Unified multichannel unitary amplitudes for hadro- and photoproduction

Extensions of Dick's Chew-Mandelstam/SAID approach

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with special thanks to Dick Arndt 1933/01/03 – 2010/04/10

Data Analysis Center
Center for Nuclear Studies
The George Washington University

Sixth International Workshop on Pion-Nucleon Partial-Wave Analysis
and the Interpretation of Baryon Resonances
The George Washington University
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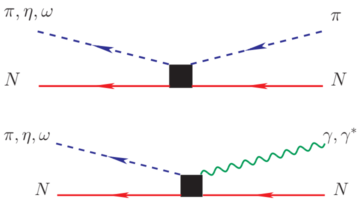
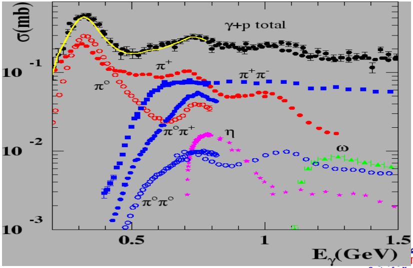
Overview of SAID

- Data Analysis Center/Center for Nuclear Studies
  - SAID[a]: suite of programs to analyze $2 \longrightarrow 2$ & 3 body data
  - Routines: database, fit, and analysis
  - Dedicated effort: analyze/interpret the terabytes of experimental data issuing from Bonn, JLab, Lund, Mainz, ...
- Reactions: $\pi N \longrightarrow \pi N, \eta N, \ldots;$ $\gamma N \longrightarrow \pi N, \eta N, \omega N, \ldots$
- Objectives: **model independent amplitudes**; unified hadro- & electo-prod; resonances & QCD$\longmapsto$
- Uses
  - Verify models vs. data
  - Experimental planning
  - Simulations/event gen: Astrophysics; Nuclear reactions; Detector design/calibration

[a] Web: `http://gwdac.phys.gwu.edu/`
ssh: `ssh -X said@said.phys.gwu.edu` `[passwordless]`





Outline

1. Analysis overview
   - Models vs. parametrizations
   - SAID PWA
2. Formalism
   - Unitarity
   - Parametrizations
3. Results $\gamma N \to \pi N, \eta N$
   - Exploratory study
   - Pion Photoproduction

Outline

Models vs. parametrizations

Working definition

Objective: determine the resonance spectrum of the nucleon by locating the poles of the T matrix

Context: What is the assumed particle content of the theory?

- **Model**: assume stable (π, η, ω, N) & unstable ($\sigma, \rho, N^*, \Delta^*$, etc.) $\rightarrow$ calculate observables $\rightarrow$ adjust resonance contribution ("independently in each partial wave") and non-resonant bare couplings
  Limitations: several hundred parameters; must assume resonance spectrum; model dependence
- **Parametrization**: assume stable (π, η, ω, N) only $\rightarrow$ fit data via unitary parametrization $\rightarrow$ deduce resonance spectrum
  Limitations: underlying Lagrangian not specified; microscopic content not unique

Complementarity: models and parametrizations are complementary approaches; each encodes dynamics of final state interactions and channel coupling

πN models and parametrizations (no particular order)

Models
Carnegie-Mellon Berkeley
Jülich
Giessen
EBAC
Chiral-Unitary

Parametrizations
Karlsruhe-Helsinki*
MAID (photoprod. only)
SAID
Bonn-Gatchina (Fit SAID $\pi N \rightarrow \pi N$ ampls.)

SAID ‘model independent’ parametrization

Dick’s work in context

- SAID = parametrization of scattering & reaction observable *data*
  SAID is distinguished as being the only active analysis to directly fit the $\pi N \longrightarrow \pi N$ data [inactive: K-H & C-M-B analyses] including fixed-t constraints

Must be remedied through parallel efforts

- Coupled-channel/multichannel Chew-Mandelstam parametrization (detailed subsequently)
  - Dick’s work in hadroproduction $\pi N \longrightarrow \pi N, \eta N, \pi\pi N$
  - Our recent extension to $\gamma N \longrightarrow \pi N$
- Each partial wave is parametrized by small number of parameters, 200 all together
  - Related by fit to data
  - Born terms included in photoproduction param.
- CM parametrization
  - Consistent with QFT
  - No non-resonant/resonant separation [Fearing/Scherer/Gegalia PRC62(2003)/EPJA44(2010)]
  - Correct analytic structure of two- and three-body unitarity branch points

Outline

SAID partial wave amplitudes

Database/parametrization

Process	Data	Channels	Comment
$\pi N \to \pi N$	55,823	πN,ηN,$\pi\Delta$,ρN	w/DR constraints; PRC 74(06)
$\pi N \to \pi\pi N$	263,343	$\pi\Delta$,ρN,σN,πN^*	indep of πN; isobar PRD 30(84)
$KN \to KN$	8,043	KN, $K\Delta$	PRD 31(85)
$\gamma N \to \pi N$	25,501	πN	Born; PRC 66(02)
$\gamma p \to \eta p$	6,151	πN	online
$\gamma p \to \eta' p$	,871	πN	online
$\gamma N \to KY$	5,828	KN	online; Web: Bennhold PRC 61(99)
$eN \to e'\pi N$	107,078	πN	online
$NN \to NN$	39,075	NN, $N\Delta$	PRD 45(92)
$\pi D \to \pi D$	1,914	$N\Delta$	PRC 50(94)
$\pi^+ D \to pp$	6,001	$N\Delta$	PRC 48(93)

Outline

Unitarity constraint on T

- S matrix definition

$$S_{\alpha\beta}(E) = \langle \mathbf{k}_\alpha \alpha | S | \mathbf{k}_\beta \beta \rangle$$
$$= \delta^{(3)}(\mathbf{k}_\alpha - \mathbf{k}_\beta)\delta_{\alpha\beta} + 2i\pi\delta(E_\alpha - E_\beta)\langle \mathbf{k}_\alpha \alpha | T | \mathbf{k}_\beta \beta \rangle$$

- Unitarity constraint on T from $S^\dagger S = SS^\dagger = 1$

$$T_{\alpha\beta} - T^\dagger_{\alpha\beta} = 2\pi i \sum_\gamma \int d^3k_\gamma \, T^\dagger_{\alpha\gamma} \delta(E_\gamma - E_\beta) T_{\gamma\beta}$$

$$T_{\alpha\beta} - T^\dagger_{\alpha\beta} = 2\pi i \sum_\gamma \int d\Omega_\gamma \int_0^\infty dk_\gamma \; T^\dagger_{\alpha\gamma} \delta[(k_\gamma^2 + m_{\gamma 1}^2)^{1/2} + (k_\gamma^2 + m_{\gamma 2}^2)^{1/2} - E_\beta] T_{\gamma\beta}$$

$$= 2i \sum_\gamma \int d\Omega_\gamma \, T^\dagger_{\alpha\gamma} \rho_\gamma T_{\gamma\beta}$$

$$\rho_\gamma = \theta(W - m_{\gamma +}) \frac{\pi \bar{k}_\gamma E_{\gamma 1} E_{\gamma 2}}{W}.$$

NB: Presence of Heaviside $\theta(W - m_{\sigma +}) \longrightarrow$ threshold branch points

- "Maximal analyticity" $\Longrightarrow$ real branch points
- Branch points $\notin \mathbb{R}$ are **model dependent**

Unitarity

Branch points

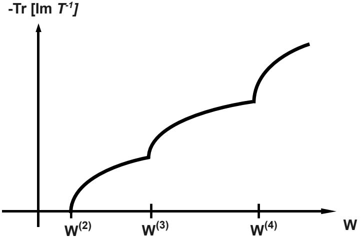
- Unitarity (conservation of probability) $\leftrightarrow$ Analyticity

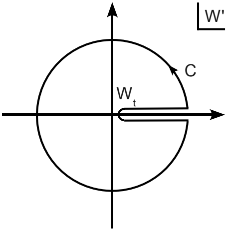
$$S = 1 + 2i\rho T$$

$$S^\dagger S = SS^\dagger = 1$$

$$T - T^\dagger = 2iT\rho T^\dagger \qquad \rho = \text{density of states}$$

$$\text{Im}\, T^{-1}(W) = -\rho \qquad \text{Disc}\ T^{-1} = -2i\rho$$





- Ignoring poles and unphysical branch points

Outline

Parametrizations

Complexity $\longrightarrow$ simplicity

Objective

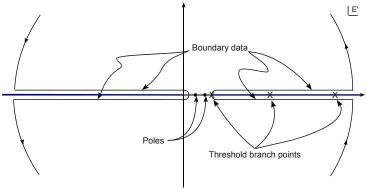
Determine simple functional form to reduce data observables to **model independent** amplitudes

Domain of analyticity

- Unitarity $\longrightarrow$ threshold branch points
- Bound-state poles, $\text{Re}\, E = W < W_t$
- Large W behavior $\longrightarrow$ subtractions

Simplify or 'implement functionally'

- Stage 1: 'remove' branch points $\longrightarrow$ Heitler K matrix
- Stage 2: 'remove' poles $\longrightarrow$ Chew-Mandelstam K matrix



Heitler K-matrix form

'Removing' branch points

Levy; Gunson/Taylor; Zimmerman; ...

Unitarity constraint determines on-shell K uniquely

$$T^{-1} = \text{Re } T^{-1} + i\text{Im } T^{-1} \qquad \text{Im } T^{-1} = -\rho$$
$$= K^{-1} - i\rho \qquad \text{Re } T^{-1} \equiv K^{-1}$$

Heitler equation

$$K^{-1} \times \{K^{-1} = T^{-1} + i\rho\} \times T^{-1}$$

$$\boxed{T = K + iK\rho T}$$

- $\text{Im } T^{-1}$ saturates unitarity, determines branch points
- $\text{Re } T^{-1} \equiv K^{-1}$ differentiable (analytic) in physical region
  $\Longrightarrow$ *must have no branch points*
- K meromorphic function of W, may possess poles

$$K = \frac{1}{T^{-1} + i\rho} = T \frac{1}{1 + i\rho T}$$

Chew-Mandelstam K-matrix form

'Removing' poles *Basdevant/Berger; Edwards/Thomas; Arndt/Ford/Roper*

- Motivation — Im $T^{-1} = -\rho$ [UCT]

$$T^{-1} = K^{-1} - i\rho$$
$$= \{K^{-1} - \mathrm{Re}\, C\} + \{\mathrm{Re}\, C - i\rho\}$$
$$= K_{CM}^{-1} - C$$
$$[UCT] \implies \mathrm{Im}\, C = \rho \text{ 'disperse' it – next slide}$$

- Chew-Mandelstam K_{CM} vs. K matrix

$$K^{-1} = K_{CM}^{-1} - \mathrm{Re}\, C$$
$$K = [1 - K_{CM} \mathrm{Re}\ C]^{-1} K_{CM}$$

- For polynomial K_{CM}, in general, has Det$[1 - K_{CM}\mathrm{Re}\ C] = 0$
- No need for explicit poles in K_{CM}, but possible to include explicit poles
- K_{CM} poles are 'dressed'
- **Query**: Are K_{CM} poles related to bare poles of dynamical models (*eg.* $S_{11}(1535)$ & $P_{33}(1232)$)? (arXiv:1101.0621 w/R. Workman)

T and K poles

*R. Workman & MP Phys. Rev. C **79**, 038201 (2009)*

ℓ_{JT}	T poles		K poles	
S_{11}	$(1500, 50)$	$(1650, 40)$	1535	1675
P_{11}	$(1360, 80)$	$(1390, 80)^{\dagger}$	–	–
P_{13}	$(1665, 175)$		–	
D_{13}	$(1515, 55)$		–	
D_{15}	$(1655, 70)$		1760	
F_{15}	$(1675, 60)$	$(1780, 130)$	–	–

Table: Pole positions in complex energy plane of T and K matrix for the $\pi N \longrightarrow \pi N$ reaction from SAID (SP06) for isospin $T = \frac{1}{2}$ partial waves. Each T pole position is expressed in terms of its real and imaginary parts $(M_R, -\Gamma_R/2)$ in MeV. Only K matrix pole positions which satisfy 1.1 GeV $< W <$ 2.0 GeV are considered. †This pole is located on the second Riemann sheet.

Chew-Mandelstam function $C_\ell(W)$

Dispersion relation representation

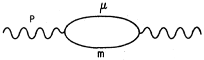


FIG. 1. Feynman graph representing the Chew-Mandelstam function for the scattering of stable particles with masses m and μ.

$$C_\ell(W) = \int_{W_t}^{\infty} \frac{dW'}{\pi} \frac{\text{Im } C_\ell(W')}{W' - W} - \int_{W_t}^{\infty} \frac{dW'}{\pi} \frac{\text{Im } C_\ell(W')}{W' - W_s}$$

$$\text{Im } C_\ell(W) = \left(\frac{W - W_t}{W - W_s} \right)^{\ell + 1/2}$$

$$C_\ell(z) = \int_0^1 \frac{dx}{\pi} \frac{x^{\ell+1/2}}{x - z}$$

$$z = \frac{W - W_t}{W - W_s}$$

- Proper threshold behavior $\sim \sqrt{W - W_t}$
- Two-particle channel cut
- Unstable (quasi two-body) particle channels $W_t = \text{Re } W_t + i\text{Im } W_t$
- NO left-hand cut

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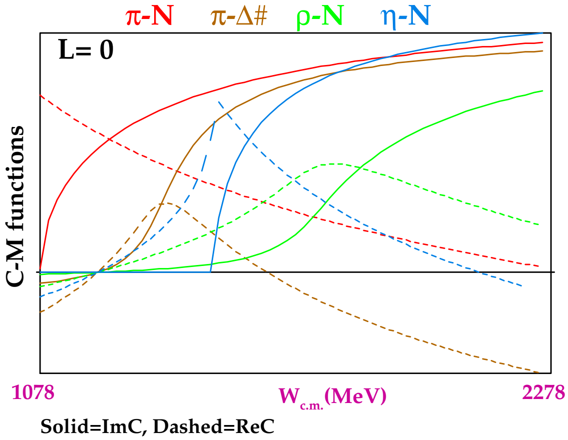
Chew-Mandelstam function $C_\ell(W)$

Dispersion relation representation



Energy-dependent solutions

Chew-Mandelstam K-matrix parametrization

Partial wave $T = \rho^{1/2} K_{CM} [1 - C K_{CM}]^{-1} \rho^{1/2}$ parametrized:

$$K_{CM} = \begin{pmatrix} K_{ee} & K_{ei} \\ K_{ei} & K_{ii} \end{pmatrix}$$

K_{ee}	$\pi N \to \pi N$	element
K_{ei}	$\pi N \to i$	vector, $i = 1, \ldots, N_{ch} - 1$
K_{ij}	$N_{ch} - 1 \times N_{ch} - 1$	matrix

Parametrization

$$K_{ee} = \sum_{n=0}^{5} p_n z^n + \frac{p_{r1}^2 + p_{r2}^2}{M_R - W} \qquad z = W - (m_\pi + m_N)$$

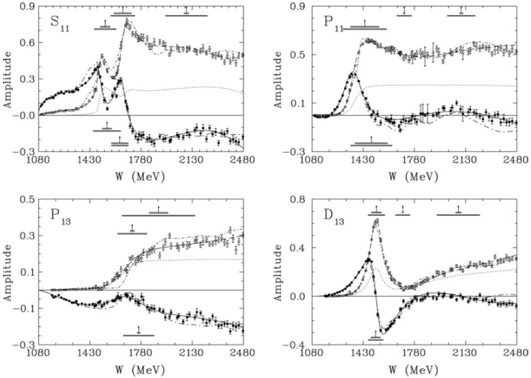
$$K_{ei} = \sum_{n=0}^{3} p_{n+5i} z^n + \frac{p_{r3} p_{r4}}{M_R - W} \qquad z = W - (2m_\pi + m_N)$$

$$K_{ij} = \delta_{ij} \left(p_{4+5i} + p_{5+5i} z + \frac{p_{r5}^2}{M_R - W} \right) \qquad z = W - (2m_\pi + m_N)$$

SAID partial waves

'SP06' Solution

Solution names: 'XX##', 'X###' — SP06 = Spring 2006, FA02 = Fall 2002, . . .



Each $\pi N \longrightarrow \pi N$ partial wave [shown here: S_{11}, P_{11}, P_{13}, D_{13}] shows

- energy-dependent
- single-energy (SE) solutions.

Single energy are solutions are sometimes called, confusingly, "energy-independent."

Outline

Unified hadro/photoproduction parametrization

Motivation *MP & R. Workman, Phys. Rev. C* ***82****, 035202 (2010)*

1. Current SAID photoproduction parametrization form lacks full multichannel unitarity
2. Despite χ^2/datum $\sim$ 2, problems with *eg.* $\gamma p \to \eta p$
3. Extend SAID Chew-Mandelstam approach used in hadronic sector to electromagnetic
   - Hadronic sector $\pi N \to \pi N$ & $\pi N \to \eta N$ (untouched)
     4 channel Chew-Mandelstam approach $\{\pi N, \eta N, \pi\Delta, \rho N\}$
   - Electromagnetic sector $\gamma N \to \pi N$ & $\gamma N \to \eta N$
     Introduce 4 channel Chew-Mandelstam approach $\{\pi N, \eta N, \pi\Delta, \rho N\}$ with same hadronic "rescattering" matrix
   - Hadronic subprocess dominant in photoproduction $\to$ 'backconstrain' hadronic amplitudes
   - Obtain $\eta-$photoproduction amplitude with *resonant* phase – various model approaches [*Green & Wycech; Kaiser et. al.; Aznauryan*] yield wide range of phases
4. Study baryon resonances in ηN channel
5. Study η-sector physics

‘Old’ SAID K-matrix formalism

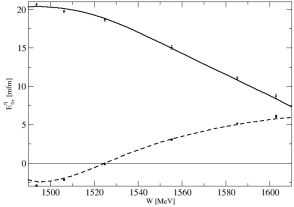
compare to *Green & Wycech PRC**55**(1997); Arndt et. al. PRC**58**(1998); Green & Wycech PRC**60**(1999)*

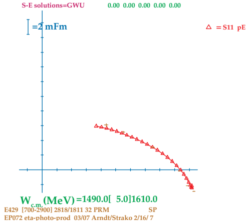
Two-channel formalism (can be generalized to N 2-body channels)

$$T_{\pi\gamma} = (1 + iT_{\pi\pi})K_{\pi\gamma} + iT_{\pi\eta}K_{\eta\gamma} \qquad T_{\eta\gamma} = (1 + iT_{\eta\eta})K_{\eta\gamma} + iT_{\eta\pi}K_{\pi\gamma}$$

Reduction via hadronic matrix to various forms

$$\begin{aligned} T_{\eta\gamma} &= \left(K_{\eta\gamma} - \frac{K_{\pi\gamma}K_{\eta\eta}}{K_{\pi\eta}} \right)(1 + iT_{\eta\eta}) + \frac{K_{\pi\gamma}}{K_{\pi\eta}} T_{\eta\eta} \\ &= A(W)(1 + iT_{\eta\eta}(W)) + B(W)T_{\eta\eta}(W) \qquad \text{Form 1} \\ &= A'(W)(1 + iT_{\pi\pi}(W)) + B'(W)T_{\pi\pi}(W) \qquad \text{Form 2} \end{aligned}$$





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Anticipate resonant phase in region 1.49 GeV $\lesssim W \lesssim$ 1.6 GeV

'New' SAID Chew-Mandelstam parametrization

$\pi-$ & $\eta-$photoproduction *MP & R. Workman Phys. Rev. C **82**, 035202 (2010)*

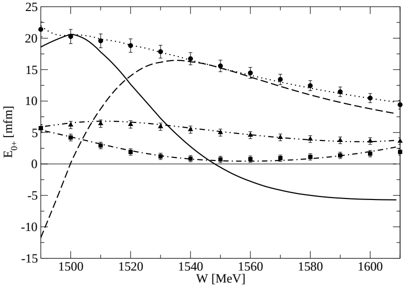
Current hadronic parametrization fits $\pi N \to \pi N, \pi N \to \eta N$, DR, ...

$$T_{\alpha\beta} = \sum_\sigma [1 - \overline{K}C]^{-1}_{\alpha\sigma} \overline{K}_{\sigma\beta} \;\; \longrightarrow \;\; \text{CM 'rescattering' matrix}$$

Generalized to photoproduction (hadronic matrix fixed by above)

$$T_{\alpha\gamma} = \sum_\sigma [1 - \overline{K}C]^{-1}_{\alpha\sigma} \overline{K}_{\sigma\gamma}$$

Perform fit at amplitude level to Re E^π_{0+}, Im E^π_{0+} & $|E^\eta_{0+}|$



Amplitudes
Solid curve: Re E^η_{0+}
Dashed curve: Im E^η_{0+}
Dot-dashed curve: Re E^π_{0+}
Double dot-dashed curve: Im E^π_{0+}
Dotted curve: $|E^\eta_{0+}|$

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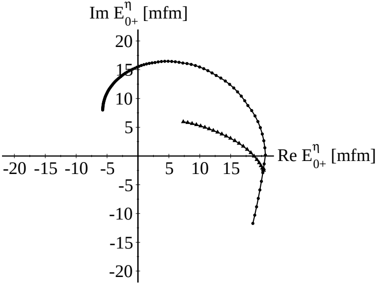
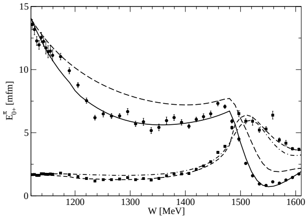


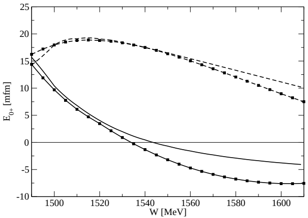
Argand diagram
Triangles – 'Old' Heitler K matrix (non-unitary) formalism
Circles – 'New' Chew-Mandelstam K matrix (non-unitary) formalism

Comparison to MAID

E^{π}_{0+} SAID and MAID solutions



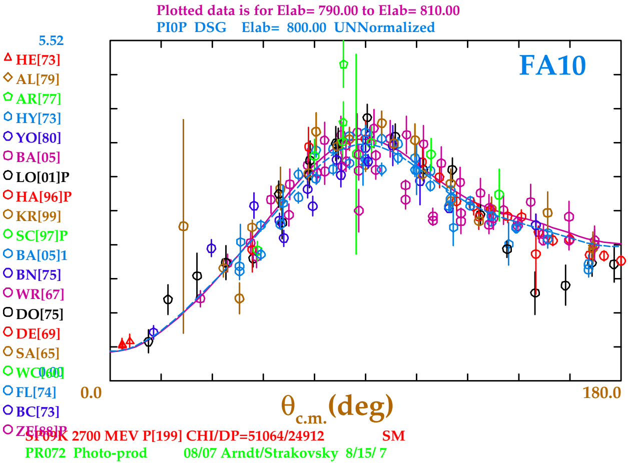
Refitting with MAID E^{π}_{0+} and same $|E^{\eta}_{0+}|$



Outline

CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



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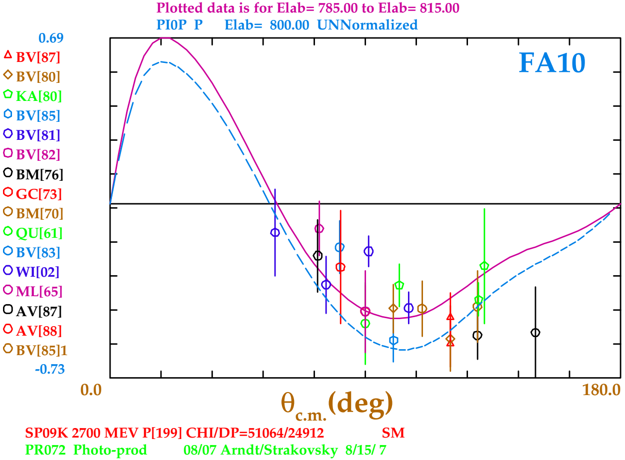

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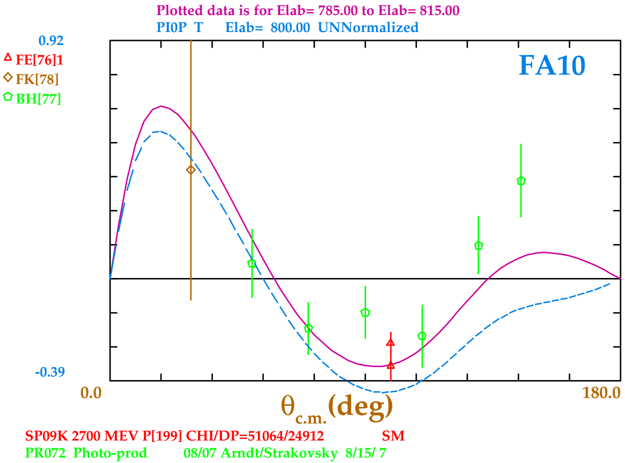

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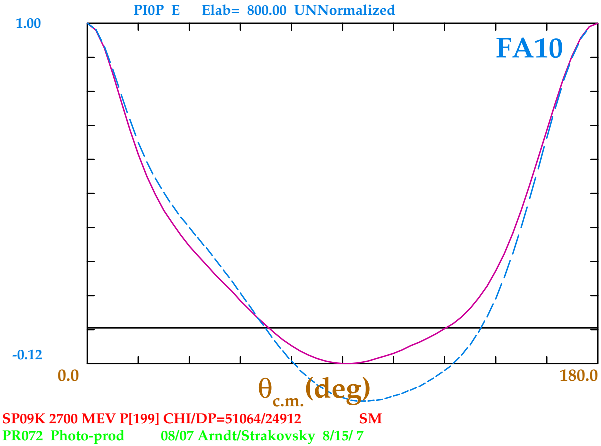

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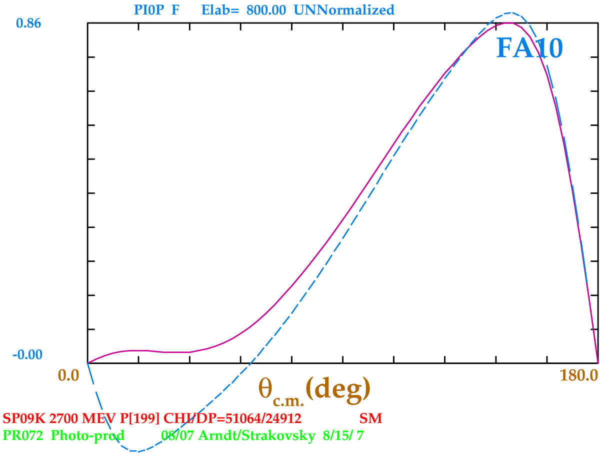

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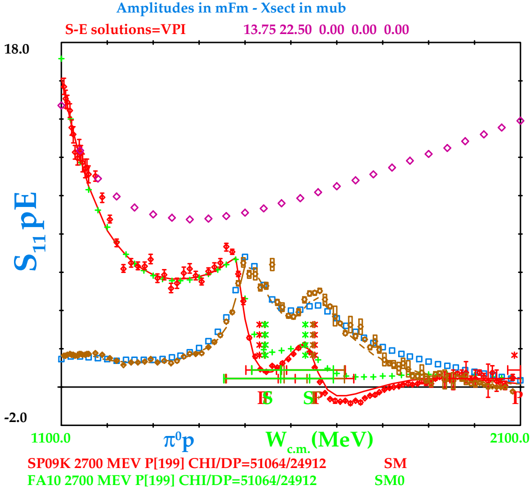

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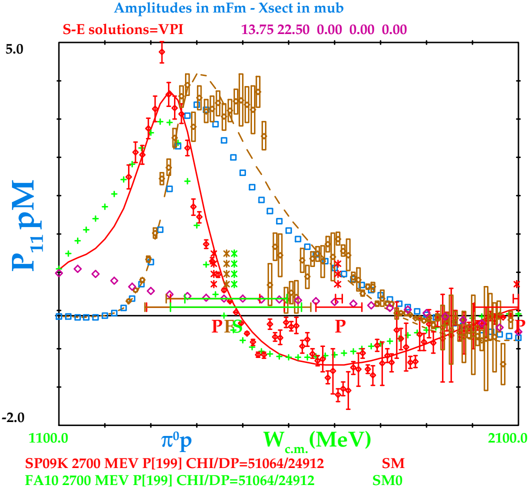

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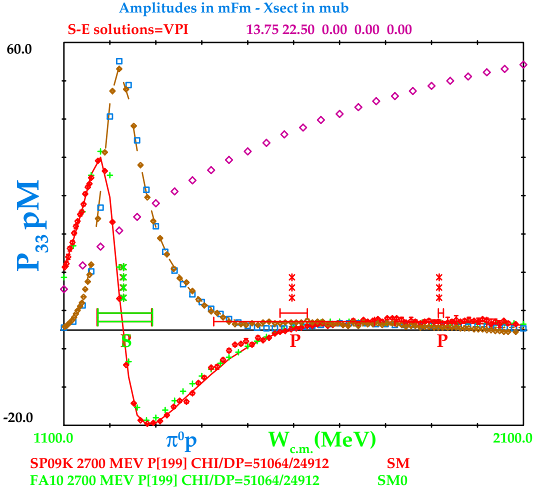

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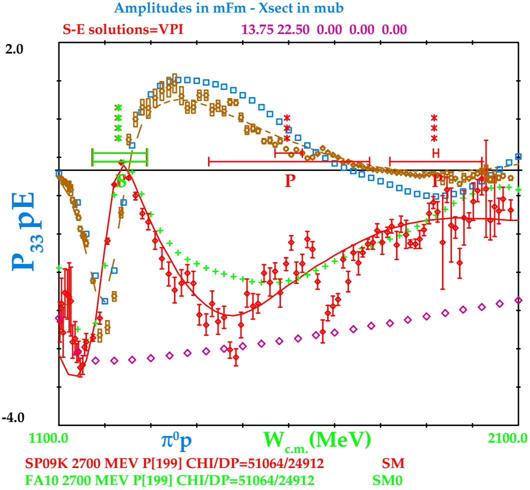

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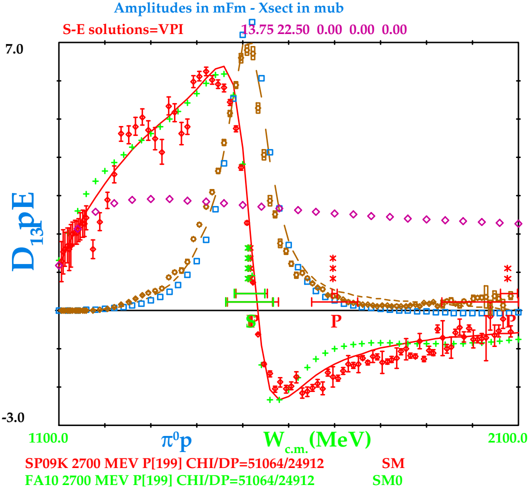

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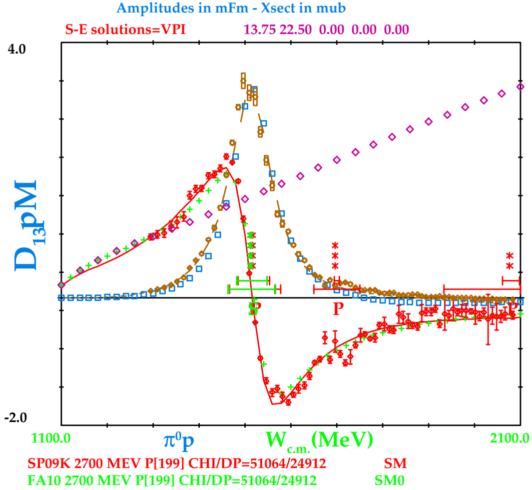

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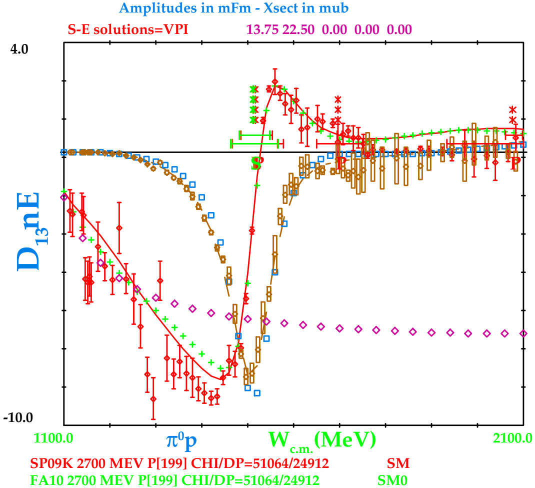
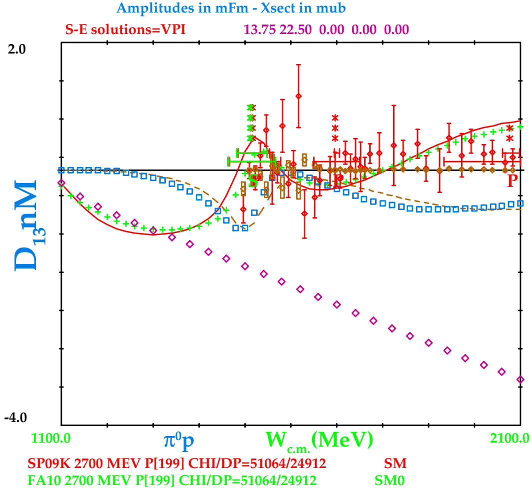


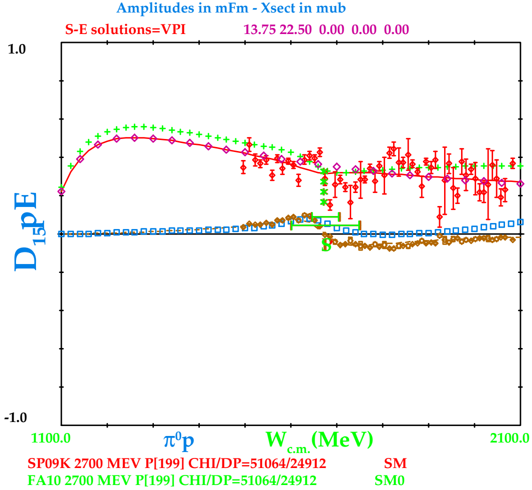
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



CM form for $\gamma N \longrightarrow \pi N$

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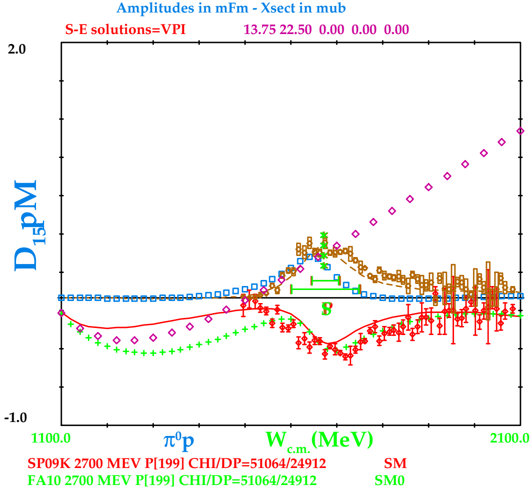
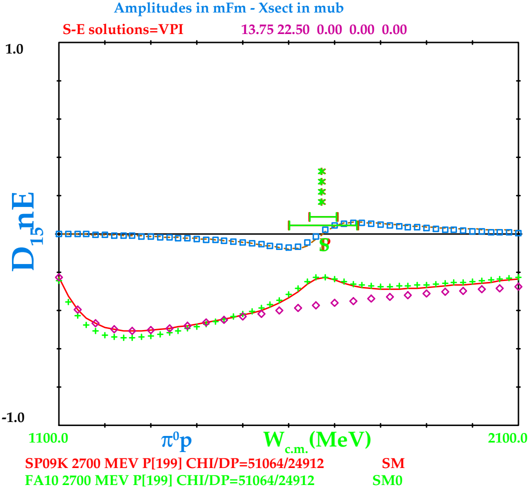


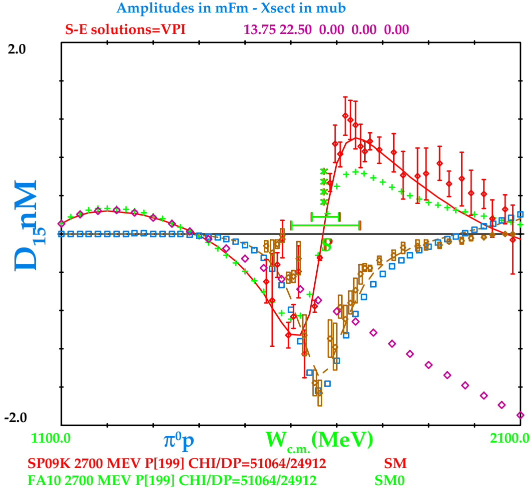
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



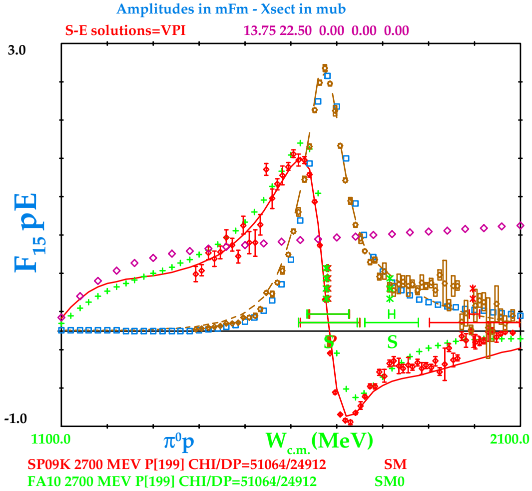
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



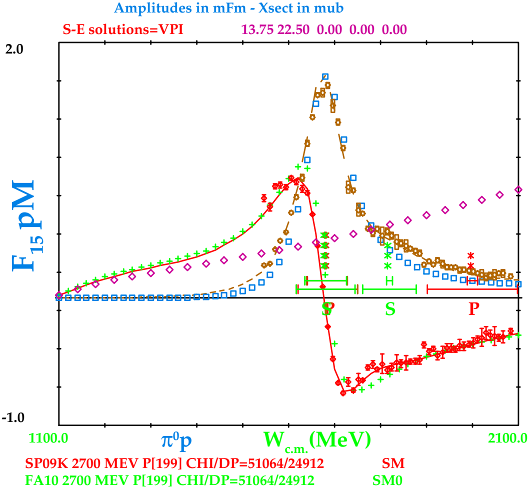
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



CM form for $\gamma N \to \pi N$

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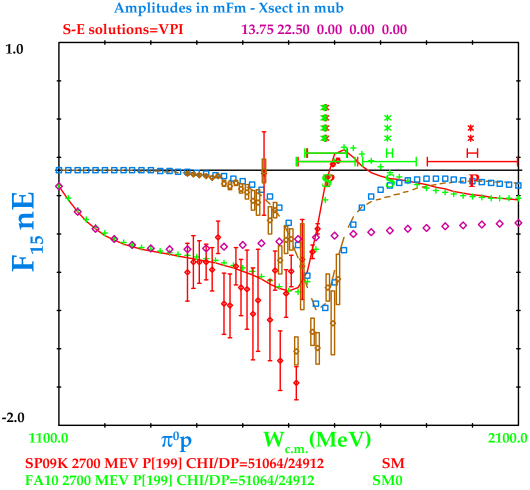
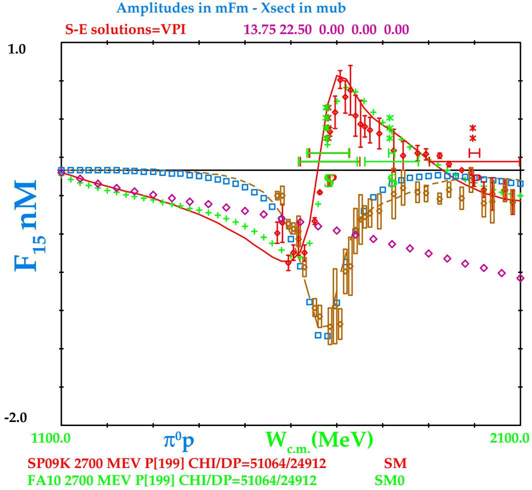


CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



Summary

- Possess a solution that describes $\pi N \to \pi N, \pi N \to \eta N, \gamma N \to \pi N$ reactions
- Unitarity determines non-analyticities in physical region, $w > m_i + m_t$
- Related Chew-Mandelstam form to N/D approach $\to$ 'left-hand cut' neglected in C-M
- Realistic description of hadroproduction data requires correct analytic form $\leftrightarrow$ unitarity
- Forthcoming polarization photoproduction data $\to$ better constraint
- Performed simultaneous coupled-channel fit of $\eta-$photoproduction S_{11} multipole modulus, $|E^{\eta}_{0+}|$ and $\pi-$photoproduction amplitude, E^{π}_{0+}
- Current approach yields resonant E^{η}_{0+} phase $\to$ encourages us to pursue the C-M approach in fits to photoproduction observables (not amplitudes)
- Performed fit to $\pi-$photoproduction *data* using C-M form, yields similar but distinct partial waves with comparable χ-squared
- Outlook
  1. Perform simultaneous fit to $\pi-$ and $\eta-$photoproduction *data* using C-M form
  2. Perform simultaneous, global fit to $\pi N \to \pi N$, $\pi N \to \eta N$, $\gamma N \to \pi N$, $\gamma N \to \eta N$ using C-M form: **offers opportunity for precision electromagnetic data to 'back-constrain' hadronic amplitudes (some of which are very poorly known)**
  3. 'Left-hand' branch points?

Dedication

To the memory of our friend and colleague, Dick Arndt, GWU Research Professor and Virginia Tech Emeritus Professor, who passed Saturday, April 10, 2010.

Supplementary material

Follow-on material

SAID: Scattering Analysis Interactive Database amplitudes

πN elastic scattering and inelastic reactions

- Chi-squared per datum compared with model calculations
- Optimize χ–squared w.r.t. $p \to K_{CM}(p)$

$$\chi^2(p) = \frac{1}{N_{data}} \sum_{i=1}^{N_{data}} \left[\frac{\Phi_{n(i)} y_i(p) - Y_i}{\Delta Y_i} \right]^2 + \frac{1}{N_{exp}} \sum_{n=1}^{N_{exp}} \left[\frac{\Phi_n - 1}{\Delta \Phi_n} \right]^2$$

χ^2/Data	SP06	FA02	KA84	EBAC	Gießen
Reaction	Norm	Norm	Norm	Norm	Norm
$\pi^+p \to \pi^+p$	2.0	2.1	5.0	13.1	10.5
$\pi^-p \to \pi^-p$	1.9	2.0	9.1	4.9	12.1
$\pi^-p \to \pi^0n$	2.0	1.9	4.4	3.5	6.3
$\pi^-p \to \eta n$	2.5	2.5			

FA02 [R. Arndt *ea* Phys Rev C **69**, 035213 (2004)]
KA84 [R. Koch, Z Phys C **29**, 597 (1985)]
EBAC [B. Julia-Diaz *ea* Phys Rev C **76**, 065201 (2007)]
Gießen [V. Shklyar *ea* Phys Rev C **71**, 055206 (2005)]

- ***Correct analytic behavior ensures realistic description (low–χ^2) of the data***

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