

Exotic Baryons And Chiral Soliton Models

Outline

- Philosophical musings on the nature of baryon models
- Chiral soliton models---what they can and cannot do
 - Lessons from the θ^+
 - A straightforward collective quantization appears to naturally give rise to exotics. However, it is not justified from large N_c QCD.
 - Proper treatment in a $1/N_c$ expansion is via meson-baryon scattering but these results are highly model dependent.

Philosophical musings on the nature of baryon models



What *are* baryon models?

- Baryon models are *more* than their lagrangians.
 - Models that are field theoretically based are usually non-renormalizable but in any even not exactly solvable as a field theory
 - To compute, one must specify some approximation scheme (eg. mean-field theory) and often some *ad hoc* prescription (eg. some cutoff scheme) .
 - Models such as quark models & bag models describe bound states and not the observables directly seen in nature (scattering amplitudes for resonances).
 - Additional *ad hoc* assumptions must be added in to connect the model calculations to observables

- The model is the lagrangian plus all of the additional assumptions about how actual observables are computed.
- In context of chiral soliton models it is very important to specify exactly what one means by the model and exactly how one computes with it.
 - The central issue is the extent to which the model is treated self-consistently in a “semi-classical” approximation justified at large N_c and the extent to which it is treated in a more *ad hoc* way.

What are baryon models good for?

- We know that QCD is the correct description of strongly interacting matter including baryons. **Why bother with models?**
 - We cannot solve QCD itself except numerically on the lattice and highly excited states are for the time being intractable on the lattice.
 - Models (NJL, Skyrme, constituent quark, bag...) are used precisely because they are tractable.
 - Models can give *qualitative insights* into the dynamics even though do not reproduce QCD in detail

The cost of tractability

- Although the QCD lagrangian is simple, its dynamics is not; the theory is fully quantum.
- Simple models aimed at QCD dynamics are of necessity rather crude.
- To accurately describe nature even in a fairly limited domain, one must build more and more detail into the models, yielding a baroque structure.



Going for baroque



- Models with considerable detail built in can **describe** nature well. However the extent to which they **explain** it can be debated.
- They are likely to be quite useful in describing a region in which much data has already been fitted.
- The qualitative insight in such models is limited: **[insight, accuracy]≠0** for baryon models.
- Moreover, *a priori* there is little reason to expect them to be useful in extrapolating to qualitatively new phenomena---such as the possible narrow states upon which this workshop focuses.

Going for insight



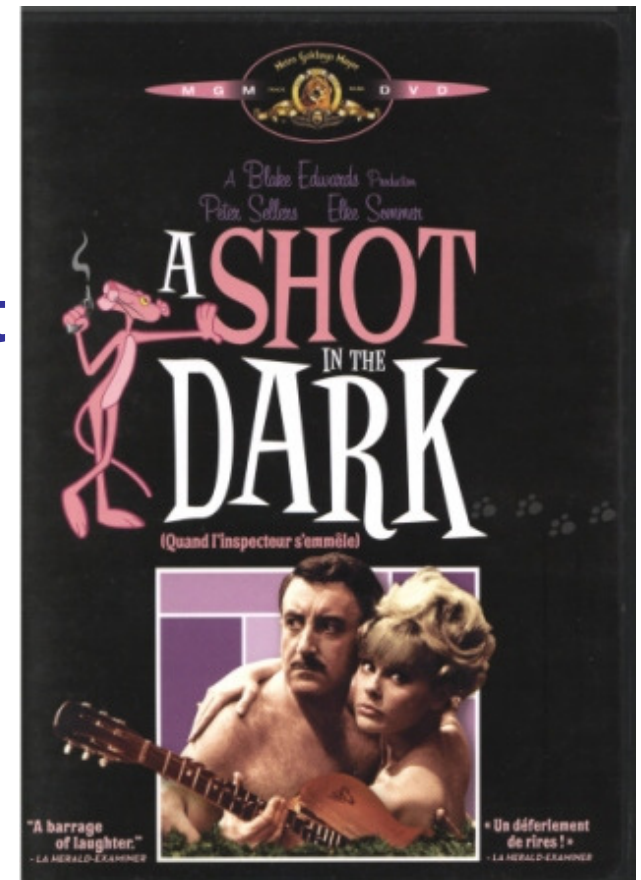
- Crude nature of the simple models means that they cannot be expected to predict accurately subtle effects involving large cancellations.
 - If this is what is going on with the apparently narrow states studied at this workshop, we are unlikely to find a good description with any simple model.
- Simple models are crude but *may* give qualitative insight. This may happen via an inspired guess about the effective degrees of freedom.
 - The constituent quark model is based on the assumption that the dominant degree of freedom are constituent quark. (Is this, in fact, so? What is the evidence?)
- Models can also encode fundamental model-independent things from QCD (or limits of QCD) in a transparent manner.

- **Classic examples:**
 - quark models simply encode isospin or more generally SU(3) symmetry; symmetry breaking is encoded in a consistent way.
 - NJL and sigma models encode approximate chiral symmetry and its spontaneous breaking.
- **Models can identify model-independent results which had not be originally recognized.**
 - Heavy quark (Isgur-Wise) symmetry---an emergent symmetry of QCD---was first seen in quark models
 - Numerous relations which were first discovered in the Skyrme model as relations which were insensitive to the detailed form of the Skyrme lagrangian (Adkins, Nappi & Witten; Adkins, Nappi) were later seen to be general results of large N_c QCD (Gervais & Sakita; Dashen&Manohar) .

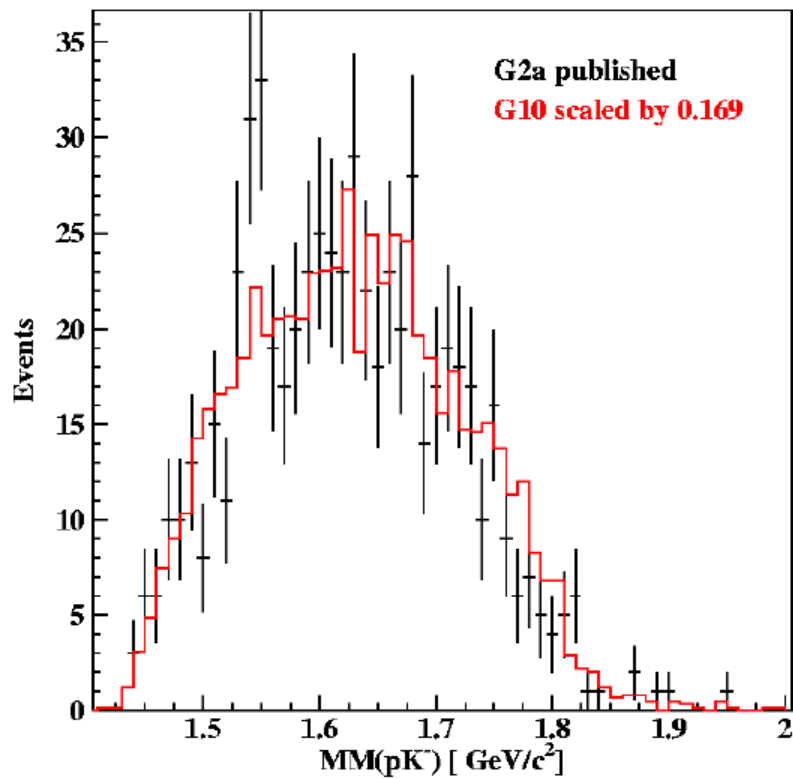
My general philosophical view about any baryon model--- including chiral solitons

- A prediction in a baryon model about new class of exotic phenomena is essentially just **a shot in the dark**, unless the model is actually capturing a general model-independent result not previously noticed.

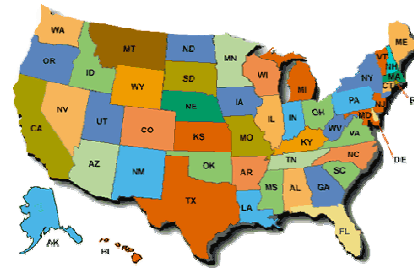
A first glance in the case of the θ^+ it might have seemed that this was what was happening. In fact, it was not.



Chiral soliton models what they can and cannot teach us: Lessons from the θ^+



Key question: what is
the nature of a putative
exotic state?



Is it collective (implying it
can be studied by collective
quantization)?

Chiral Solitons Models



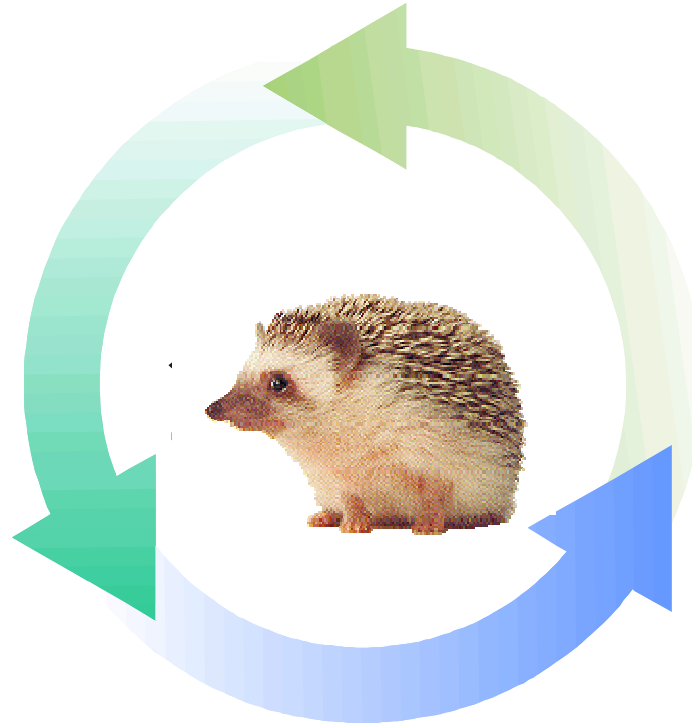
- Early analysis predicting the θ^+ used collective quantization for almost all calculations
- Result for the mass is almost completely insensitive to details of model.
 - Details of profile completely irrelevant to prediction. Only structure of model plus parameters of SU(3) breaking and the identification of the nucleon state in the multiplet

Good news if collective quantization is legitimate

- Is it?
- At a formally level based on the $1/N_c$ expansion, the answer is...
 - Yes for non-exotic states
 - No for exotic states (TDC [PLB531 175 \(2004\)](#); TDC [hep-ph/031219 PRD 70 014011 \(2004\)](#), Princeton Mafia [hep-ph/0309305 NPB684 264 \(2004\)](#)).



- Collective quantization amounts to quantizing the motion of a slowly rotating hedgehog.



- Only legitimate if the motion described is slow at large N_c : there is a Born-Oppenheimer type separation between the collective and intrinsic degrees of freedom. (Moreover the model is only justified at large N_c . Large N_c justifies the classical treatment of the soliton profile as well as collective quantization)

Semiclassical Quantization of SU(3) Solitons

- Assume exact SU(3) Symmetry (m_s perturbatively)

- Hedgehog solution
(assume in u-d subspace)
- $$U = \begin{pmatrix} \overbrace{U_h}^{2 \times 2 \text{ hh}} & 0 \\ 0 & 1 \end{pmatrix}$$

- Follow ANW approach
(Guadagnini 1984...)
- $$U(\vec{r}, t) = A^+(t) U_o(\vec{r}) A^+(t)$$

- Constraint due to
Wess-Zumino term: $\mathbf{J}'_8 = -\frac{N_c B}{2\sqrt{3}}$

- Analog of intrinsic angular momentum for monopole problem.
- Derivable at quark level

- Hamiltonian:

$$H = M_0 + \sum_{A=1,2,3} \frac{J_A'^2}{2I_1} + \sum_{A=4,5,6,7} \frac{J_A'^2}{2I_2}$$

- Two moments of inertia (in SU(2) space and out.)
- No kinetic energy in 8 direction (leave hedgehog unchanged. Note analogy to monopole.)

- **Energies:**

$$M = M_0 + \frac{C_2}{2I_2} + \frac{(I_2 - I_1)J(J+1)}{2I_1I_2} - \frac{N_c^2}{2I_2}$$

$$C_2 = \frac{p^2 + q^2 + pq + 3(p+q)}{3}$$

- **Constraint:**

- Representation must have $Y=N_c/3$
- $(2J+1) = \#$ of states with $S=0$

- **For $N_c=3$ lowest representations**

(p, q)	$\underline{\text{rep}}$	\underline{J}
(1,1)	8	1/2
(3,0)	10	3/2
(0,3)	$\overline{10}$	1/2

- The anti-decuplet is manifestly exotic
- Masses:

$$M_8 = M_0 + \frac{3}{8I_1}$$

$$M_{10} = M_0 + \frac{15}{8I_1}$$

$$M_{\overline{10}} = M_0 + \frac{3}{8I_1} + \frac{3}{2I_2}$$

- SU(3) symmetry breaking added perturbatively.

Problems with rigid rotor quantization for exotic excitations?

Is semi-classical rigid-rotor quantization Kosher for exotic states?

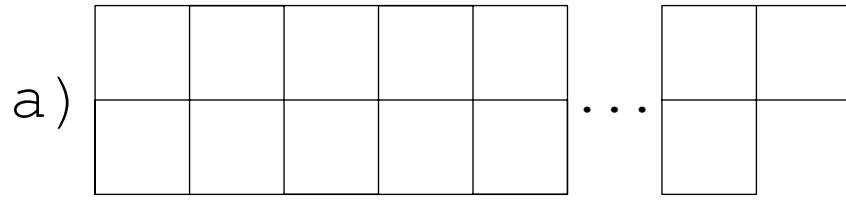
- Superficially yes. It depends on an adiabatic scale separation between collective motion and intrinsic motion, *i.e.* $\tau_{\text{collective}} \gg \tau_{\text{intrinsic}}$
 - Standard semi-classical relation $\tau \sim 1/(\Delta E)$
 - Intrinsic (vibrational motion) $\Delta E \sim N_c^0$
 - For exotic (nonexotic) motion $\Delta E \sim 1/I_1$ ($1/I_2$)
so in both cases $\tau_{\text{collective}} \sim N_c$
 - For both cases $\tau_{\text{collective}} \gg \tau_{\text{intrinsic}}$

- Actually this argument is a complete swindle. To test whether the exotic motion is slow at large N_c we must go to large N_c limit. But $N_c = 3$ was built in to constraint condition!!!
- Redo analysis for arbitrary N_c and take large N_c limit.
 - Issue in identifying states as all representations are larger than for $N_c=3$. (Issue does not arise in $SU(2)$ models)

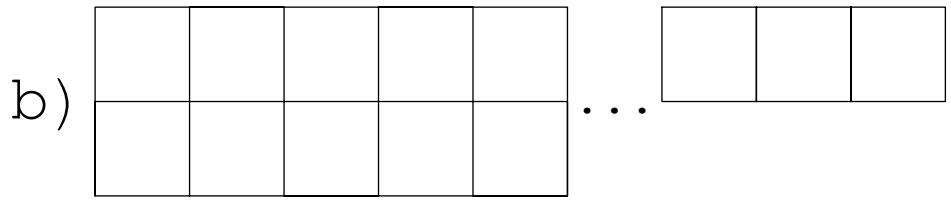
(Standard approach identify representations whose lowest members match on to $N_c=3$ and dismiss other states as large N_c artifacts)

- **Lowest representation** $(p, q) = \left(1, \frac{N_c - 1}{2}\right)$
 (analog of octet) $J = 1/2$ denoted "8"
- **Next representation** $(p, q) = \left(3, \frac{N_c - 3}{2}\right)$
 (analog of decuplet) $J = 3/2$ denoted "10"
- **Lowest representation** $(p, q) = \left(0, \frac{N_c + 3}{2}\right)$
 containing $s = +1$ state
 (analog of antidecuplet) $J = 1/2$ denoted " $\overline{10}$ "

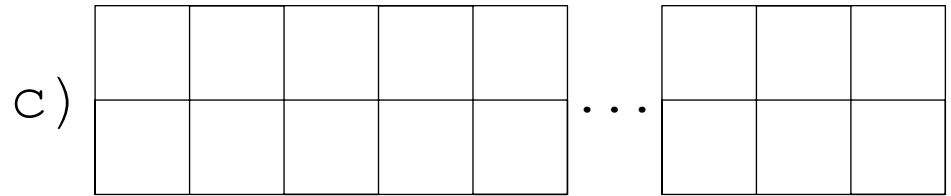
"8"



"10"



"10"



- Use mass formula from before:

- Nonexotic excitations

$$M_{"10"} - M_{"8"} = \frac{3}{2I_1} \sim \frac{1}{N_c} \quad \tau_{\text{collective}} \sim N_c \gg \tau_{\text{intrinsic}}$$

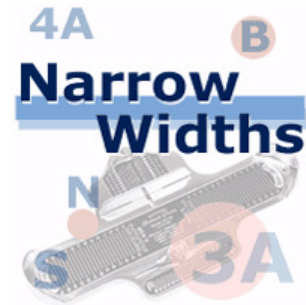
Adiabatic : collective quantization justified.

- Exotic excitations

$$M_{"10\bar{0}"} - M_{"8"} = \frac{3 + N_c}{4I_2} \sim N_c^0 \quad \tau_{\text{collective}} \sim N_c^0 \sim \tau_{\text{intrinsic}}$$

Nonadiabatic: collective quantization not justified!!!

There are other ways to see this:



Widths

- The early analysis stress narrow numerical width to justify approach self-consistently.
- **there is still a fundamental *formal* issue.**
 - If approach is legitimate it should give exact mass at large N_c . Otherwise ad hoc corrections need to be added.
 - This implies width must be zero at large N_c .
If not, the state doesn't really exist and concept of an exact mass is silly. Alternatively, view width as an imaginary contribution to mass

- Width computed from coupling constant which in turn depends on asymptotic profile function and collective wave function. Explicit computation in the context of rigid rotor quantization was done by Praszalowicz:

$$\left| \langle \theta^+, s = \downarrow | \hat{O}_{K3} | N, s = \downarrow \rangle \right|^2 = \frac{9(N_c + 1)}{(M_N + M_{\theta^+})^2 (N_c + 3)(N_c + 7)}$$

$$\times \left[G_0 - \frac{N_c + 1}{4} G_1 \right]^2 p^2$$

$$G_0 \sim N_c^{3/2} \quad G_1 \sim N_c^{1/2}$$

Where the operator gives the coupling to a Kaon in direction J.

- Including phase space and scaling one deduces that

$$\Gamma \sim N_c^0$$

- This indicates a formal inconsistency
- Whether the width calculated this way is large or small is a question of detail in the model---not a general principle
- At a formal level this is not a well-defined narrow state---it is a resonance and should be computed via a scattering amplitude.

Large N_c Consistency

- Reason to chiral soliton prediction seriously in first place was model insensitivity; this typically means that relation derivable directly by large N_c consistency rules.
- These rules known for three flavor QCD.
 - Give exactly the same states as in a large N_c Quark model. (Dashen, Jenkins Manohar 94).
 - Exotic collective states not predicted by this model independent approach. But all nonexotic ones are.
- Existence of exotic states and their properties are NOT model independent. But semi-classical quantization is only justified for model-independent quantities.

- Previous arguments show that at large N_c , the rigid rotor quantization fails for exotic states but works for nonexotic states. Why?
- Fundamental reason---mixing of collective and intrinsic (vibrational) modes at leading order in N_c due to Wess-Zumino term. Collective and vibration modes not orthogonal.
- This can be illustrated in toy models---analog of collective quantization works only when vibrations and rotations decouple for reasons other than N_c .
- **The formal failure at large N_c of collective quantization for exotic states should not be controversial at this point.**

Do chiral soliton models have a spectrum of narrow exotics?



- Problem was with collective quantization for exotic states---this does **not** mean that this class of models are incapable of describing exotic states.
 - One can study meson-baryon scattering amplitudes in the models and look for resonances. This is completely consistent with $1/N_c$ expansion at leading order.
- However, when you do this the question of whether one gets narrow exotics or not is a question of model dependent detail. It does not automatically follow from the structure of the model or anything tying the model to QCD.
 - The properties of exotics if they exist are similarly model-dependent

- Diakonov & Petrov ([arXiv:0812.3418](https://arxiv.org/abs/0812.3418)) have recently been advocating approaches that yield narrow exotics:
 - **K-N scattering** The leading order approach of the Princeton mafia was reanalyzed. While the phase shift does not go thru π , it is claimed that there is pole well off axis. Thus there is a **broad** exotic resonance
 - $1/N_c$ corrections are claimed to be able to greatly reduce this width
 - It has also been argued that more generally including **SOME** $1/N_c$ effects (after all N_c is only 3!!) in some variant of collective quantization can yield narrow pentaquarks

- While one may quibble about various aspects of this attempt (*and I do*), it is important to realize that they illustrate my main point:

The results for exotics depend sensitively on details of the model in this case in terms of the *ad hoc* rules by which one calculates $1/N_c$ corrections.

- Thus the models in the exotic sector do not simply encode some underlying general features of QCD requiring exotics with rather particular properties.