frac{g_A^m M_N}{f_m}$$

$$g_{\pi NN} = 13.48,$$

$$g_{\eta NN} = 0.81$$

$$g_{a_0 NN} g_{a_0 \pi \eta} = 100$$

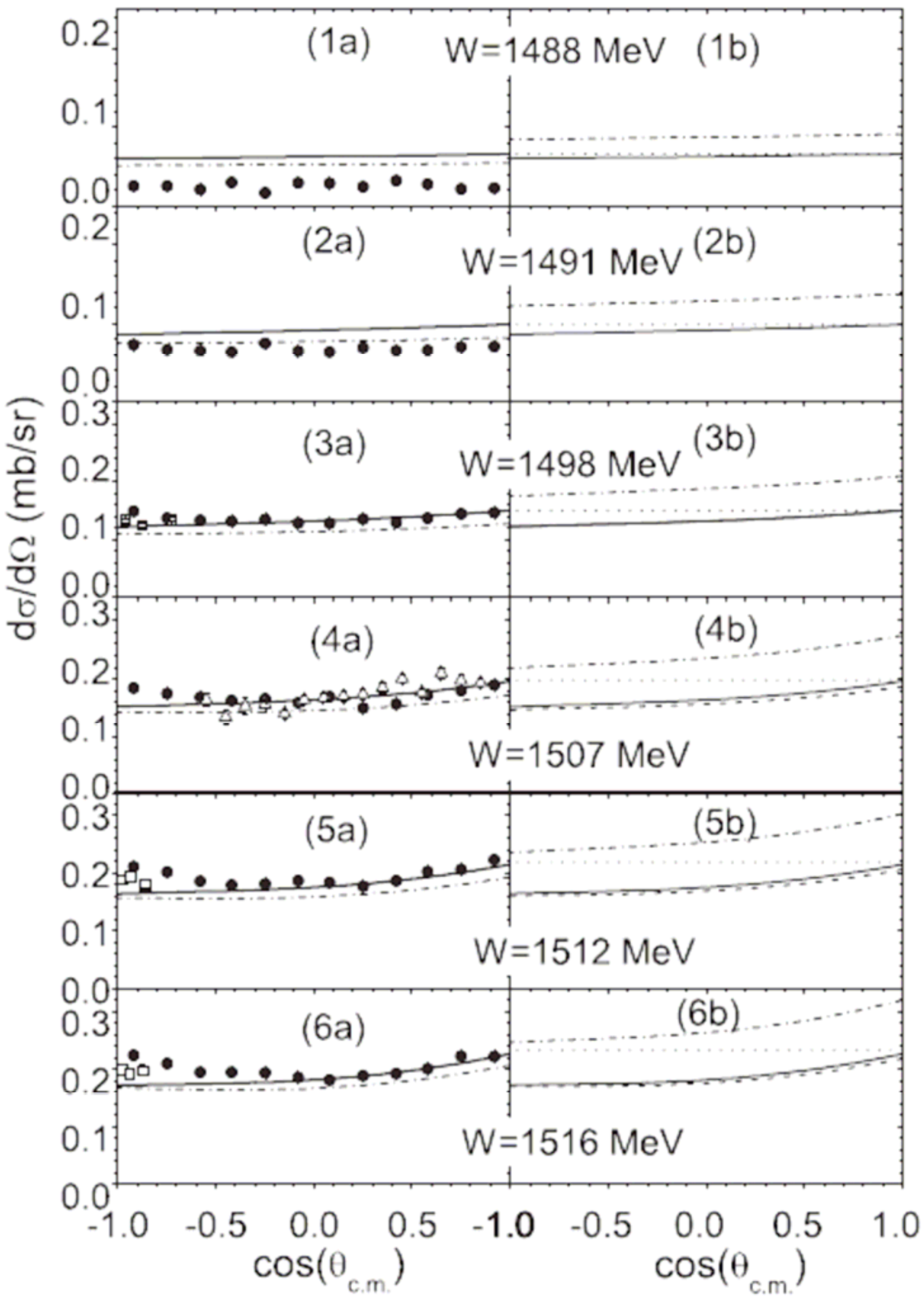
$$m_q = 330 \text{ MeV},$$

$$\alpha^2 = 0.16 \text{ GeV}^2.$$

TABLE II. Breit-Wigner masses M_R (in MeV) and widths Γ_R (in MeV) for the resonances. $n = 1$ and $n = 2$ stand for the degenerate states with quantum number $n = 1$ and $n = 2$ in the u channel.

Resonance	M_R	Γ_R	Resonance	M_R	Γ_R
$S_{11}(1535)$	1535	150	$P_{11}(1440)$	1440	300
$S_{11}(1650)$	1655	165	$P_{11}(1710)$	1710	100
$D_{13}(1520)$	1520	115	$P_{13}(1720)$	1720	200
$D_{13}(1700)$	1700	115	$P_{13}(1900)$	1900	500
$D_{15}(1675)$	1675	150	$P_{11}(2100)$	2100	150
$n = 1$	1650	230	$F_{15}(1680)$	1685	130
$n = 2$	1750	300	$F_{15}(2000)$	2000	200
–	–	–	$F_{17}(1990)$	1990	350

Differential cross sections

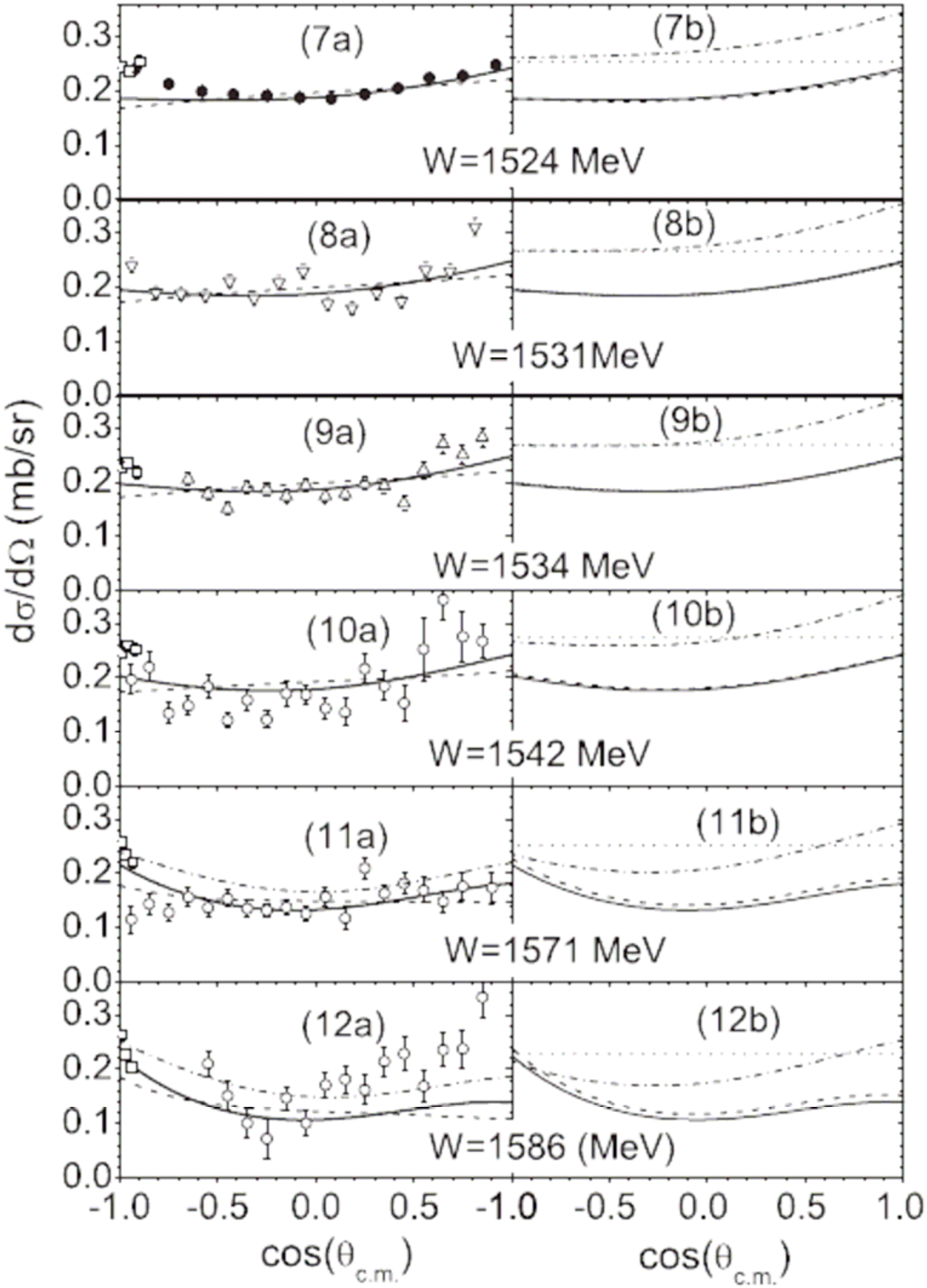


Left panel:

- Solid: full calculation
- Dot-dashed: without nucleon Born term

Right panel:

- Solid: full calculation
- Dotted lines: exclusive S11(1535)
- Dot-dashed: without S11(1650)
- Dashed: without t-channel



Left panel:

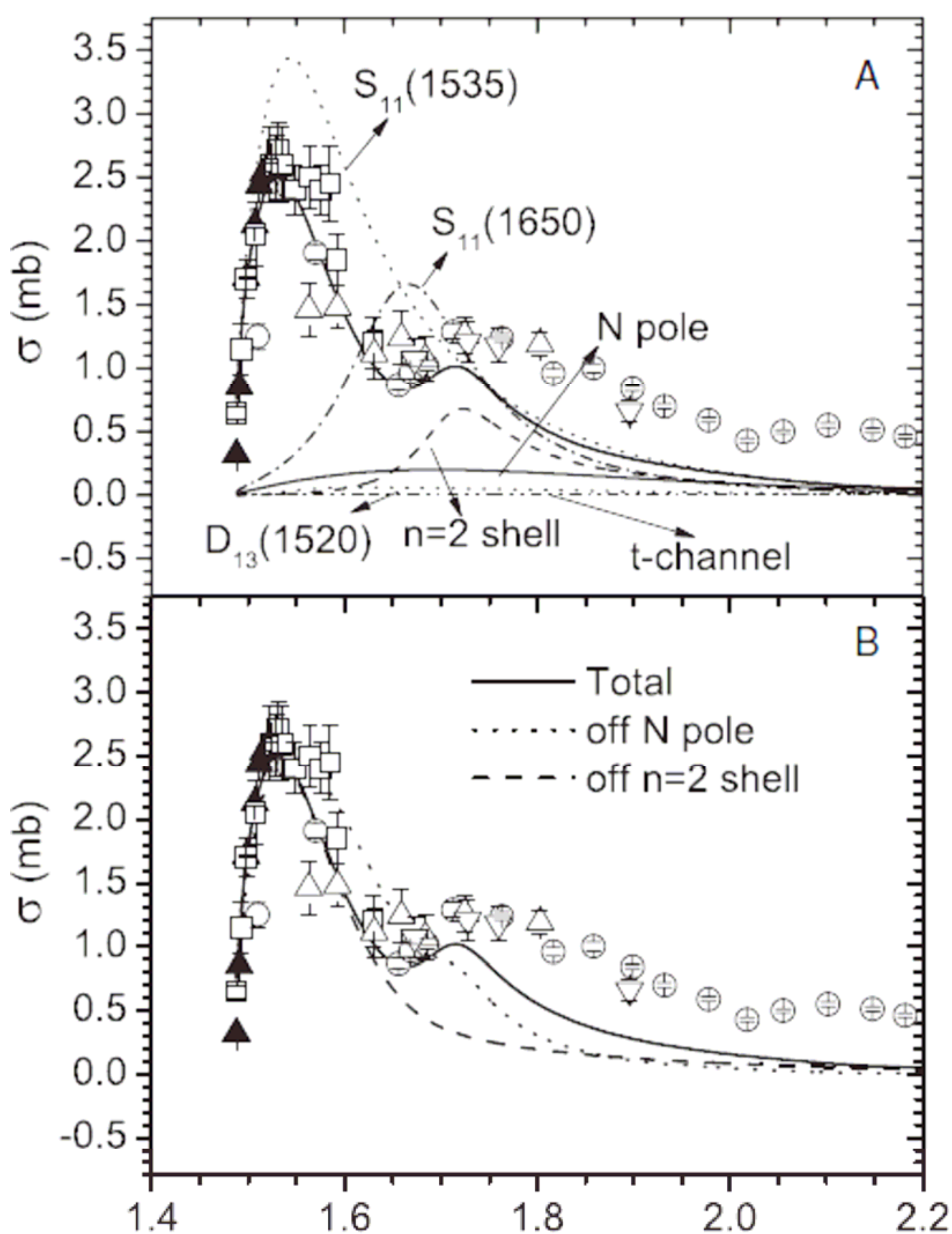
- Solid: full calculation
- Dot-dashed: without nucleon Born term
- Dashed: without D13(1520)

Right panel:

- Solid: full calculation
- Dotted lines: exclusive S11(1535)
- Dot-dashed: without S11(1650)
- Dashed: without t-channel

Total cross sections

- S11(1535) is dominant near threshold. The exclusive cross section is even larger than the data.
- S11(1650) has a destructive interference with the S11(1535), and appears to be a dip in the total cross section.
- States from n=2 shell account for the second enhancement around 1.7 GeV.



Zhong, Zhao, He, and Saghai, PRC76, 065205 (2007)

❑ S-channel resonance excitations in $K^-p \to \Sigma^0 \pi^0$

$$\mathcal{O}_S = [g_{S_{01}(1405)} + g_{S_{01}(1670)}]\mathcal{O}_S,$$
$$\mathcal{O}_D = [g_{D_{03}(1520)} + g_{D_{03}(1690)}]\mathcal{O}_D,$$

$$\frac{g_{S_{01}(1405)}}{g_{S_{01}(1670)}} = \frac{\langle N_f | I_3^\pi \sigma_3 | S_{01}(1405)\rangle \langle S_{01}(1405) | I_3^K \sigma_3 | N_i\rangle}{\langle N_f | I^\pi \sigma_3 | S_{01}(1670)\rangle \langle S_{01}(1670) | I_3^K \sigma_3 | N_i\rangle}$$

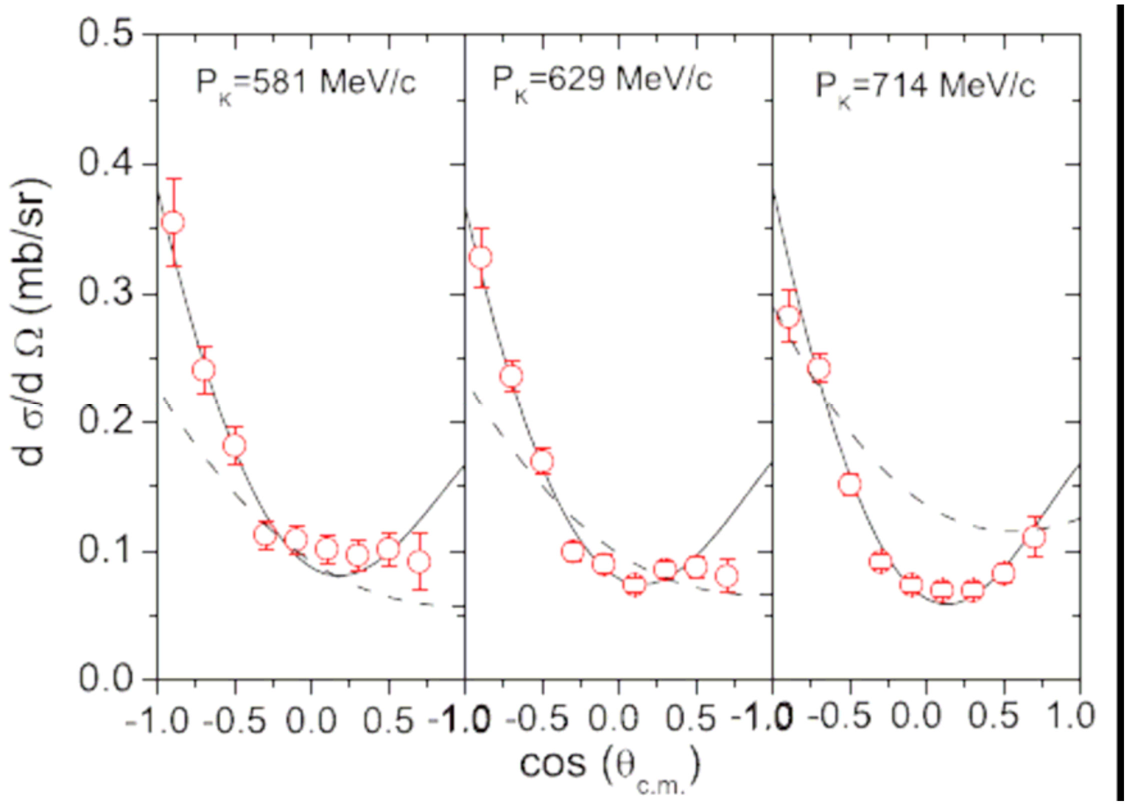
$$|S_{01}(1405)\rangle = \cos(\theta)|\mathbf{70},{}^2\mathbf{1}\rangle - \sin(\theta)|\mathbf{70},{}^2\mathbf{8}\rangle$$
$$|S_{01}(1670)\rangle = \sin(\theta)|\mathbf{70},{}^2\mathbf{1}\rangle + \cos(\theta)|\mathbf{70},{}^2\mathbf{8}\rangle$$

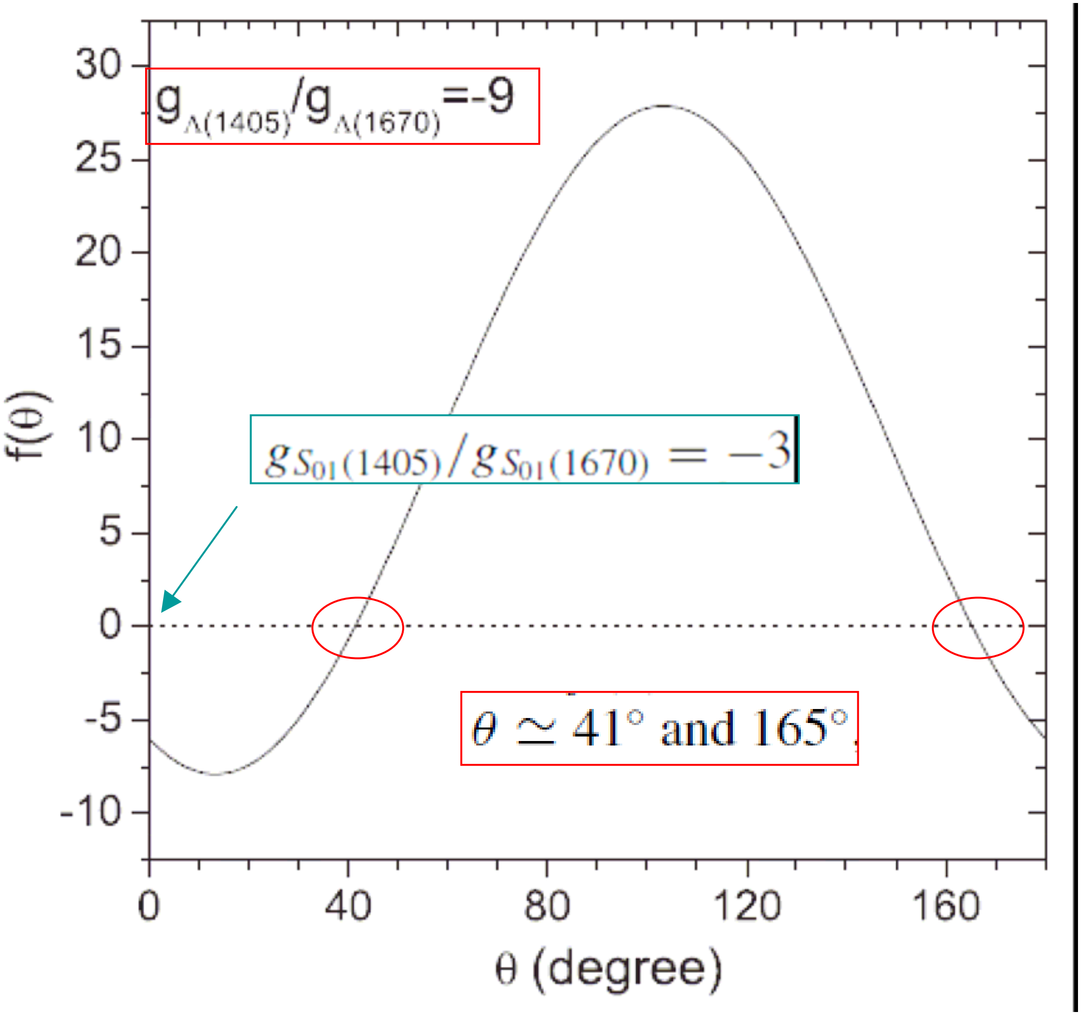
$$\frac{g_{S_{01}(1405)}}{g_{S_{01}(1670)}} = \frac{[3\cos(\theta) - \sin(\theta)][\cos(\theta) + \sin(\theta)]}{[3\sin(\theta) + \cos(\theta)][\sin(\theta) - \cos(\theta)]}$$

$g_{S_{01}(1405)}/g_{S_{01}(1670)} = -3$ leads to $\theta = 0^\circ$, i.e., no configuration mixing between $[\mathbf{70},{}^2\mathbf{1}]$ and $[\mathbf{70},{}^2\mathbf{8}]$.

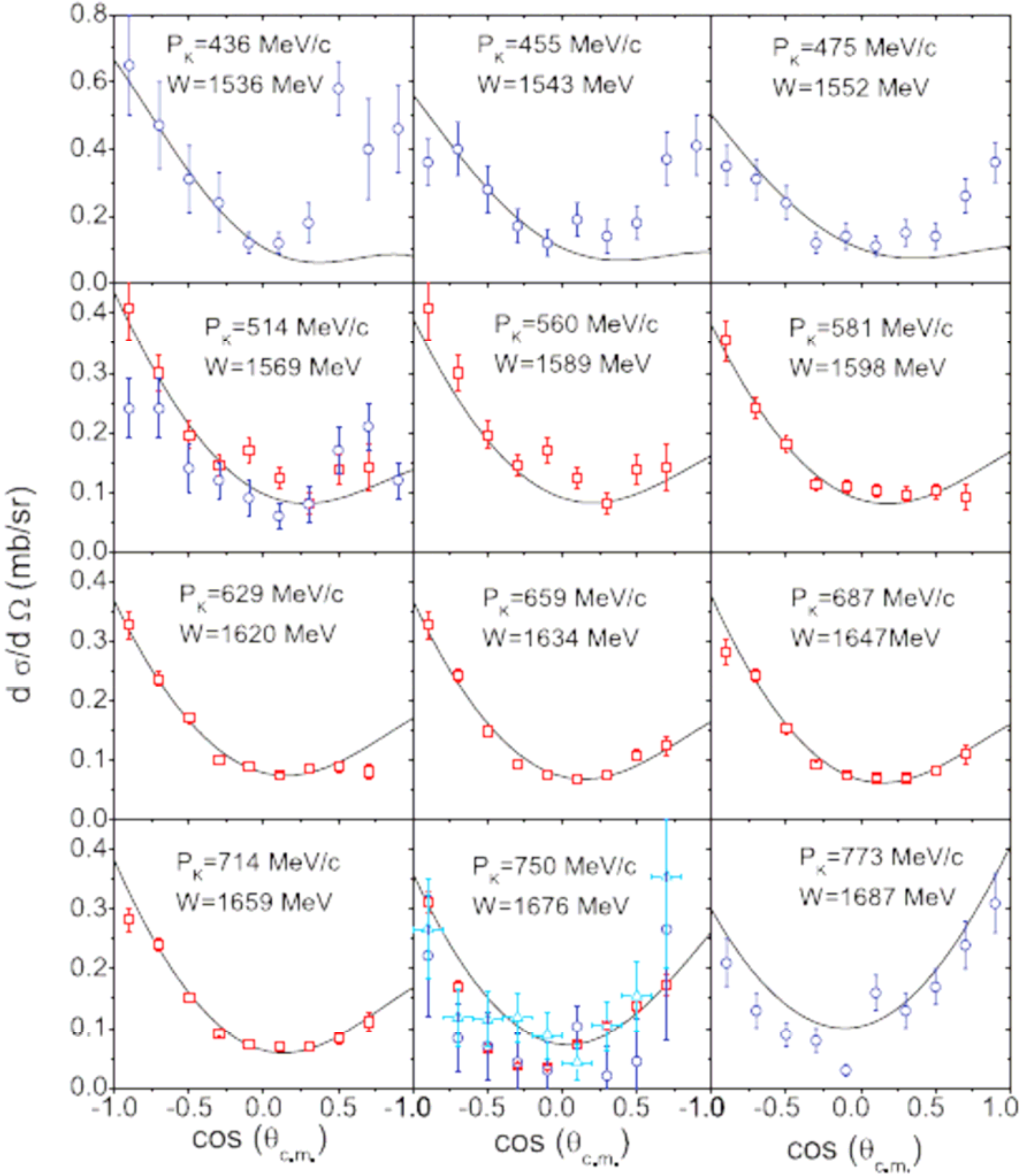
Ref. Zhong and Zhao, PRC79, 045202 (2009)

We thus determine the mixing angle by experimental data which requires $g_{S_{01}(1405)}/g_{S_{01}(1670)} \simeq -9$



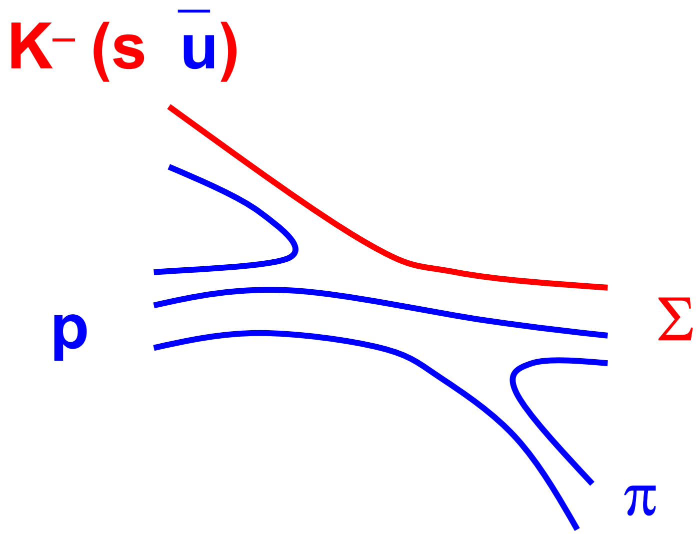


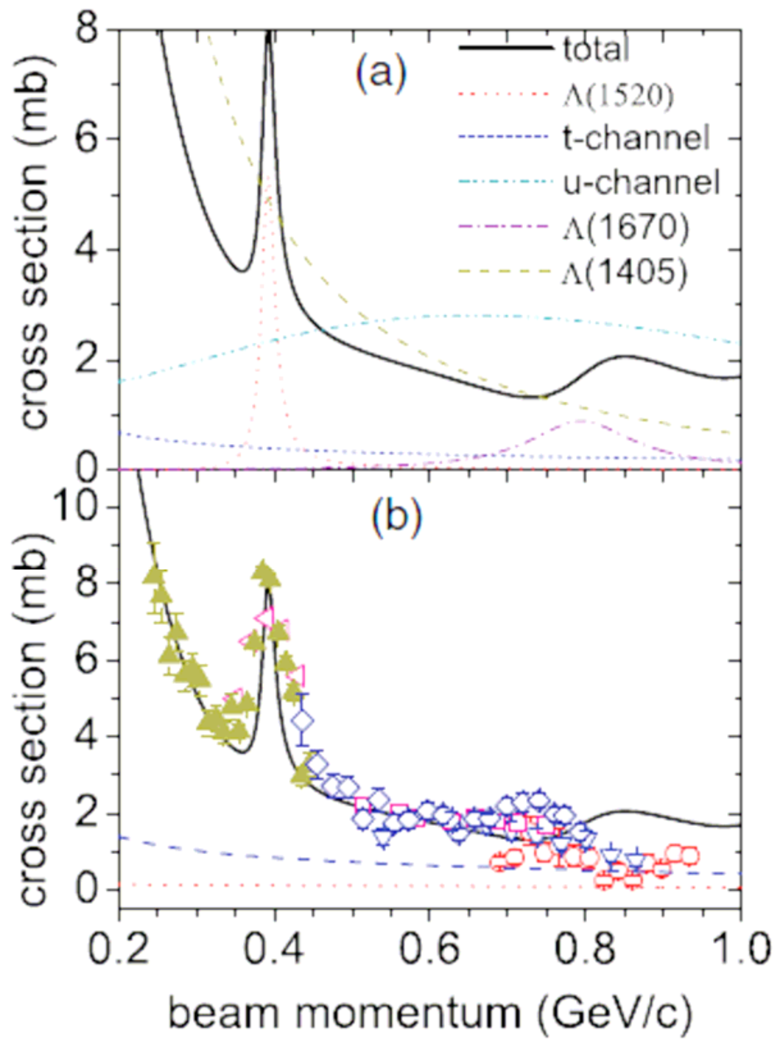
30
25
20
15
10
5
0
-5
-10
f(θ)
0
40
80
120
160
θ (degree)
$g_{\Lambda(1405)}/g_{\Lambda(1670)}$=-9
$g_{S_{01}(1405)}/g_{S_{01}(1670)} = -3$
$\theta \simeq 41^\circ$ and 165°,



P_K=436 MeV/c
W=1536 MeV
P_K=455 MeV/c
W=1543 MeV
P_K=475 MeV/c
W=1552 MeV
P_K=514 MeV/c
W=1569 MeV
P_K=560 MeV/c
W=1589 MeV
P_K=581 MeV/c
W=1598 MeV
P_K=629 MeV/c
W=1620 MeV
P_K=659 MeV/c
W=1634 MeV
P_K=687 MeV/c
W=1647MeV
P_K=714 MeV/c
W=1659 MeV
P_K=750 MeV/c
W=1676 MeV
P_K=773 MeV/c
W=1687 MeV
d σ/d Ω (mb/sr)
cos (θ_c.m.)

s-channel



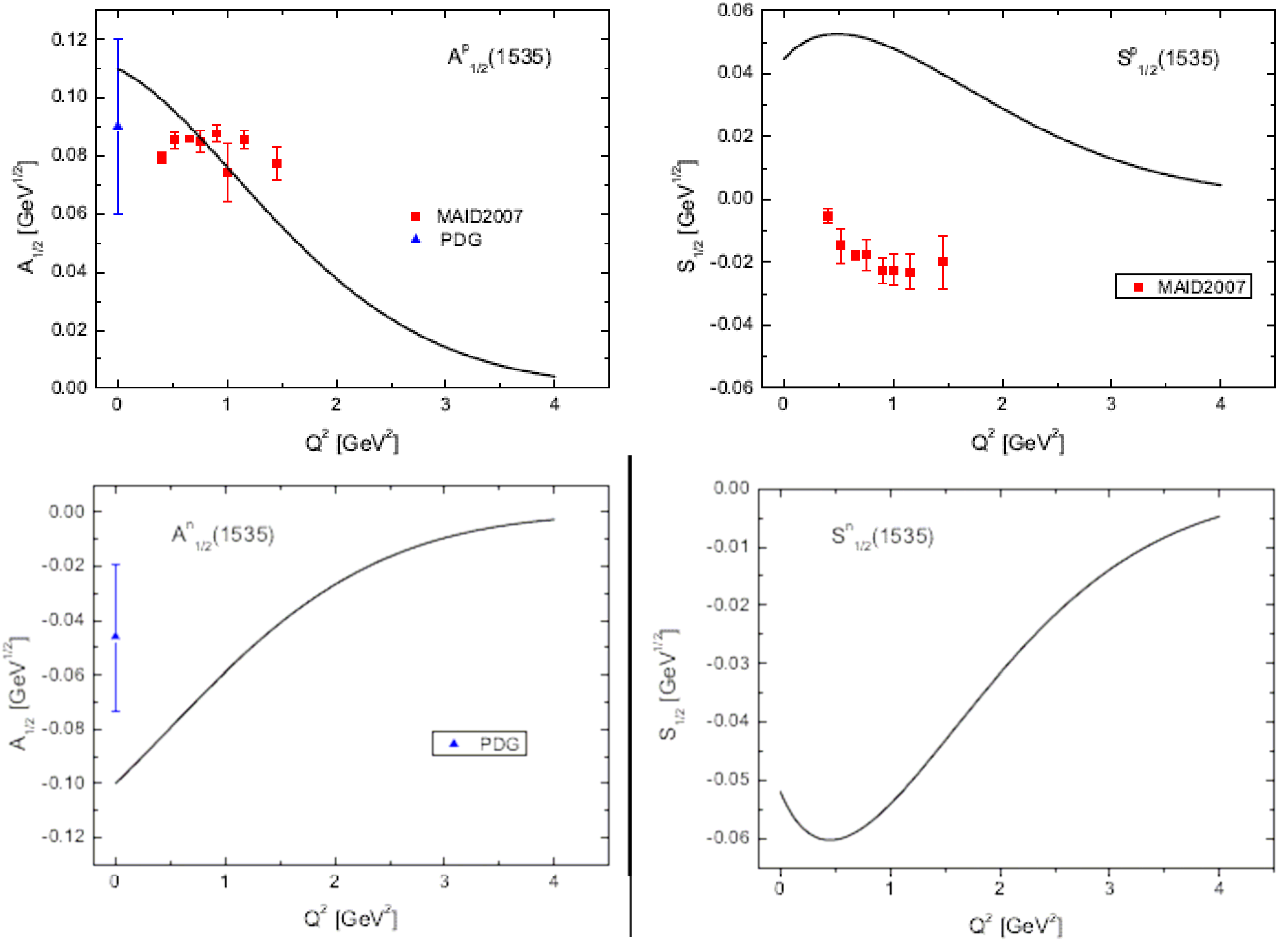


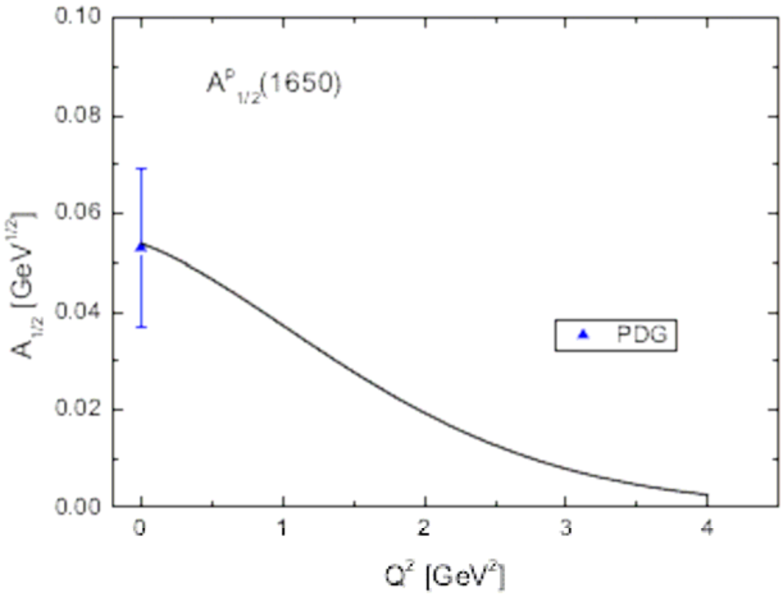
$\mathcal{M}_3^s$ =0

$\mathcal{M}_2^s$ **is the only s-channel amplitude**

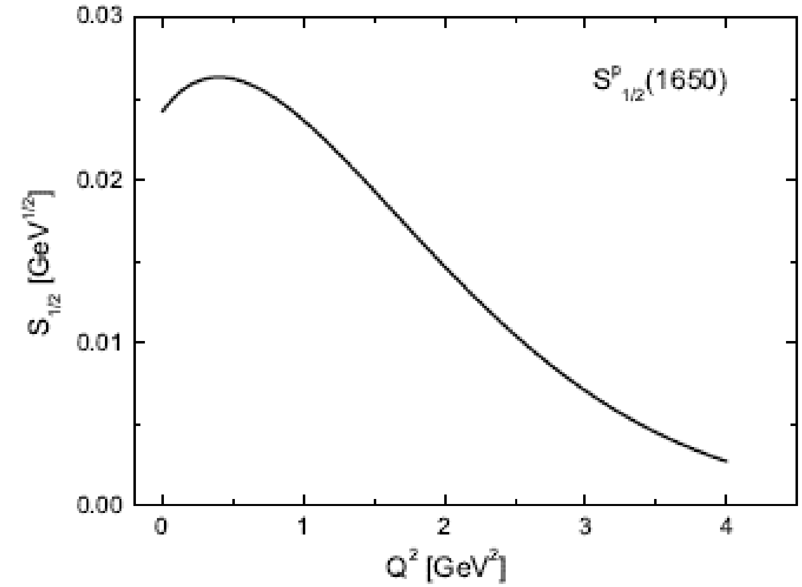
U-channel turns to be important

❑ EM helicity amplitudes after mixing

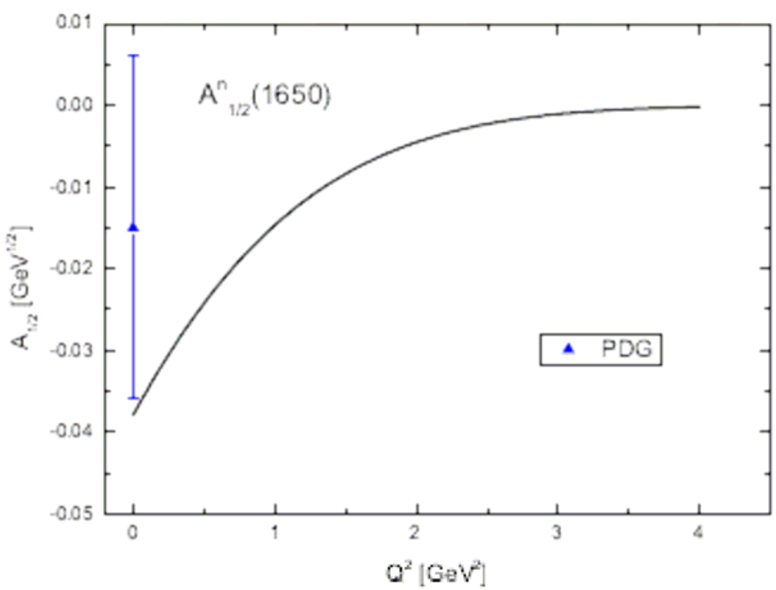




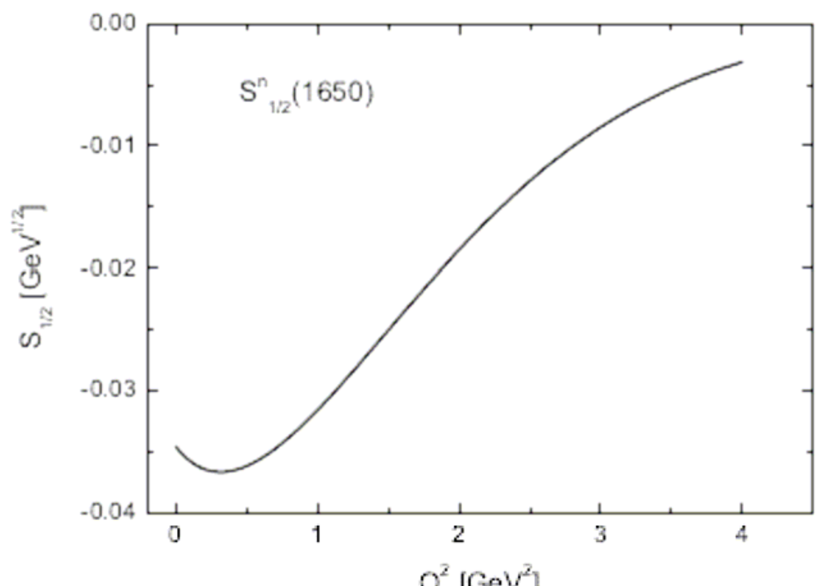
$A^p_{1/2}(1650)$
PDG
$A_{1/2}$ [$GeV^{1/2}$]
Q^2 [GeV^2]



$S^p_{1/2}(1650)$
$S_{1/2}$ [$GeV^{1/2}$]
Q^2 [GeV^2]



$A^n_{1/2}(1650)$
PDG
$A_{1/2}$ [$GeV^{1/2}$]
Q^2 [GeV^2]



$S^n_{1/2}(1650)$
$S_{1/2}$ [$GeV^{1/2}$]
Q^2 [GeV^2]

3. Summary

The mixing between the quark model representations can explain the large S11 couplings to ηN, πN, $K\Lambda$. But not as large as that proposed for 5-quark scenario by Zou and Riska.

The S11(1535) and S11(1650) appear to have destructive interferences in photo and hadronic productions of which the relative sign can be given by the quark model. This seems to be consistent with the coupled-channel results by Shklyar et al.

In the real photon limit, the transverse helicity amplitudes seem to be consistent with the mixings determined in hadronic decays. But the magnitude at large Q^2 region is much lower than the data just as been found in many other studies.

The longitudinal one has a reversed sign which is impossible to be explained by the leading EM operator in 3q framework and/or the mixing due to Moorhouse selection rule.

→ Some mechanisms seem indeed to have been missed by the NRCQM

Thanks for your attention !