What is a Resonance?

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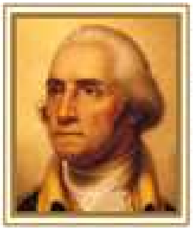

What is a Resonance?

Or: Why parents love their children best, and other thoughts...

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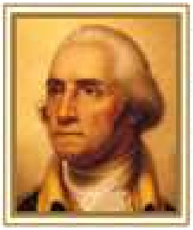

Thanks to: S. Krewald, K. Nakayama, R. Workman

Outline

- Resonance Basics
- Scattering-Theory Basics for Narrow Structures
- Approximations
- Elementary vs. Dynamical Resonances
- Can one define an interface between quark and hadronic models at the level of bare masses and bare vertices? [A: Not without a lot of work.]
- Summary

Resonance Basics

Experiment

- ‘Bump’ in the cross section
- Phase shifts show rapid change through $\pi/2$
- Time delay
- Exponential decay law

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Theory

Resonant structures arise from three types of mechanisms:

- Poles of the S-matrix corresponding to elementary resonances
- Poles of the S-matrix corresponding to dynamic resonances
- Structures that produce the usual signals of resonances (see above) *without* accompanying poles of the S-matrix [Calucci/Ghirardi, Phys. Rev. **169**, 1339 (1968)]

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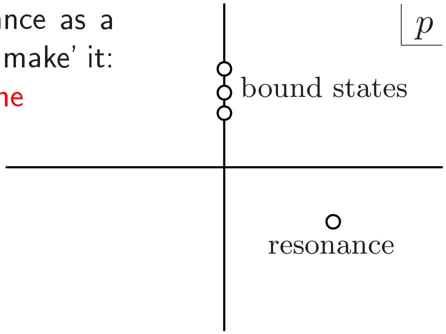
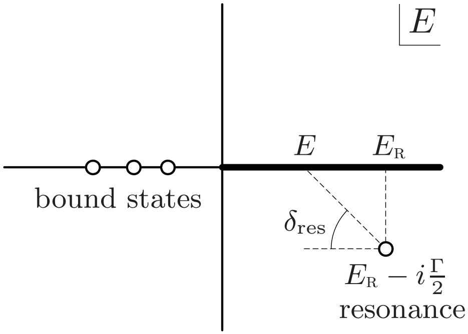
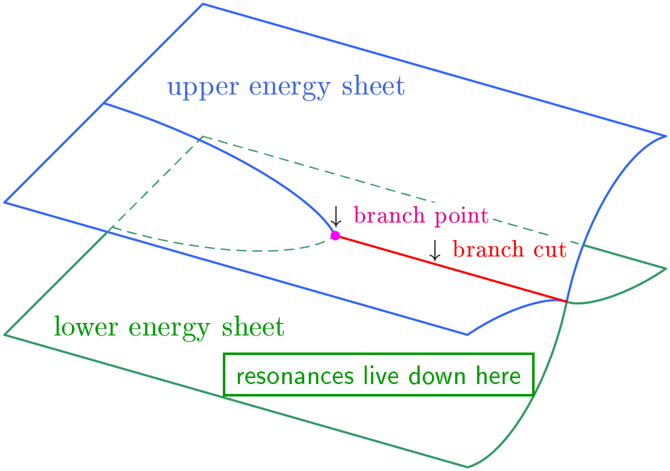
- **Will ignore last item because it cannot be treated generically. Caveat: Its experimental manifestation may lead to erroneous phenomenological pole-type description.**

Resonance Basics

- Usually, we think of a resonance as a bound state that didn't quite 'make' it:
  $\Longrightarrow$ Pole in the complex plane







Scattering phase shifts:

$$\delta(E) = \delta_{\mathsf{res}}(E) + \delta_{\mathsf{bg}}(E)$$

Resonance phase shift:

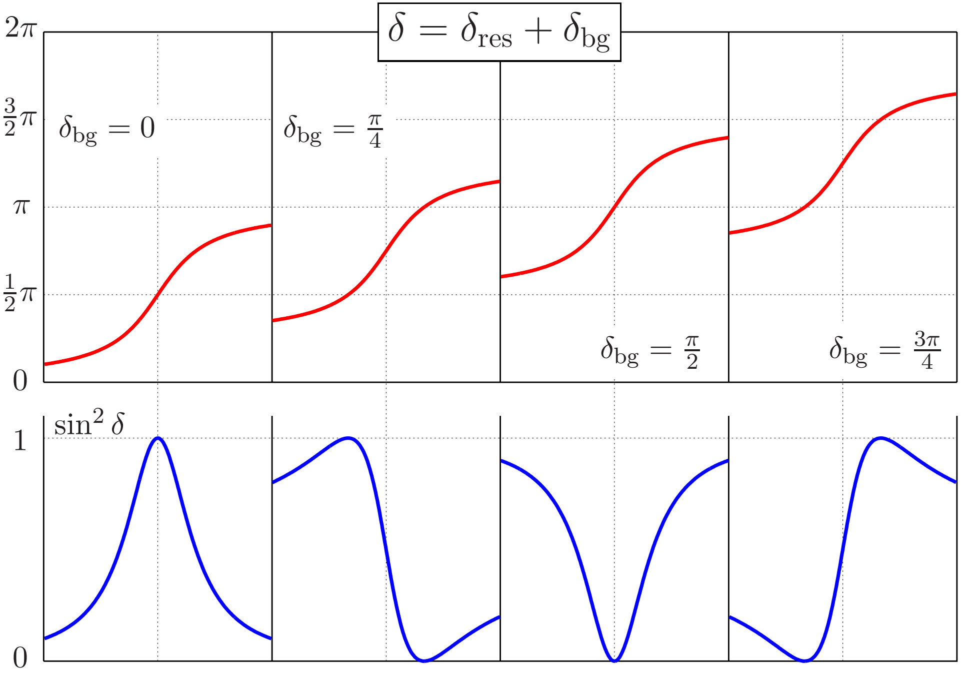
$$\tan\delta_{\mathsf{res}} = -\frac{\Gamma}{2(E - E_R)}$$

Breit–Wigner peak:

$$\sin^2\delta_{\mathsf{res}} = \frac{\Gamma^2}{4(E - E_R)^2 + \Gamma^2}$$

Resonance Basics — Wide Resonance

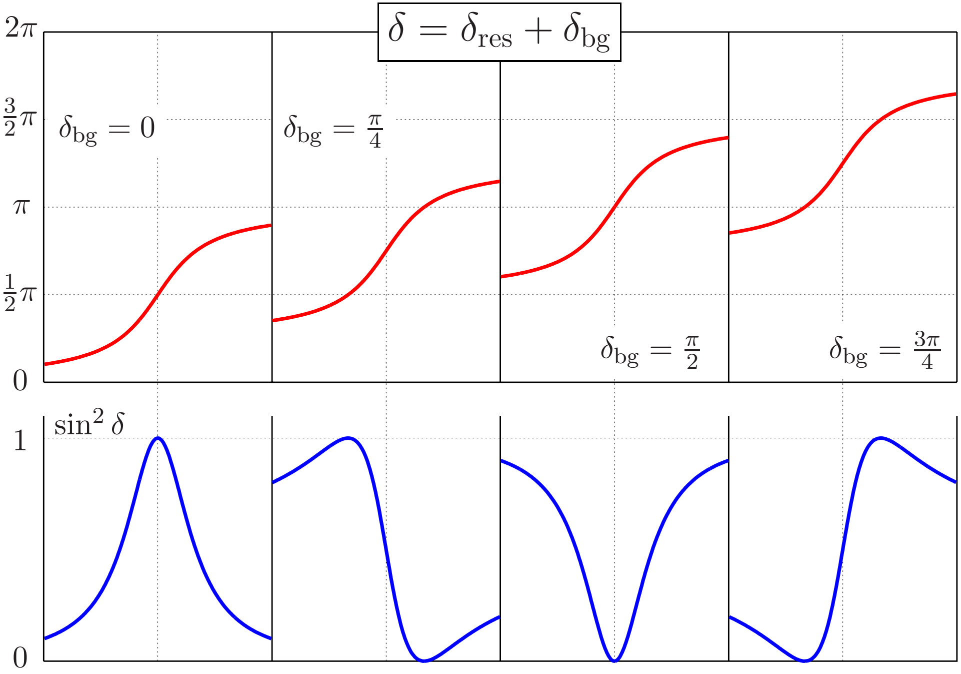
Resonant phase shifts with various constant background contributions



Each tile $300\,\text{MeV}$ wide; width $\Gamma = 100\,\text{MeV}$

Resonance Basics — Wide Resonance

Resonant phase shifts with various constant background contributions



Assume

$$\Delta E \ll \Gamma$$

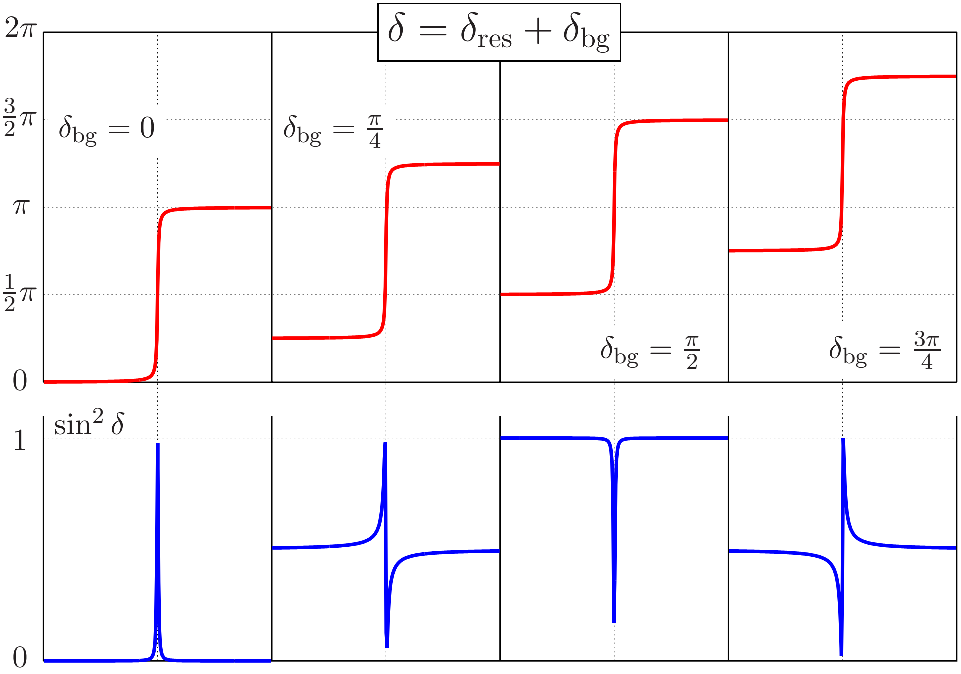
Then

$$\frac{d\sigma}{d\Omega} = |T|^2$$

Each tile $300\,\text{MeV}$ wide; width $\Gamma = 100\,\text{MeV}$

Resonance Basics — Narrow Resonance

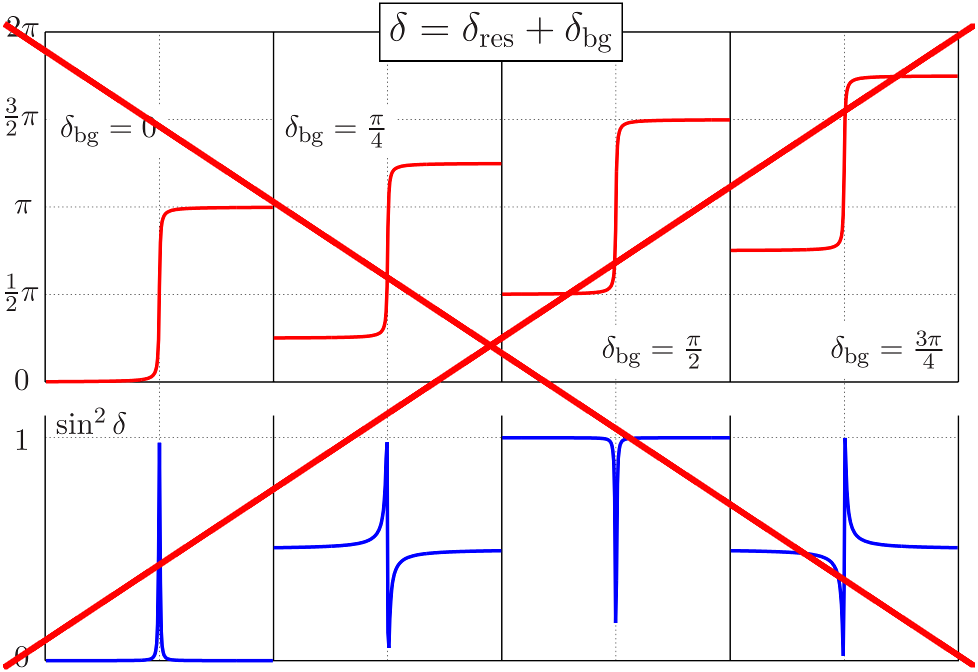
Resonant phase shifts with various constant background contributions



Each tile 300 MeV wide; width $\Gamma = 2$ MeV

Resonance Basics — Narrow Resonance

Resonant phase shifts with various constant background contributions



If

$$\Delta E \gtrsim \Gamma$$

Then

$$\frac{d\sigma}{d\Omega} \neq |T|^2$$

Each tile 300 MeV wide; width $\Gamma = 2$ MeV

Some Basics of Scattering Theory

Time-independent scattering:

$$\psi^{(+)}(\mathbf{r}) \quad \xrightarrow{r\to\infty} \quad \mathrm{e}^{i\mathbf{p}\cdot\mathbf{r}} + T\,\frac{\mathrm{e}^{ipr}}{r}$$

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Time-dependent scattering:

$$\Psi^{(+)}(\mathbf{r},t) \quad \xrightarrow{r\to\infty} \quad \int d^3p\,\phi(\mathbf{p})\,\mathrm{e}^{-iE_pt}\left[\mathrm{e}^{i\mathbf{p}\cdot\mathbf{r}} + T\,\frac{\mathrm{e}^{ipr}}{r}\right] = \Psi_{\mathsf{in}}(\mathbf{r},t) + \Psi_{\mathsf{sc}}(\mathbf{r},t)$$

$\phi(\mathbf{p})$: experimental momentum distribution peaked at $\mathbf{p}_0$

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$\phi(\mathbf{p})$: experimental momentum distribution peaked at $\mathbf{p}_0$

IF structures in T smooth compared to width of $\phi(\mathbf{p})$, then

$$\Psi_{\mathsf{sc}}(\mathbf{r},t) = \frac{T}{r}\int d^3p\,\phi(\mathbf{p})\,\mathrm{e}^{i(pr-E_pt)} \qquad \Rightarrow \qquad \frac{d\sigma}{d\Omega} = |T|^2$$

Some Basics of Scattering Theory

However:

IF T varies rapidly across the width of $\phi(\mathbf{p})$, then

$$\Psi_{\mathsf{sc}}(\mathbf{r},t) \neq \frac{T}{r}\int d^3p\,\phi(\mathbf{p})\,\mathrm{e}^{i(pr-E_pt)} \qquad \text{and} \qquad \frac{d\sigma}{d\Omega} \neq |T|^2$$

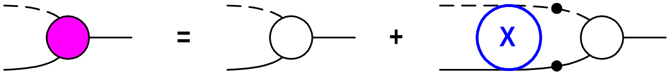
- Usual scattering-theoretical relations do not apply.
- **Needed: Correct theoretical description of narrow structures.**

Elementary vs. Dynamic Resonances

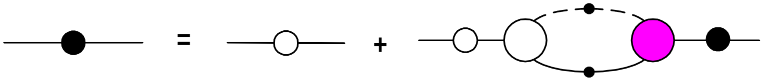
PRC56,2041(1997)

$\pi N \to \pi N$

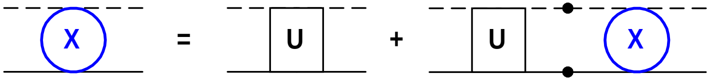
dressed vertex



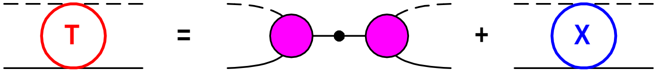
dressed propagator



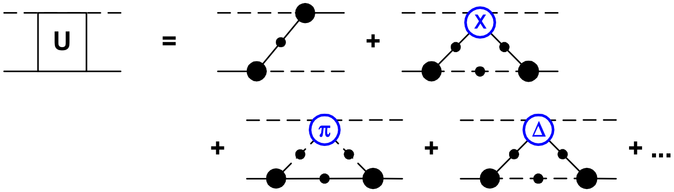
nonpolar T matrix



full T matrix



two-particle irreducible kernel



Elementary vs. Dynamic Resonances

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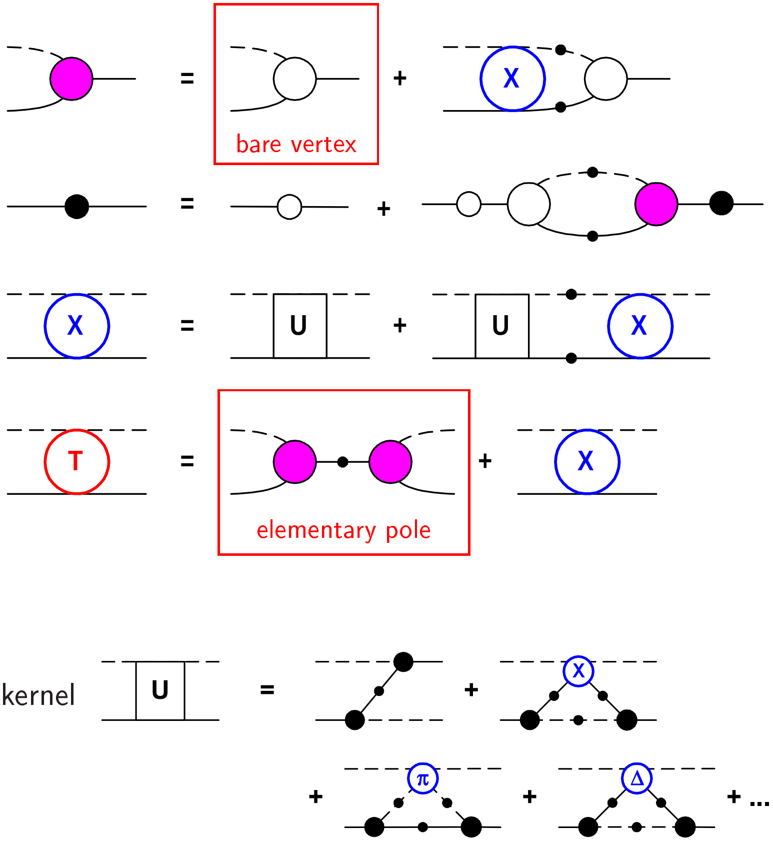

dressed vertex

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Elementary vs. Dynamic Resonances

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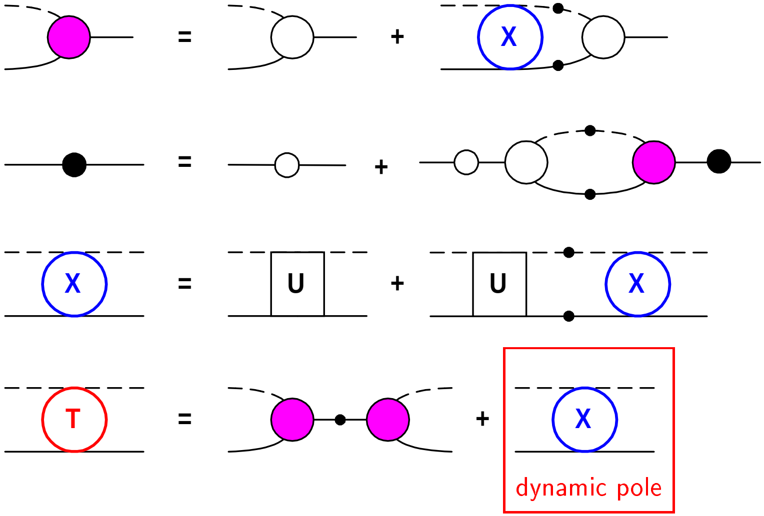
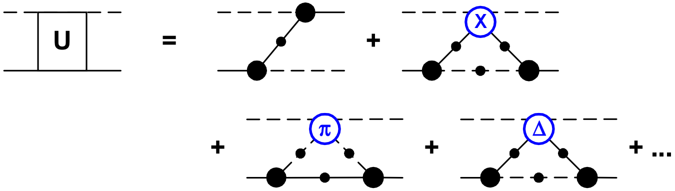


dynamic pole arises from
two-particle irreducible kernel



Elementary vs. Dynamic Resonances

PRC56,2041(1997)

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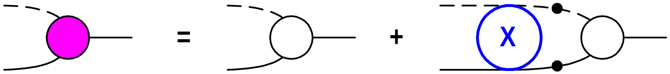
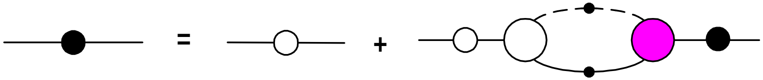
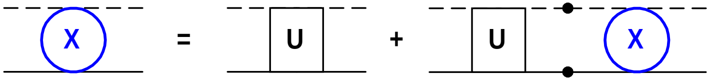
dressed vertex



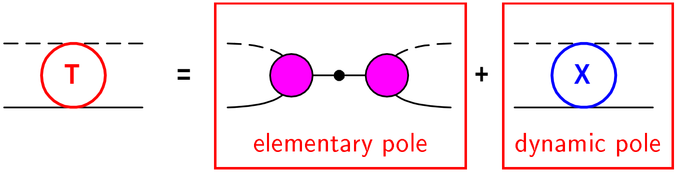
dressed propagator



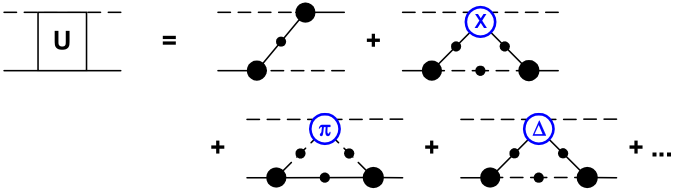
nonpolar T matrix



full T matrix



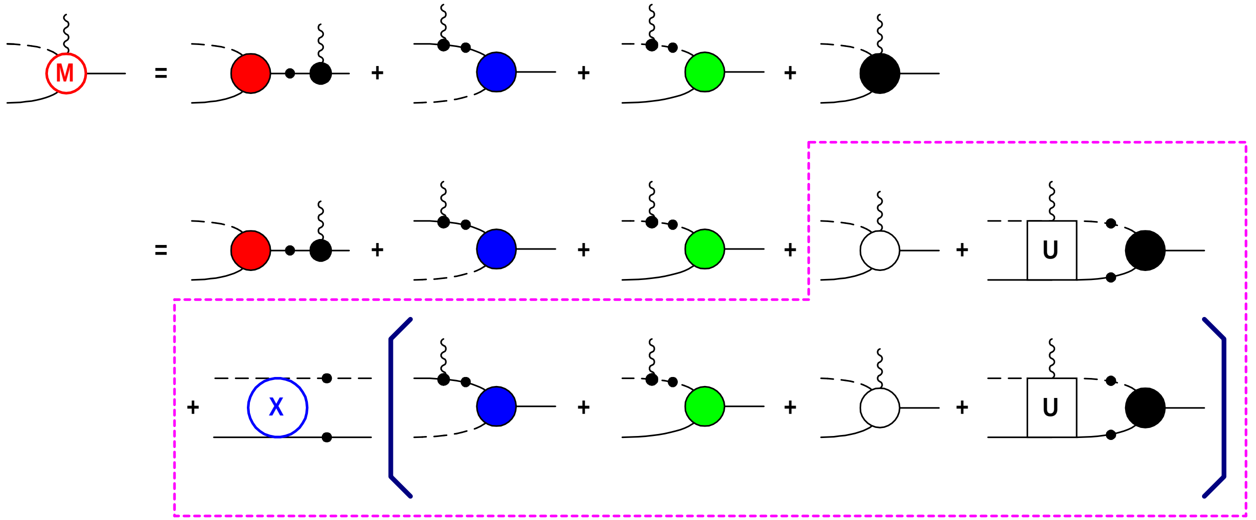
two-particle irreducible kernel



Elementary vs. Dynamic Resonances

PRC56,2041(1997)

$\gamma N \to \pi N$



interaction current

Elementary vs. Dynamic Resonances

PRC56,2041(1997)

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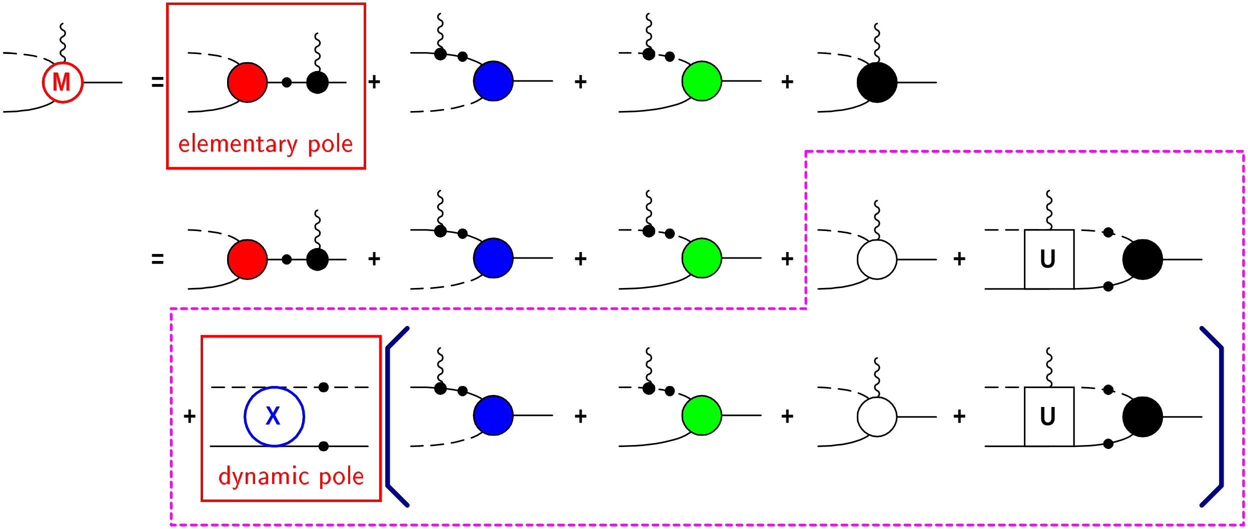
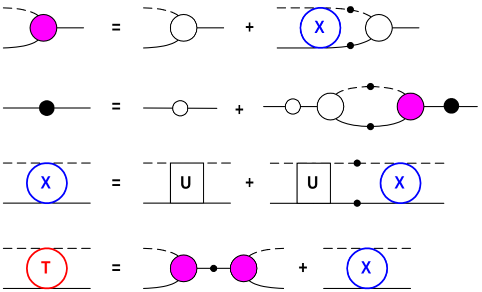
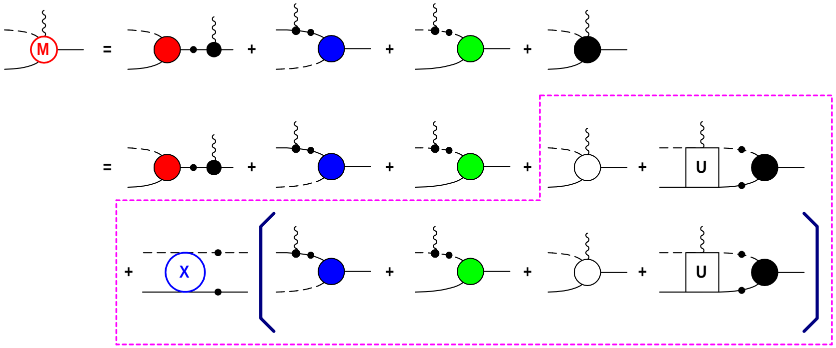


- Same mechanisms as in hadronic reaction.

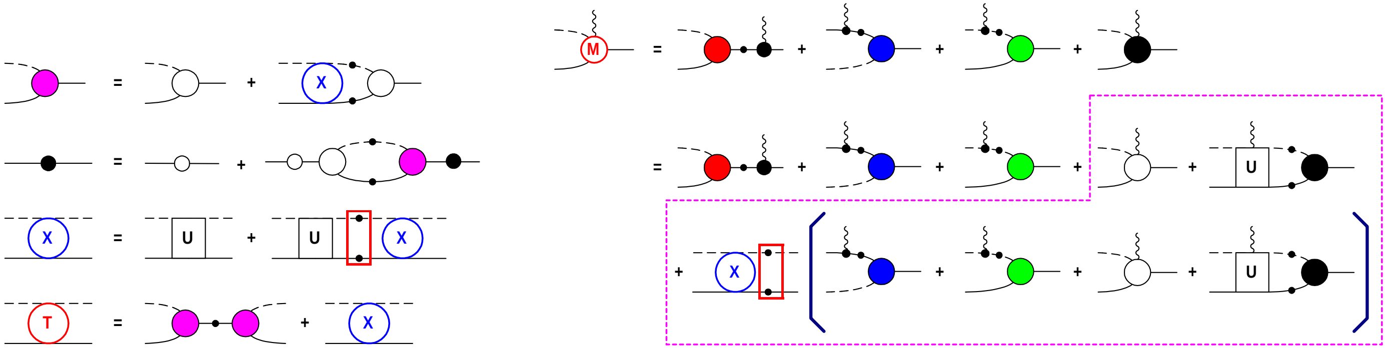
Approximations





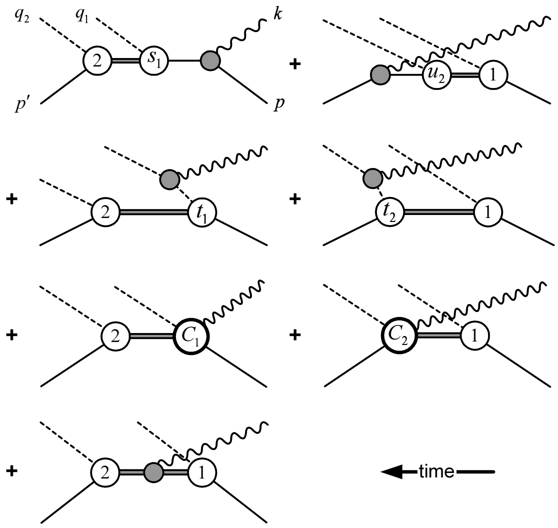
- Approximations necessary to make equations manageable.
- **However: Approximations often violate basic theoretical constraints.** (But the parents of the corresponding models love their children anyway...)

Approximations

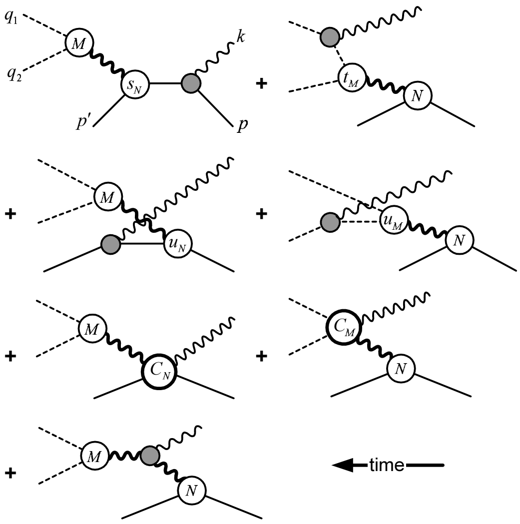


- Approximations necessary to make equations manageable.
- **However: Approximations often violate basic theoretical constraints.** (But the parents of the corresponding models love their children anyway...)
- **Case in point:** K-matrix Born approximation destroys gauge invariance. (Constructing a conserved current is not enough for a microscopic model!)

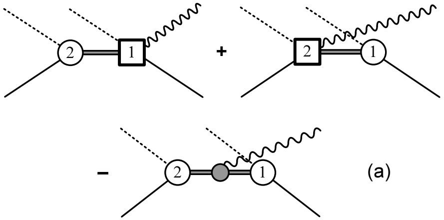
Basic Two-pion Production Mechanisms



(a)



(b)



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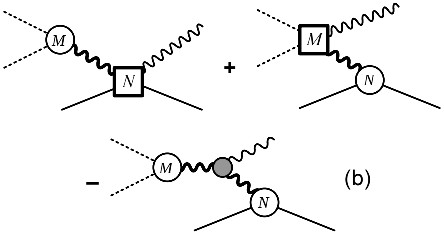
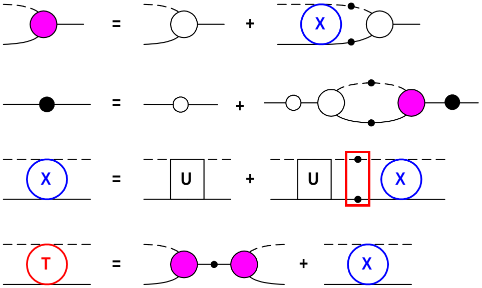
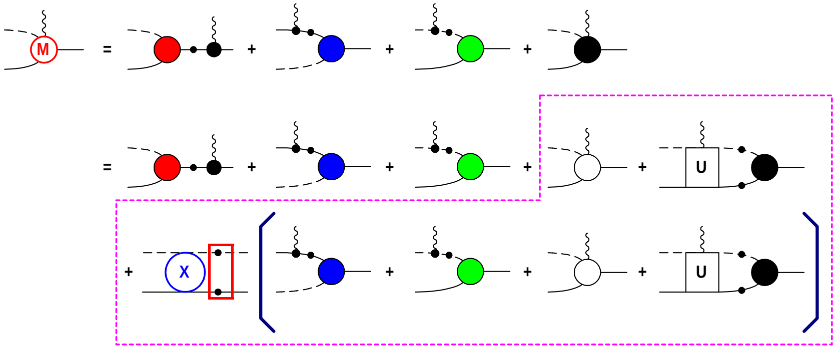


(b)

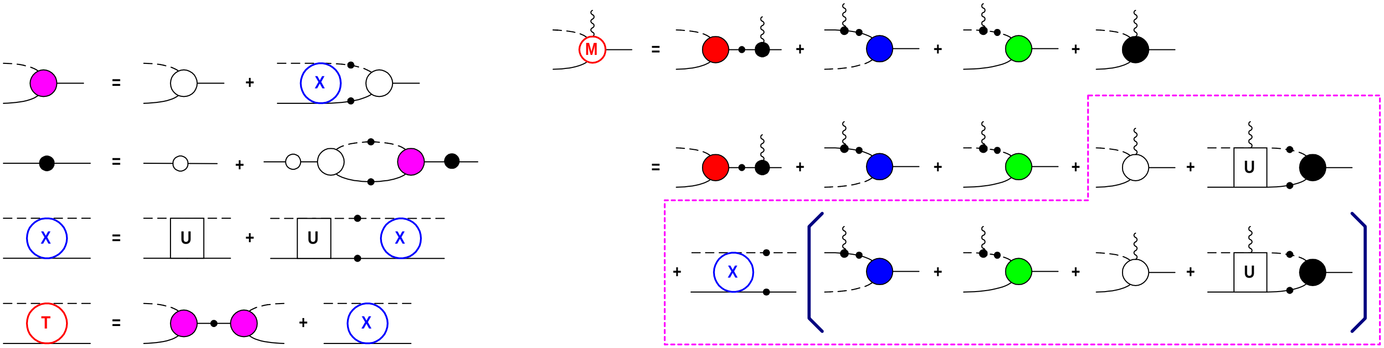
Approximations





- Approximations necessary to make equations manageable.
- **However: Approximations often violate basic theoretical constraints.** (But the parents of the corresponding models love their children anyway...)
- **Case in point:** K-matrix Born approximation destroys gauge invariance. (Constructing a conserved current is not enough for a microscopic model!)
- It is relatively easy and straightforward to fix the gauge-invariance problem in any approximation.
  HH, Nakayama, Krewald, PRC **74**, 045202 (2006)

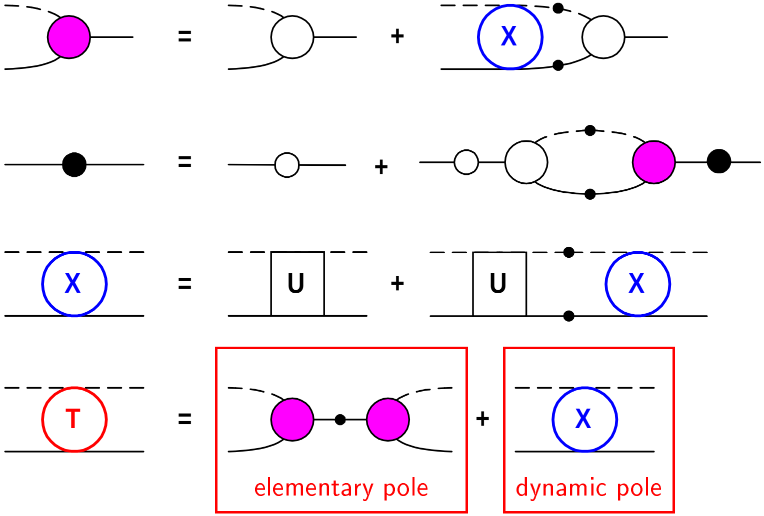
Approximations



- Approximations necessary to make equations manageable.
- **However: Approximations often violate basic theoretical constraints.** (But the parents of the corresponding models love their children anyway...)
- If things go wrong, oftentimes model builders are too quick to look for alternative physical mechanisms instead of blaming the deficiencies of their models.

Elementary vs. Dynamic Resonances

$\pi N \to \pi N$



dressed vertex

dressed propagator

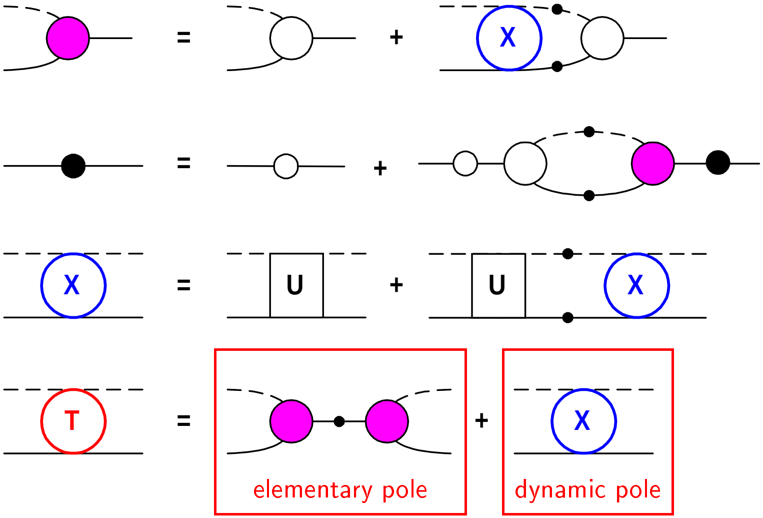
nonpolar T matrix

full T matrix

- **Phenomenologically, one cannot distinguish between elementary and dynamic resonances.**

Elementary vs. Dynamic Resonances

$\pi N \to \pi N$



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- **Theoretically, it makes a huge difference whether a resonance is elementary or dynamic.**

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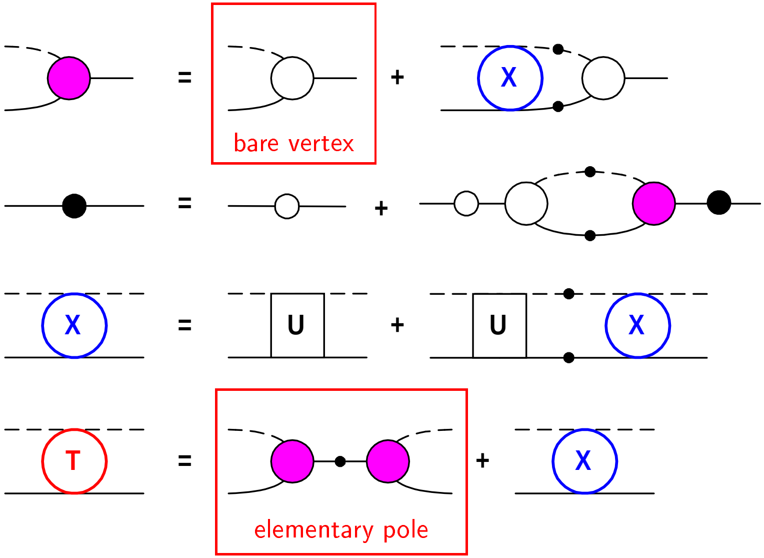
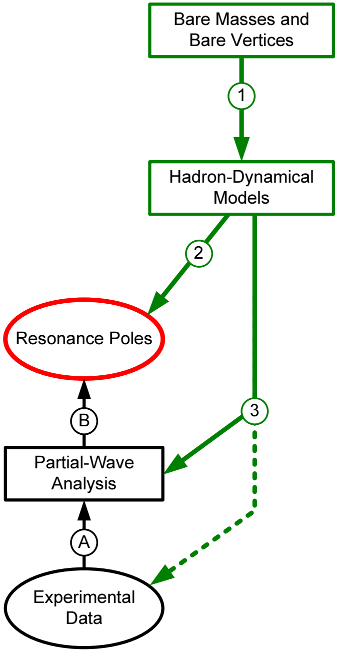


- **Phenomenologically, one cannot distinguish between elementary and dynamic resonances.**
- **Theoretically, it makes a huge difference whether a resonance is elementary or dynamic.**

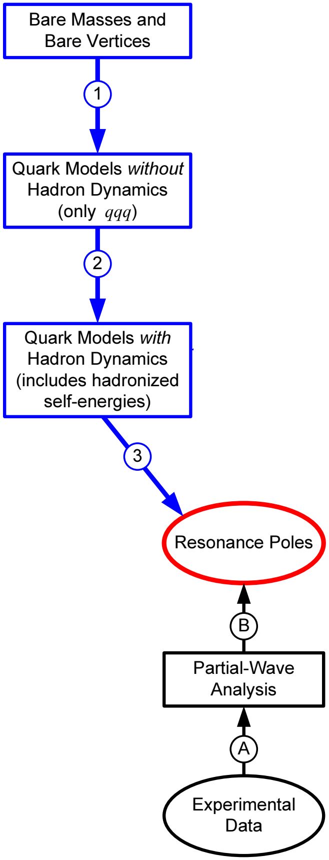
Quark dynamics $\Longrightarrow$ Bare hadronic vertex $\Longrightarrow$ Elementary resonance

Elementary Resonances and Hadron-Dynamical Models



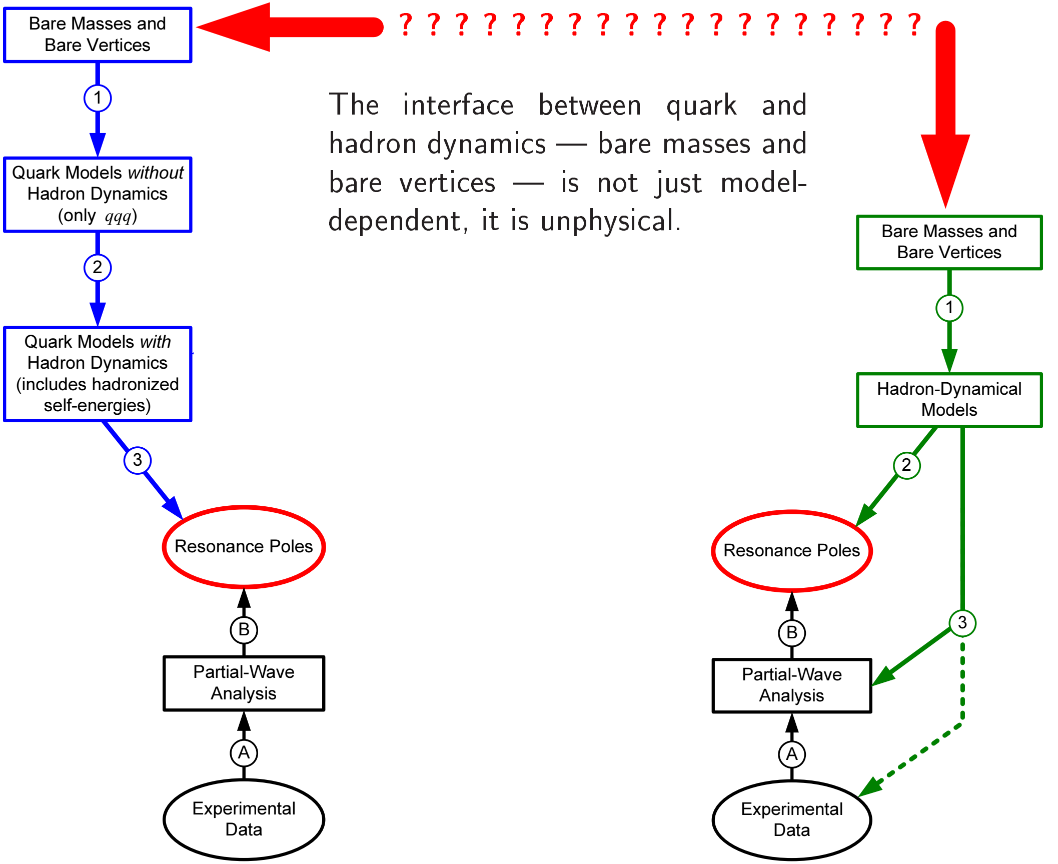
Hadron-Dynamical Model

Elementary Resonances and Constituent Quark Models



Constituent Quark Model

Elementary Resonances

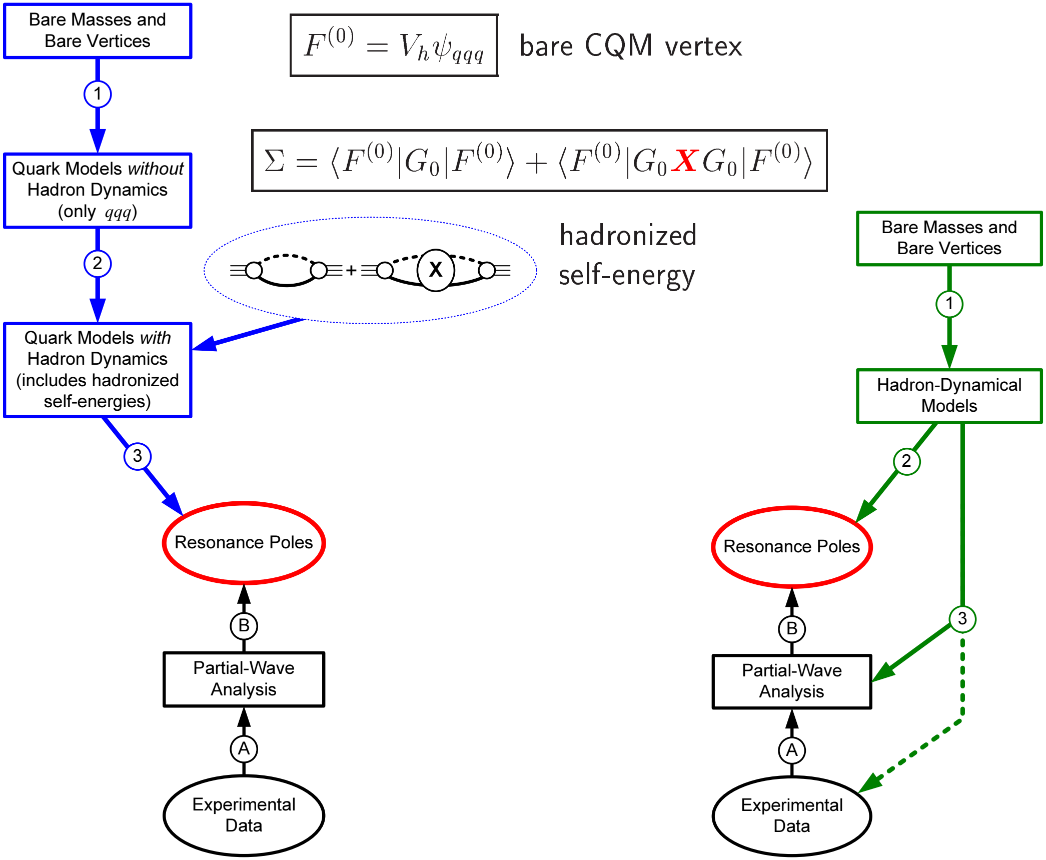


The interface between quark and hadron dynamics — bare masses and bare vertices — is not just model-dependent, it is unphysical.

Constituent Quark Model

Hadron-Dynamical Model

Elementary Resonances



Bare Masses and Bare Vertices

①

Quark Models *without* Hadron Dynamics (only qqq)

②

Quark Models *with* Hadron Dynamics (includes hadronized self-energies)

③

Resonance Poles

Ⓑ

Partial-Wave Analysis

Ⓐ

Experimental Data

$$\boxed{F^{(0)} = V_h \psi_{qqq}}$$ bare CQM vertex

$$\boxed{\Sigma = \langle F^{(0)} | G_0 | F^{(0)} \rangle + \langle F^{(0)} | G_0 \, X \, G_0 | F^{(0)} \rangle}$$

hadronized self-energy

Bare Masses and Bare Vertices

①

Hadron-Dynamical Models

②

③

Resonance Poles

Ⓑ

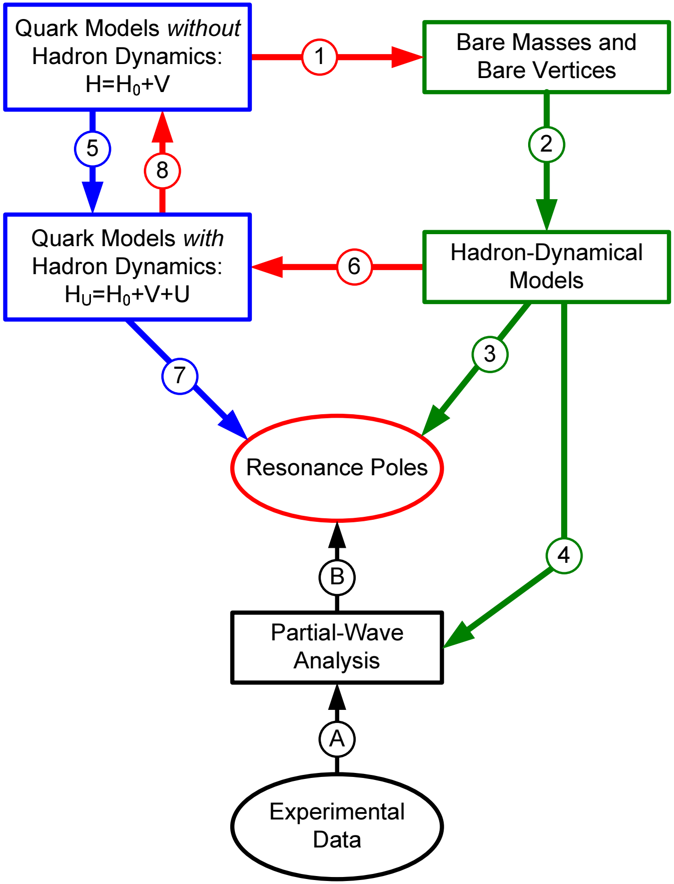
Partial-Wave Analysis

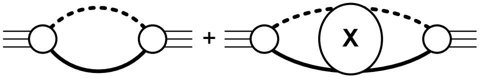
Ⓐ

Experimental Data

Constituent Quark Model **Hadron-Dynamical Model**

Elementary Resonances — Self-Consistency

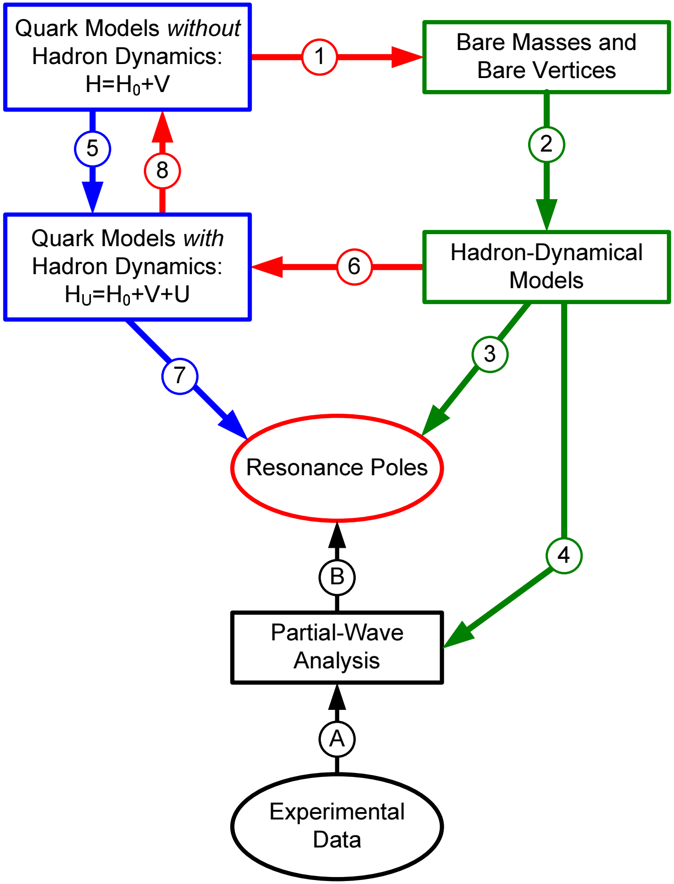




Optical potential

Schematic connection between quark-model calculations (top two boxes on the left) and field-theory-inspired hadron-dynamical models (top two boxes on the right). Both quark-models *with* explicit hadron degrees of freedom and hadron-dynamical models can be used to directly extract resonance-pole parameters (masses and widths). The experimental data are linked to these parameters via partial-wave analyses. The lines labeled 1–4, taken by themselves, describe an approach where there is no feedback between the hadronic dynamics that link to the data and the quark model. The feedback mechanism enters via lines 6 and 8: Line 6 supplies the optical potential into the quark model which may then be used to calculate the physical resonance-pole parameters directly. Comparison of the corresponding values obtained via the hadronic or quark routes 3 or 7, respectively, provide a feedback that, via line 8, can be used to improve, along line 1, the bare input for the hadronic approach.

Elementary Resonances — Self-Consistency



Quark Models *without* Hadron Dynamics: $H=H_0+V$

Bare Masses and Bare Vertices

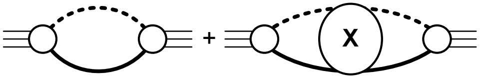
Quark Models *with* Hadron Dynamics: $H_U=H_0+V+U$

Hadron-Dynamical Models

Resonance Poles

Partial-Wave Analysis

Experimental Data



Optical potential

- Self-consistency scheme very elaborate
- Presumably not very practical
- Serves only to show that idea of *defining* an interface between CQM and hadronic models without any correction mechanism is ill-advised

Summary

- There are many structures — dynamical or otherwise — that produce signatures usually attributed to resonant behavior.
- For a truly resonant state one should establish that there is a (positive) time delay, i.e., that the reacting particles spend an enhanced period of time in the interaction region.
- For narrow structures with $\Delta E \gtrsim \Gamma$, the basics of scattering theory need to be revisited.
- Model builders need to be more critical of their own models when assessing the physical consequences of their findings.
- Whether poles of T- or S-matrices are elementary or dynamic in origin cannot be unambiguously decided at the phenomenological level.
- Bare input for hadron-dynamical models cannot be directly related to quark constituent models. (At least not without a *lot* of work.)

Thank You!