On the determination of the parity quantum number of narrow resonances

- Motivation.
- Spin cross sections in NN reactions and the parity of narrow resonances.
- How to measure the spin cross sections and the actual feasibility to determine the parity.
- Summary.

Motivation

The search for narrow resonances, especially the so-called exotic resonances has been always of special interest in hadron spectroscopy [see, e.g., E. Klempt, hep-ph/0404270 for a recent review; see also L. G. Landsberg, Phys. Rep. 320, 223(1999)]. Apart from establishing their existence, the determination of their basic properties are of extreme importance. Among these properties, the parity is of particular importance in connection with the substructure of these resonances.

However, contrary to the situations encountered with ordinary resonances, it is often the case that no theoretical predictions can provide a conclusive result for the parity and other basic properties of these resonances [recent example: the pentaquark Θ^+] .

This calls for a model-independent way of determining these properties.

Bohr's theorem: a general result for spin-parity relation *(Bohr, NPA'59, Satchler, Direct Nuclear Reactions, '83)*

reflection symmetry in the scattering plane

product of all intrinsic parities

sum of the spin-projections onto the axis ⊥ to the reaction plane

$$\pi_{if} = (-)^{M_i - M_f}$$

- allows for an unambiguous determination of the parity.

[see: Nakayama&Love, PRC70, 012201(R)(2004)]

- drawback: requires the measurement of the polarization of all the particles in the initial and final states.

In (ps-meson) photoproduction:

(Nakayama & Love, PRC70,'04)

A number of spin observables can be directly related to the parity of the resonance:

$$\Sigma_{yy}(i,i) = \frac{\sigma_y^{\perp}(i,i) - \sigma_y^{\parallel}(i,i)}{\sigma_y^{\perp}(i,i) + \sigma_y^{\parallel}(i,i)} = -\Sigma_{yy}(i,-i) = \pi(R)$$

beam asymmetry (pol.: $\varepsilon^{\perp}$, $\varepsilon^{\parallel}$)
in conjunction with
polarization transfer along the y-axis (i=+,-)

$$K_{yy}^{\perp} = -K_{yy}^{\parallel} = \pi(R)$$

polarization transfer coefficient with the linearly polarized photon beam

$$K_{yy} = \pi(R)\Sigma$$

polarization transfer coefficient & beam asymmetry

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-
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the observables require measuring the spin of the produced resonance

In NN → YΘ⁺ :

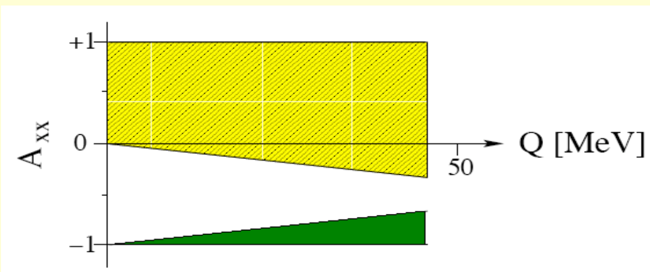
- Pauli principle + parity conservation ⇒ spin-parity relation can pin down $\pi(\Theta^+)$ in $\vec{p}\vec{p} \to \Sigma^+\Theta^+$. [Thomas, Hicks, Hosaka, PTP111 '04]

- At threshold:

  $\pi(\Theta^+) = +$: $A_{xx} = -1$ [Rekalo&T-Gustafson, PLB591 '04;

  $\pi(\Theta^+) = -$: $A_{xx} \geq 0$ Uzikov, PLB595 '04]

- Energy dependence of A_{xx} in conjuntion with the "naturalness" assumption [Hanhart et al., PLB590,'04]



- Energy dependence of the spin-triplet cross section: [Hanhart, Haidenbauer, Nakayama, Meiβner, PLB606 '05]

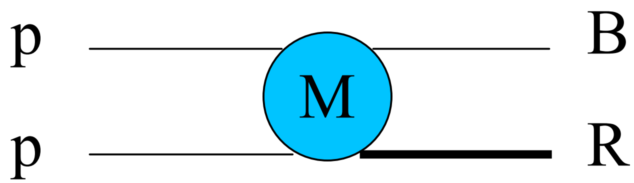
$$\pi(\Theta^+) = + : \qquad \frac{{}^3\sigma}{p'} = \beta_1 \left(\frac{p'^2}{\Lambda^2}\right) + O\left(\frac{p'^6}{\Lambda^6}\right)$$

$$\pi(\Theta^+) = - : \qquad \frac{{}^3\sigma}{p'} = \beta_0' + O\left(\frac{p'^4}{\Lambda^4}\right)$$

Pauli principle + parity conservation in NN reactions

(A. Thomas et al., PTP111 '04)

$^{(2S+1)}L_J$



$^{(2S'+1)}L'_J$

Pauli principle + parity conservation : $(-)^{S+L'+T} = +/-1,\ \pi_B\pi_R = -/+$

$\pi_B\pi_R = +$	$\pi_B\pi_R = -$
$S=0 \rightarrow L'=\text{even}$	$S=0 \rightarrow L'=\text{odd}$
$S=1 \rightarrow L'=\text{odd}$	$S=1 \rightarrow L'=\text{even}$

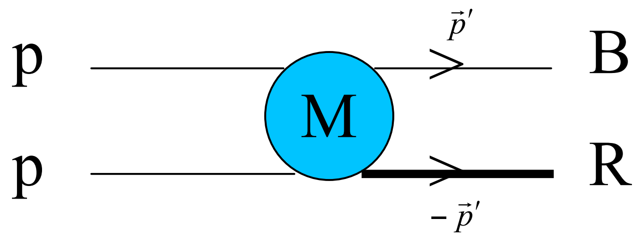
$J_B=J_R=1/2$

$^1S_0 \rightarrow {}^1S_0$	$^3P_0 \rightarrow {}^1S_0$
$^3P_1 \rightarrow {}^1P_1$	$^3P_1 \rightarrow {}^3S_1$
$^3P_{0,1,2} \rightarrow {}^3P_{0,1,2}$	$^1S_0 \rightarrow {}^3P_0$
$^3F_2 \rightarrow {}^3P_2$	$^1D_2 \rightarrow {}^3P_2$
$\vdots$	$\vdots$

Energy dependence of the spin cross section near threshold:

(Hanhart et al., PLB606, '05)

$^{(2S+1)}L_J$



$^{(2S'+1)}L'_J$

$L' = S,P$

centrifugal barrier : $(p')^{L'}$

$\pi_B\pi_R = +$

$$\frac{{}^3\sigma}{p'} = \beta_1\left(\frac{p'^2}{\Lambda^2}\right) + O\left(\frac{p'^6}{\Lambda^6}\right)$$

$\pi_B\pi_R = -$

$$\frac{{}^3\sigma}{p'} = \beta'_0 + O\left(\frac{p'^4}{\Lambda^4}\right)$$

phase space factor

$p'^2 \propto Q$ (excess energy)

$\Lambda^2 \sim -t = m_p(m_B+m_R - 2m_p)$
→ higher p.w. suppression: $(p'/\Lambda)^{L'}$

(e.g., $m_B \sim 1$GeV, $m_R \sim 1.5$GeV
→ $\Lambda \sim 900$ MeV)

$p'^2/\Lambda^2 \sim 0.1 \Rightarrow Q \sim 60$ MeV.

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sources of energy dependence of β_i

- BR fsi
- pp isi
- transition operator

Sources of energy dependence:

1=pp , 2=BR

$$T_{21} = V_{21} + V_{21} G_1 T_{11} + V_{22} G_2 T_{21}$$

$$T_{21} = \underbrace{\left(\frac{1}{1 - V_{22} G_2} \right)}_{\boxed{\text{BR fsi}}} V_{21} \underbrace{\left(1 + G_1 T_{11}\right)}_{\boxed{\text{pp isi}}}$$

pp→BR transition potential

effective range approx. :

$$T_{21} \cong \frac{1}{\alpha(p')} \left(\frac{1}{1 + iap' - ar_0 p'^2/2} \right) V_{21} \left(1 + G_1 T_{11}\right),$$

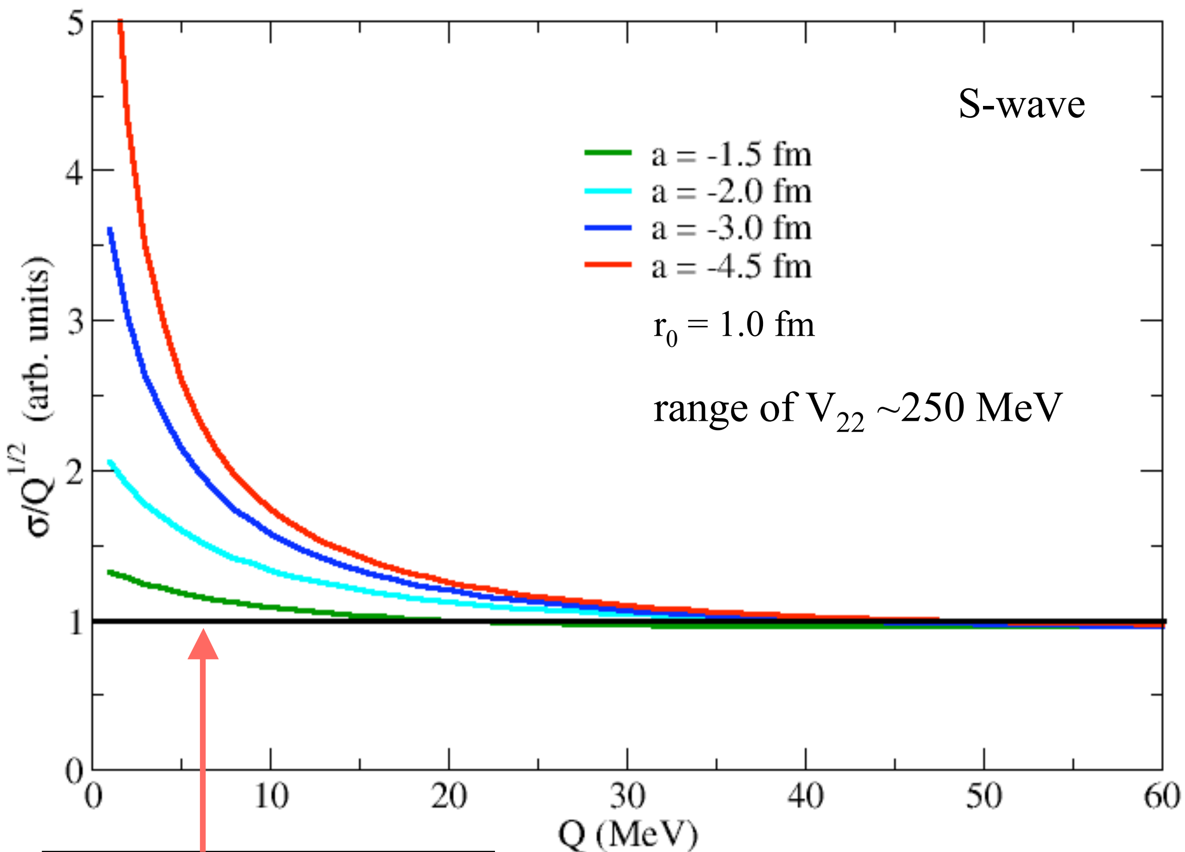
$$\alpha(p') \equiv \frac{\pi}{a} \left(\frac{m_B m_R}{\sqrt{s}} \right) V_{22}(p', p')$$

for heavy particle production (e.g., m_B~1GeV, m_R~1.5GeV → r ~ 0.2 fm) :

- pp isi → weak energy dependence (at high energies)
- V_{21} → weak energy dependence (short-range process)
- BR fsi → may introduce an energy dependence

Hanhart & Nakayama, PLB454 '99

A rough estimate of BR fsi effects :



(phase space)/p′

$Q \propto p'^2$

Energy dependence of $^3\sigma_\Sigma$:

(a model calculation: Hanhart et al., PLB606 '05)

$\underline{L' = 0,1:}$

$$\frac{{}^3\sigma}{p'} = \beta_1 \left(\frac{p'^2}{\Lambda^2}\right) + O\left(\frac{p'^6}{\Lambda^6}\right)$$

$\pi_B\pi_R = +$

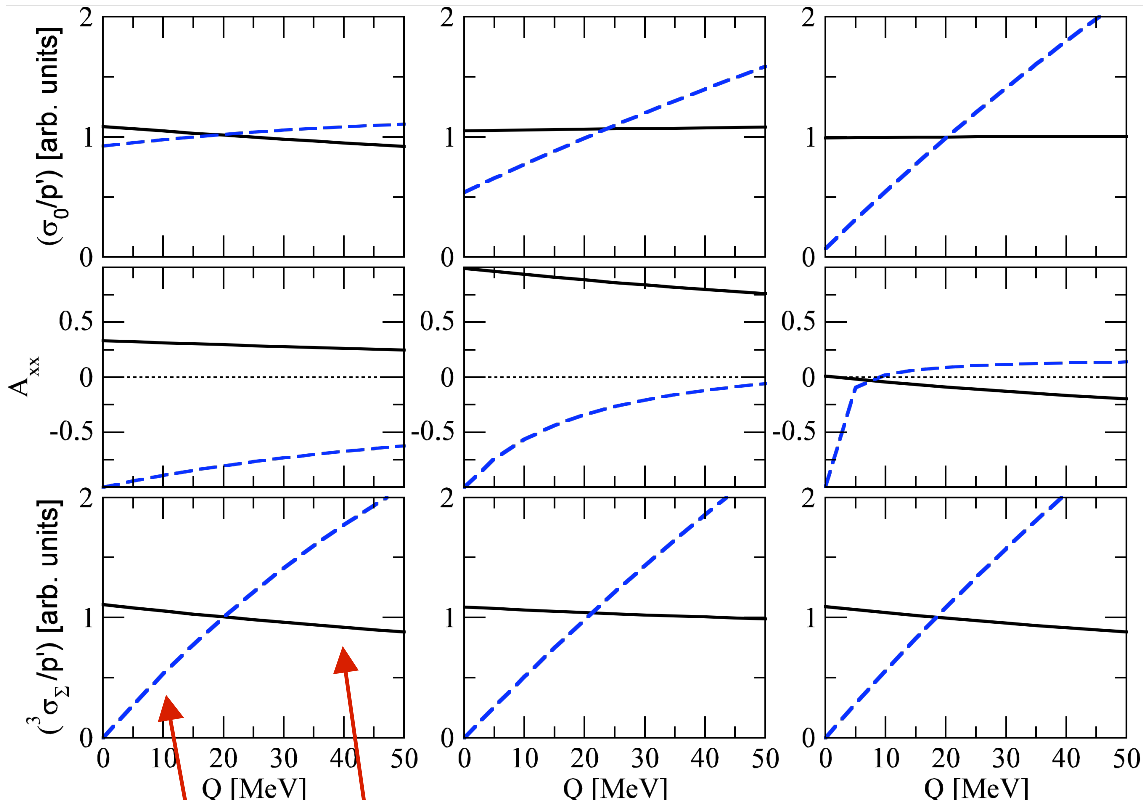
$$\frac{{}^3\sigma}{p'} = \beta_0 + O\left(\frac{p'^4}{\Lambda^4}\right)$$

$\pi_B\pi_R = -$

$B = \Sigma^+$
$R = \Theta^+$

$J_{\Theta+}=1/2$

no isi & fsi

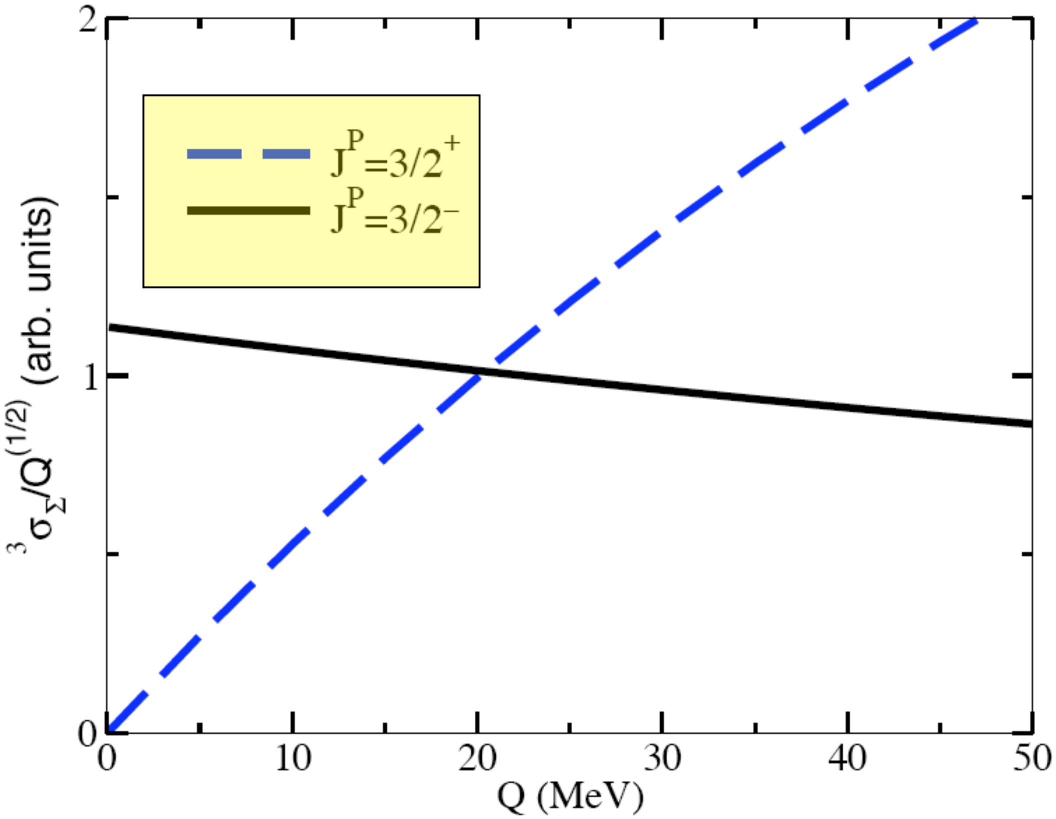


$\pi_{\Theta+} = +$

$\pi_{\Theta+} = -$

$Q \propto p'^2$

Energy dependence of ${}^3\sigma_\Sigma$: ($J_{\theta+}$=3/2 case)



$$Q \propto p'^2$$

Angular dependence of the spin cross section near threshold:

$L' = 0,1$:

completely general (may be used for $Q < 30$ MeV)

$\pi_B \pi_R = +$

$$\frac{1}{p'} \frac{d(^3\sigma)}{d\Omega} = \alpha_0 + \alpha_1 \cos^2(\theta)$$

only P-waves

α_1 may be zero (e.g., $J_B = J_R = 1/2$: $^3P_0$)

$\pi_B \pi_R = -$

$$\frac{1}{p'} \frac{d(^3\sigma)}{d\Omega} = \alpha_0'$$

only S-waves

- strong θ dependence $\Rightarrow$ $\pi_B \pi_R = +$
- weak θ dependence $\Rightarrow$ inconclusive

Features of the spin triplet cross sections:

	energy dependence	angular dependence
$\pi_B\pi_R = +$	$\frac{^3\sigma}{p'} = \beta_1 \left(\frac{p'^2}{\Lambda^2} \right)$	$\frac{1}{p'} \frac{d(^3\sigma)}{d\Omega} = \alpha_0 + \alpha_1 \cos^2(\theta)$
$\pi_B\pi_R = -$	$\frac{^3\sigma}{p'} = \beta'_0$	$\frac{1}{p'} \frac{d(^3\sigma)}{d\Omega} = \alpha'_0$

How to measure the spin cross sections ?

(Bilenky&Ryndin, PL6 '63; Meyer et al., PRC63 '01; Hanhart et.al., PLB590 '04)

spin cross section : $$\frac{d\left({}^{(2S+1)}\sigma_{M_S}\right)}{d\Omega} \equiv \frac{1}{4}\sum_{S'M_{S'}} |\langle S'M_{S'}|\widehat{M}|SM_S\rangle|^2$$

spin correl. coeff. : $$\frac{d\sigma}{d\Omega}A_{ii} \equiv \frac{1}{4}Tr\left[\widehat{M}\vec{\sigma}_i(1)\vec{\sigma}_i(2)\widehat{M}^+\right] = \left[\sigma(\uparrow\uparrow)+\sigma(\downarrow\downarrow)\right]-\left[\sigma(\uparrow\downarrow)+\sigma(\downarrow\uparrow)\right]$$

$$\frac{d({}^1\sigma_0)}{d\Omega} = \frac{1}{4}\frac{d\sigma}{d\Omega}\left(1 - A_{xx} - A_{yy} - A_{zz}\right)$$

$$\frac{d({}^3\sigma_0)}{d\Omega} = \frac{1}{4}\frac{d\sigma}{d\Omega}\left(1 + A_{xx} + A_{yy} - A_{zz}\right)$$

$$\frac{d({}^3\sigma_{+1})}{d\Omega} = \frac{d({}^3\sigma_{-1})}{d\Omega} = \frac{1}{4}\frac{d\sigma}{d\Omega}\left(1 + A_{zz}\right)$$

$$\frac{d\sigma}{d\Omega}A_{xx} = \frac{1}{4}\sum(-)^{1+S-2M_S}\langle S'M_{S'}|\widehat{M}|SM_S\rangle\langle S'M_{S'}|\widehat{M}|S-M_S\rangle^*$$

$$\frac{d\sigma}{d\Omega}A_{yy} = \frac{1}{4}\sum(-)^{1+S-M_S}\langle S'M_{S'}|\widehat{M}|SM_S\rangle\langle S'M_{S'}|\widehat{M}|S-M_S\rangle^*$$

$$\frac{d\sigma}{d\Omega}A_{zz} = \frac{1}{4}\sum(-)^{1+M_S}|\langle S'M_{S'}|\widehat{M}|SM_S\rangle|^2$$

$$\boxed{\frac{d({}^3\sigma_\Sigma)}{d\Omega} \equiv \frac{d({}^3\sigma_0)}{d\Omega} + \frac{d({}^3\sigma_{+1})}{d\Omega} = \frac{1}{4}\frac{d\sigma}{d\Omega}\left(2 + A_{xx} + A_{yy}\right)}$$

Feasibility to use $^3\sigma_\Sigma$ for parity determination

(Rekalo&T-Gustafsson, EPJA22,'04; Uzikov, hep-ph/0402216)

$$\frac{d\sigma}{d\Omega}A_{xx}=\frac{1}{4}\sum(-)^{1+S-2M_S}\langle S'M_{S'}|\widehat{M}|SM_S\rangle\langle S'M_{S'}|\widehat{M}|S-M_S\rangle^*$$

$$\frac{d\sigma}{d\Omega}A_{yy}=\frac{1}{4}\sum(-)^{1+S-M_S}\langle S'M_{S'}|\widehat{M}|SM_S\rangle\langle S'M_{S'}|\widehat{M}|S-M_S\rangle^*$$

$\boxed{\text{if } L'=0 \Rightarrow M_{S'}=M_S}$

$$\langle S'M_{S'}|\widehat{M}(\vec{p}\,',\vec{p})|SM_S\rangle=\sum i^{L-L'}\overbrace{(S'M_{S'}L'M_{L'}|JM_J)}(SM_SL0|JM_J)$$
$$\times\; M_{L'L}^{S'SJ}(p',p)Y_{L'M_{L'}}(\widehat{p}\,')Y_{L0}^*(\widehat{p}=\widehat{z})$$

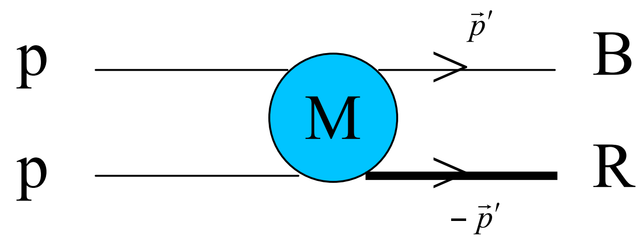
<u>At threshold (L′=0) :</u>

$\pi_B\pi_R=+:\quad S=0\ \rightarrow\ A_{xx}=A_{yy}=-1\ \Rightarrow\ \frac{d(^3\sigma_\Sigma)}{d\Omega}=\frac{1}{4}\frac{d\sigma}{d\Omega}\left(2+A_{xx}+A_{yy}\right)=0$

:

$\pi_B\pi_R=-:\quad S=1\ \rightarrow\ A_{xx}=A_{yy}\geq 0\ \Rightarrow\ \frac{d(^3\sigma_\Sigma)}{d\Omega}=\frac{1}{4}\frac{d\sigma}{d\Omega}\left(2+A_{xx}+A_{yy}\right)\geq\frac{1}{2}\frac{d\sigma}{d\Omega}$

Conclusion :

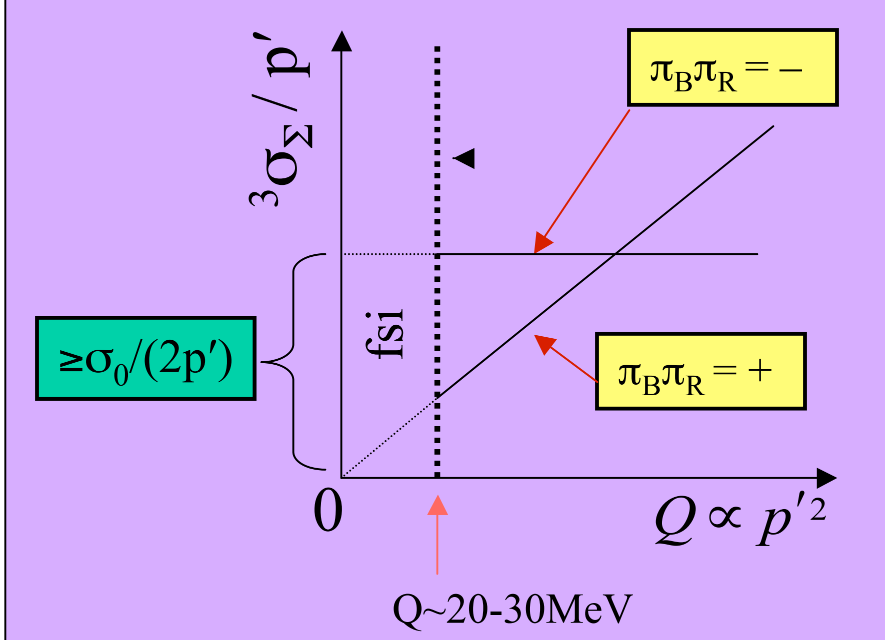


$$^{3}\sigma_{\Sigma} = \frac{\sigma_0}{4}\left(2 + A_{xx} + A_{yy}\right)$$

$^{3}\sigma_{\Sigma}(Q)$ and/or $d(^{3}\sigma_{\Sigma})/d\Omega$ in proton-proton collisions offers a possibility to determine the parity of a narrow baryon resonance with an arbitrary spin and isospin in a model independent way.

[Feasible at COSY !!!]

essential features near threshold:



only P-waves

$$\frac{1}{p'}\frac{d(^{3}\sigma_{\Sigma})}{d\Omega} = \tilde{\alpha}_0 + \tilde{\alpha}_1 \cos^2(\theta)$$

$\pi_B\pi_R = +$

may be 0

$$\frac{1}{p'}\frac{d(^{3}\sigma_{\Sigma})}{d\Omega} = \tilde{\alpha}'_0$$

$\pi_B\pi_R = -$

only S-waves

The End