

# JUSTUS-LIEBIG-



# UNIVERSITÄT GIESSEN

## Giessen coupled-channel PWA

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## What we want to learn ?

- examine the nucleon structure
  - nucleon excitation spectra
- scattering process  $\pi N, \gamma N$ 
  - meson-baryon interaction
  - phenomenological Lagrangians
  - reaction mechanisms
- Nucleon resonance properties
  - link to quark model calculations

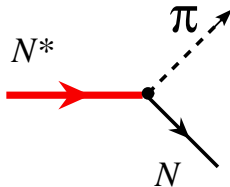
## Scattering process - how to describe ?

### Scattering $T$ -matrix:

- phenomenological models  
+ dispersion analysis: GW(SAID), MAINZ, CMB(Zagreb), KHA
- effective Lagrangian approach: Giessen, Jülich, Valencia, EBAC, Bonn-Gatchina PWA

## Effective Lagrangian approach

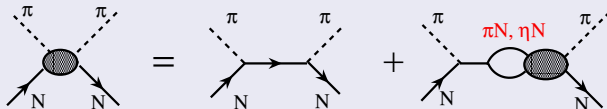
- physical degrees of freedom are mesons and baryons but not quarks and gluons
- maintains symmetries of underlying fundamental theory (QCD)
- use methods of Quantum Field Theory
- $N^*$  properties are defined from interaction Lagrangian



- difficulties: divergences, formfactors

take rescattering in the  $\pi N$  and  $\eta N$  channels into account

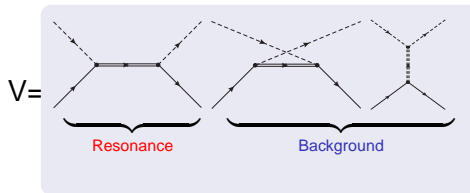
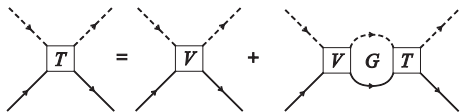
The interaction potentials  $V_{\pi N \rightarrow \eta N}$  and  $V_{\pi N \rightarrow \pi N}$  enter to



Coupled-channel problem for  $\pi N \rightarrow \pi N$  scattering:

$$\begin{aligned}
 T_{\pi N \rightarrow \pi N} &= V_{\pi N \rightarrow \pi N} + \int d^4 p V_{\pi N \rightarrow \pi N} G_{\pi N}(p) T_{\pi N \rightarrow \pi N} \\
 &\quad + \int d^4 p V_{\pi N \rightarrow \eta N} G_{\eta N}(p) T_{\eta N \rightarrow \pi N} \\
 T_{\eta N \rightarrow \pi N} &= V_{\eta N \rightarrow \pi N} + \int d^4 p V_{\eta N \rightarrow \pi N} G_{\pi N}(p) T_{\pi N \rightarrow \pi N} \\
 &\quad + \int d^4 p V_{\eta N \rightarrow \eta N} G_{\eta N}(p) T_{\eta N \rightarrow \pi N}
 \end{aligned}$$

Bethe-Salpeter in  $K$ -matrix:  $\Gamma_{N^*}$  is dynamically generated



Imaginary part of the self energy  $\Rightarrow$  resonance width.



$$T = \begin{pmatrix} T_{\gamma\gamma} & T_{\gamma\pi} & T_{\gamma\eta} & T_{\gamma\omega} & \dots \\ T_{\pi\gamma} & T_{\pi\pi} & T_{\pi\eta} & T_{\pi\omega} & \dots \\ T_{\eta\gamma} & T_{\eta\pi} & T_{\eta\eta} & T_{\eta\omega} & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

Unitarity – Which data should be included into the analysis?

- |                                 |                              |
|---------------------------------|------------------------------|
| $\gamma N \rightarrow \gamma N$ | $\pi N \rightarrow \pi N$    |
| $\gamma N \rightarrow \pi N$    | $\pi N \rightarrow 2\pi N$   |
| $\gamma N \rightarrow \eta N$   | $\pi N \rightarrow \eta N$   |
| $\gamma N \rightarrow \omega N$ | $\pi N \rightarrow \omega N$ |
| $\gamma N \rightarrow K\Lambda$ | $\pi N \rightarrow K\Lambda$ |
| $\gamma N \rightarrow K\Sigma$  | $\pi N \rightarrow K\Sigma$  |

To solve Bethe-Salpeter equation take the imaginary part of the propagator:

$$\int dq \frac{1}{q^2 - m^2 \pm i\epsilon} = P \int dq \frac{1}{q^2 - m^2} \mp i\pi \int dq \delta(q^2 - m^2)$$

The Bethe-Salpeter equation becomes:

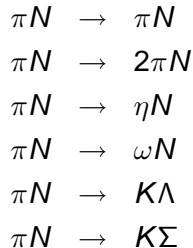
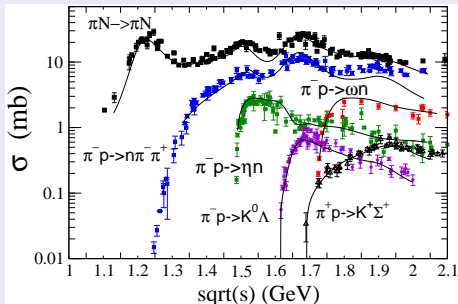
$$T_{\pi N, \pi N}(p' q'; p q) = V_{\pi N, \pi N}(p' q'; p q) - i \frac{|\mathbf{k}|}{8(2\pi)^2 \sqrt{s}} \int d\Omega_k T_{\pi N, \pi N}(p' q'; k) (k + m_N) V_{\pi N, \pi N}(k; p q)$$

where all intermediate particles are **on-shell**.

**Calculations done in Minkowsky space**

# Results for pion-induced reactions

## Giessen Model



The data are linked: **unitarity**  $\rightarrow$  **Optical theorem**

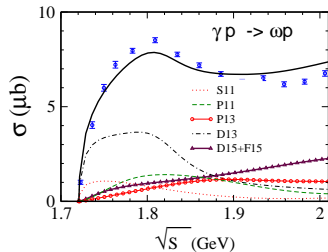
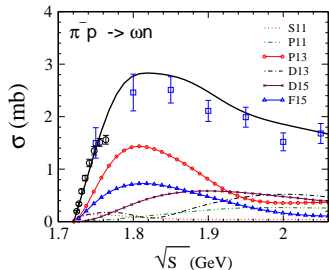
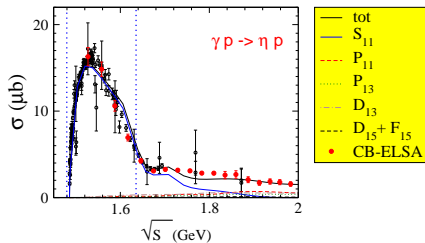
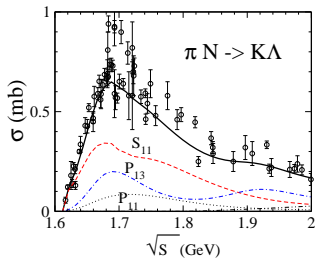
$$\begin{aligned} \text{Im}T_{\pi N \rightarrow \pi N}(0) = \frac{k^2}{4\pi} &(\sigma_{\pi N \rightarrow \pi N} + \sigma_{\pi N \rightarrow 2\pi N} + \sigma_{\pi N \rightarrow \eta N} \\ &+ \sigma_{\pi N \rightarrow \omega N} + \sigma_{\pi N \rightarrow K\Lambda} + \sigma_{\pi N \rightarrow K\Sigma}) \end{aligned}$$

$\rightarrow$  **need for multichannel analysis.**



# Giessen model. $\eta N$ and $\omega N$ final states

Giessen Model PRC72 015210 (2005), PRC71,055206(2005).



$$(\gamma/\pi)N \rightarrow \eta N$$

Strong  $S_{11}(1535)$  contribution in 1.48...1.6 GeV

Which mechanism above 1.6 GeV.?

Fix  $N^*$  properties, e.g.  $S_{11}(1535)$

- branching ratio  $\frac{\Gamma(\eta N)}{\Gamma_{tot}}$  and the full width
- $S_{11}(1535)$  from  $\gamma N \rightarrow \pi N$ :  $A_n^{\frac{1}{2}}/A_p^{\frac{1}{2}} \approx -0.5$
- $S_{11}(1535)$  from  $\gamma N \rightarrow \eta N$ :  $A_n^{\frac{1}{2}}/A_p^{\frac{1}{2}} \approx -0.84$   
(Krusche et al)

Look for 'hidden'/narrow  $N^*$  resonances:

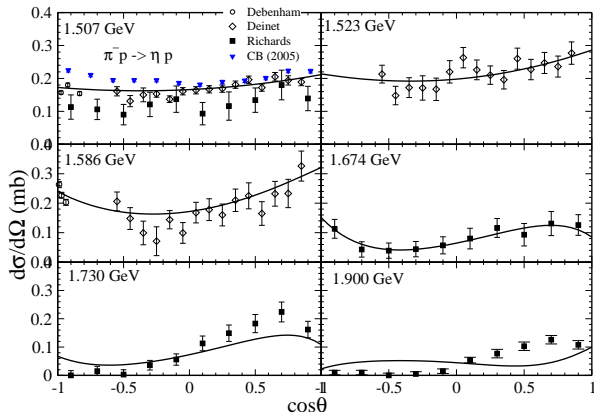
observed resonance-like structure in  $\gamma n \rightarrow \eta n$

(V. Kuznetsov; Krusche et al)

Polyakov, Diakonov, Petrov, Arndt, Strakovsky et al

- hidden narrow state or conventional mechanism

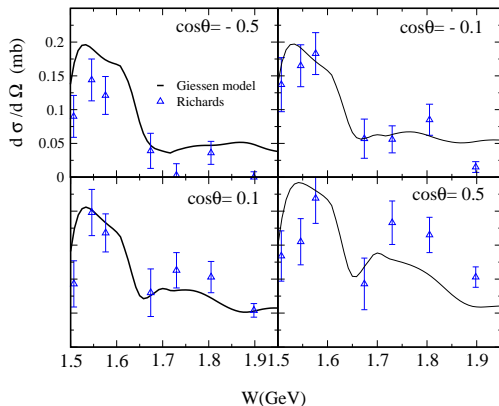
# Results for the $\pi^- p \rightarrow \eta n$ production



$\pi^- p \rightarrow \eta n$ : Solution from the Giessen coupled-channel analysis V.Shklyar et al, PRC.71. 055206 (2005).

# Results for the $\pi^- p \rightarrow \eta n$ production

Old data: W.B. Richards et al., PRD1 10 (1970).

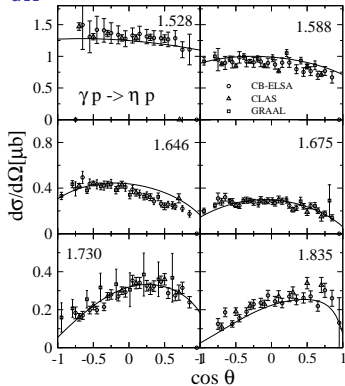


Old  $\pi^- p \rightarrow \eta n$

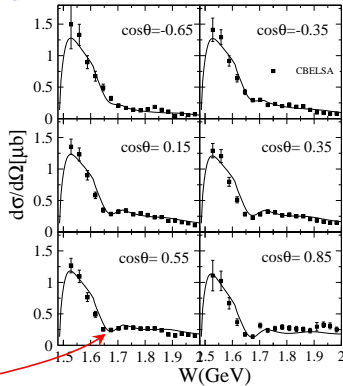
data reveals some structure above 1.7 GeV and forward angles.

# Results for the $\gamma p \rightarrow \eta p$

$\frac{d\sigma}{d\Omega}$  as a function of  $\cos(\theta)$

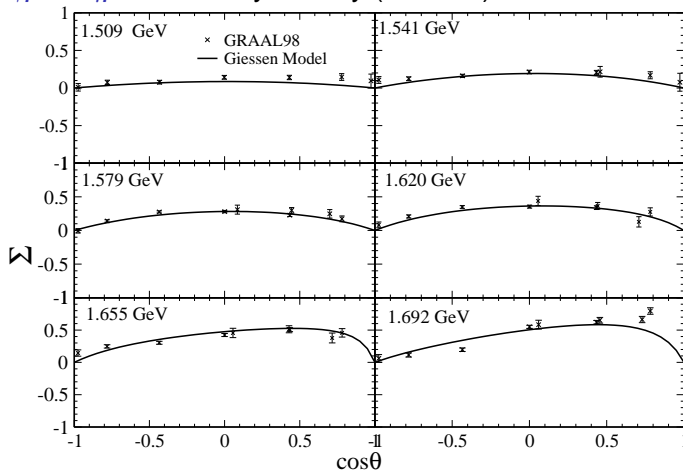


$\frac{d\sigma}{d\Omega}$  as a function of  $W$

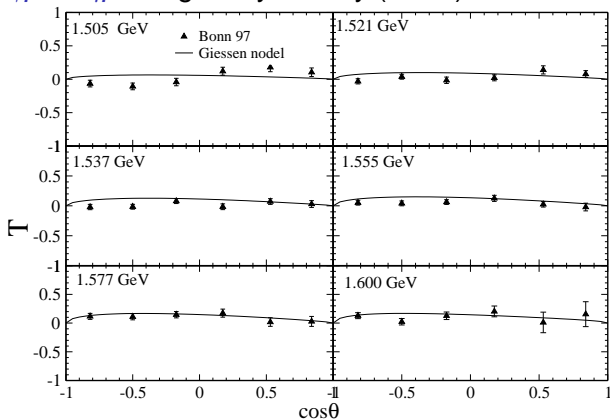


The structure at 1.67 GeV in  $\gamma p \rightarrow \eta p$  is due to  $S_{11}(1650)$   
 no need for any exotic state!

## $\gamma p \rightarrow \eta p$ : Beam asymmetry (GRAAL) - Giessen calculations



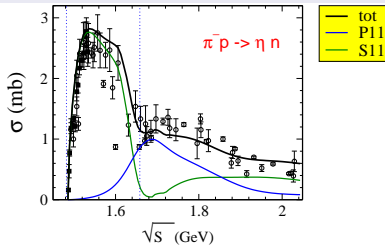
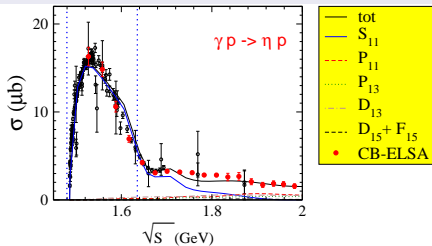
$\gamma p \rightarrow \eta p$ : Target asymmetry (Bonn) vs Giessen results



Disagreement at low energies.

Same situation in other models (MAID etc).

$S_{11}(1535)$  dominates  
 both  $\gamma p \rightarrow \eta p$  and  $\pi^- p \rightarrow \eta n$  reactions



- strong  $S_{11}(1535)$  excitation
- seems no room for other contributions

- destructive effect from  $S_{11}(1650)$
- above 1.6 GeV -  $P_{11}(1710)$



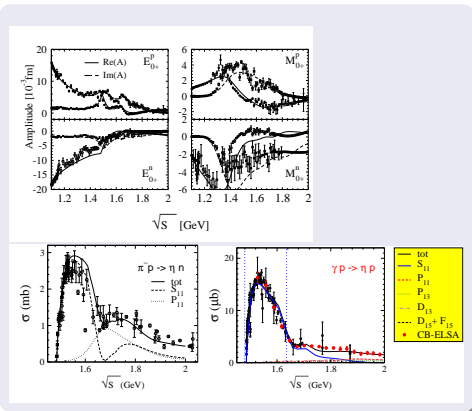
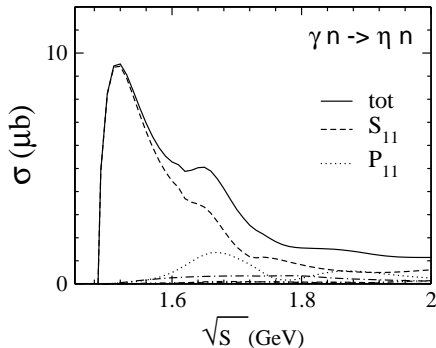
# Resonance parameters

$N^*$	$\Gamma_{\eta N}$	$A_{\frac{1}{2}}^p$	$A_{\frac{1}{2}}^n$	$A_{\frac{3}{2}}^p$	$A_{\frac{3}{2}}^n$
$S_{11}(1535)$ (Giessen)	56.2	95	-74	—	—
(PDG)	$53 \pm 1$	$90 \pm 30$	$-46 \pm 27$	—	—
$S_{11}(1650)$ (Giessen)	2.5	57	-9	—	—
(PDG)	$2.3 \pm 2$	$53 \pm 16$	$-15 \pm 21$	—	—
$P_{11}(1710)$ (Giessen)	42	-50	+24	—	—
(PDG)	$21 \pm 16$	$+9 \pm 22$	$-2 \pm 14$	—	—

**Table:** Helicity amplitudes (in  $10^{-3}\text{GeV}^{-\frac{1}{2}}$ ) and branching ratios to  $\eta N$ . First line: parameters obtained in the present calculations. Second line: PDG values.

# Results for the $\gamma n \rightarrow \eta n$

PLB650(2007)172.  $\gamma n \rightarrow \eta n$  Total partial wave cross sections



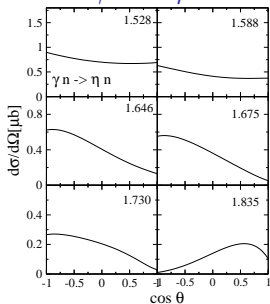
- constrained by  $\gamma(p/n) \rightarrow \pi(p/n)$  data
- constrained by  $\gamma p \rightarrow \eta p$  data
- constrained by  $\pi - p \rightarrow \eta n$  data
- both  $S_{11}(1650)$  and  $P_{11}(1710)$  contribute at 1.67 GeV.

# Results for the $\gamma n \rightarrow \eta n$ vs. $\gamma p \rightarrow \eta p$

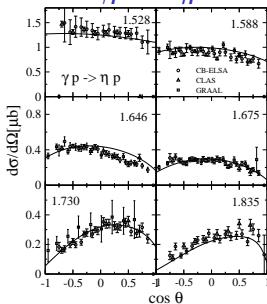
Comparison of  $\gamma n \rightarrow \eta n$  and  $\gamma p \rightarrow \eta p$   
differential cross sections

prediction

$\gamma n \rightarrow \eta n$

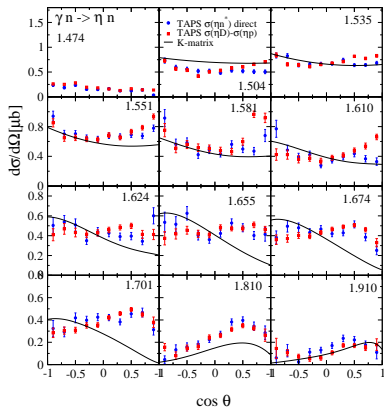


$\gamma p \rightarrow \eta p$



- $\gamma n \rightarrow \eta n$  mostly backward directions up to 1.75 GeV  
-result of S- and P-wave interference.
- above 1.75 GeV  $\gamma n \rightarrow \eta n$  looks very similar to  $\gamma p \rightarrow \eta p$

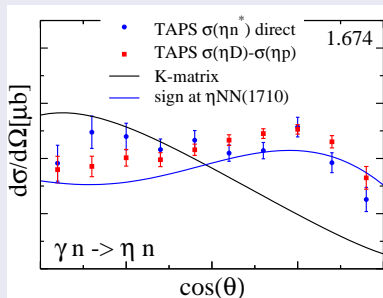
# $\gamma n \rightarrow \eta n$ comparison with the TAPS data



- good agreement in the region 1.490 ... 1.620 GeV and at high energies
- disagreement in the more interesting region around 1,7 GeV

- backward peaking - result of the  $S$ - and  $P$ -interference  
- the phase between the partial waves is crucial.

## Interference between S and P waves



- $S_{11}(1650)$  and  $P_{11}(1710)$ : properties are not well known.
- Changing the sign of  $g_{\eta NN}^{N(1710)}$  modify the interference pattern
- $P_{11}(1710)$  - strong  $2\pi N$  decay: a proper treatment of the  $2\pi$  channel is needed

- $K$ -matrix approximation to the BSE provide a simple and powerful tool to analyze experimental data on pion- and photon induced reactions.
- combined description of the reaction  
 $\gamma N \rightarrow \gamma N, \pi N, \eta N, \omega N, K\Lambda, K\Sigma$   
 $\pi N \rightarrow \pi N, 2\pi N, \eta N, \omega N, K\Lambda, K\Sigma$
- Old  $\pi N \rightarrow \eta N$  data shows a promising structure around 1.7 GeV Unfortunately the data is pure.
- $\gamma n \rightarrow \eta n$ : the resonance like structure seen in the TAPS data can be explained the excitations of  $S_{11}(1650)$  and  $P_{11}(1710)$  states. Interference effects might be crucial - further analyses is needed.
- more detailed description of the  $2\pi$  channel is necessary to understand the reaction mechanism

Born and  $t$ -channel terms:

$$\begin{aligned}
 L_{Born} + L_t = & -\bar{u}_{B'}(p') \left[ g_{\tilde{\varphi}} \gamma_5 \gamma_\mu (\partial^\mu \tilde{\varphi}) + g_\eta i \gamma_5 \eta + g_S S \right. \\
 & \left. + g_V \left( \gamma_\mu V^\mu + \frac{\kappa_V}{2m_N} \sigma_{\mu\nu} V^{\mu\nu} \right) \right] u_B(p) \\
 & - \frac{g_S}{2m_\pi} (\partial_\mu \varphi') (\partial^\mu \varphi) S - g_V \varphi' (\partial_\mu \varphi) V^\mu - \frac{g}{4m_\varphi} \epsilon_{\mu\nu\rho\sigma} V^{\mu\nu} V'^{\rho\sigma} \varphi.
 \end{aligned}$$

positive-parity **spin- $\frac{1}{2}$**  resonances, PV coupling is used:

$$L_{\frac{1}{2}B\varphi}^{PV} = -\frac{g_{RB\varphi}}{m_R \pm m_B} \bar{u}_R \begin{pmatrix} \gamma_5 \\ i \end{pmatrix} \gamma_\mu u_B \partial^\mu \varphi. \quad (1)$$

negative-parity **spin- $\frac{1}{2}$**  resonances, PS coupling is used:

$$L_{\frac{1}{2}B\varphi}^{PS} = -g_{RB\varphi} \bar{u}_R \begin{pmatrix} 1 \\ -i\gamma_5 \end{pmatrix} u_B \varphi. \quad (2)$$

# Nucleon resonances in effective field theory

## To solve the scattering problem

- Bethe-Salpeter equation
  - Dyson-Schwinger equation
- etc

## Background and resonance contributions in EFT

Solving the Dyson-Schwinger equation for the potential

$V = V(\text{resonance}) + V(\text{background})$

$$T = \frac{\Gamma(V_R, V_B)}{s - m^2 + \Sigma(V_R, V_B)} \approx \frac{\Gamma(V_R)}{s - m^2 + \Sigma(V_R)} + V_B$$

Breit-Wigner form

similar in QM:

$$\left(\frac{\hat{p}^2}{2m} + V_1 + V_2\right)\Psi_{12} = E\Psi_{12}$$

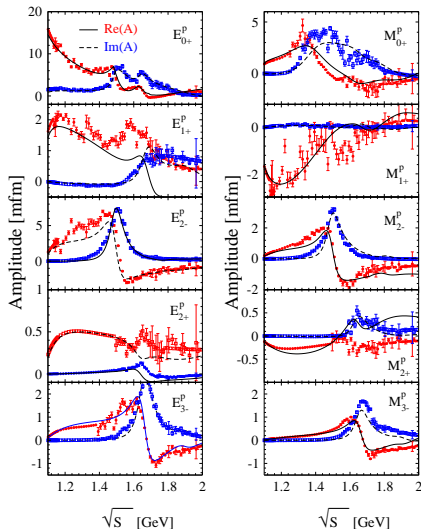
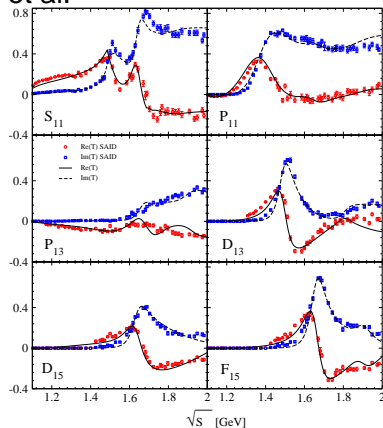
$$\Psi_{12} \neq \Psi_1 + \Psi_2 \quad (\text{only in perturb. } \Psi_{12} \approx \Psi_1 + \Psi_2)$$

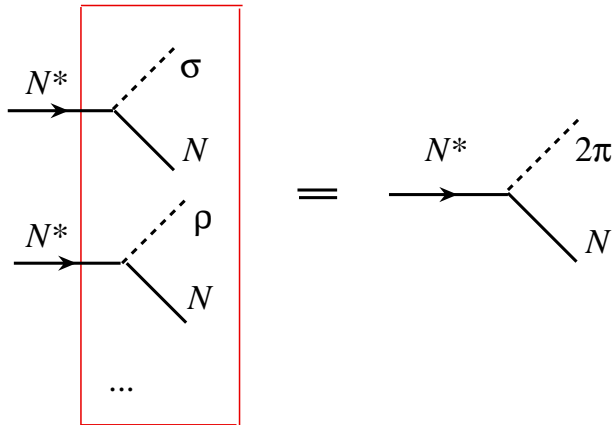
**Resonance and background contributions are generally not separable**

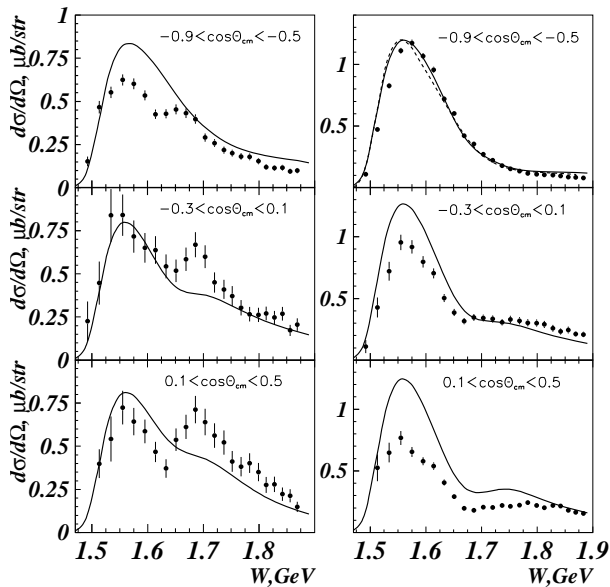


# Giessen model. Results for $\pi N \rightarrow \pi N, 2\pi N$ scattering

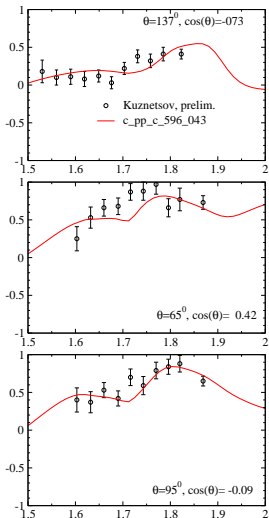
$\pi N$  scattering amplitudes  
PRC72,015210(2005) .vs. SAID  
partial wave analysis FA02 Arndt  
et al.







Beam asymmetry:  $\Sigma = \frac{d\sigma_{\perp} - d\sigma_{\parallel}}{d\sigma_{\perp} + d\sigma_{\parallel}}$



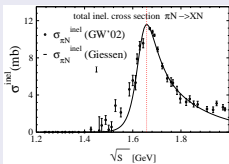
- asymmetry is not fitted
- take care: the calculations done without Fermi-motion

# Why $\Gamma_{N(1675) \rightarrow \eta N} = 17\%$ has problems

## Optical theorem for $\pi N \rightarrow \pi N$ scattering

$$(J + \frac{1}{2}) \text{Im} T_{\pi N \rightarrow \pi N}^{\frac{5}{2} + \frac{1}{2}} = \frac{k^2}{4\pi} (\sigma_{\pi N \rightarrow \pi N}^{\frac{5}{2} + \frac{1}{2}} + \sigma_{\pi N \rightarrow 2\pi N}^{\frac{5}{2} + \frac{1}{2}} + \sigma_{\pi N \rightarrow \eta N}^{\frac{5}{2} + \frac{1}{2}})$$

$$\sigma_{inel.}^{\frac{5}{2} + \frac{1}{2}} = \sigma_{\pi N \rightarrow 2\pi N}^{\frac{5}{2} + \frac{1}{2}} + \sigma_{\pi N \rightarrow \eta N}^{\frac{5}{2} + \frac{1}{2}} \approx 12 \text{mb}$$



## Assuming dominant $D_{15}(1675)$

$$T_{\pi N \rightarrow 2\pi N} \sim \frac{\Gamma_{\pi N}^{N(1675)} \Gamma_{2\pi N}^{N(1675)}}{s - m_{1675}^2 - i\Gamma_{tot}/2}$$

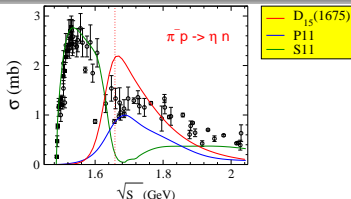
$$T_{\pi N \rightarrow \eta N} \sim \frac{\Gamma_{\pi N}^{N(1675)} \Gamma_{\eta N}^{N(1675)}}{s - m_{1675}^2 - i\Gamma_{tot}/2}$$

$$\frac{\sigma_{\pi N \rightarrow \eta N}}{\sigma_{\pi N \rightarrow 2\pi N}} = \left( \frac{\Gamma_{\eta N}^{1675}}{\Gamma_{2\pi N}^{1675}} \right)^2$$

$\eta$ -MAID L. Tiator (hep-ex/0601002):

$$\Gamma_{\eta N} \approx 17\%, \Gamma_{2\pi N} \approx 40\%$$

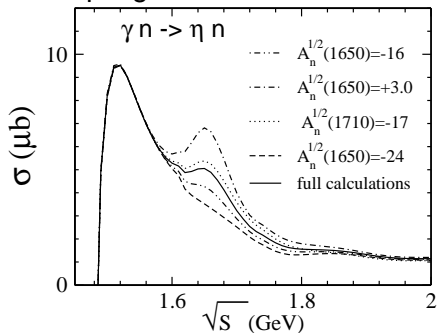
$$\sigma_{\pi N \rightarrow \eta N}^{\frac{5}{2}} |_{1675 \text{MeV}} = 12 \left( \frac{17}{40} \right)^2 \approx 2.2 \text{mb}$$



# Results for the $\gamma n^* \rightarrow \eta n$

- some fitted data are not of good quality
- some of data is still preliminary

How sensitive our calculation to a variation of  $N^* \rightarrow \gamma n$  couplings ?



The variation of the  $N^* \rightarrow \gamma n$  couplings strongly affect the magnitude of the peak