EXTRACTION OF RESONANCE PROPERTIES
CAN WE MEASURE S-MATRIX POLES?

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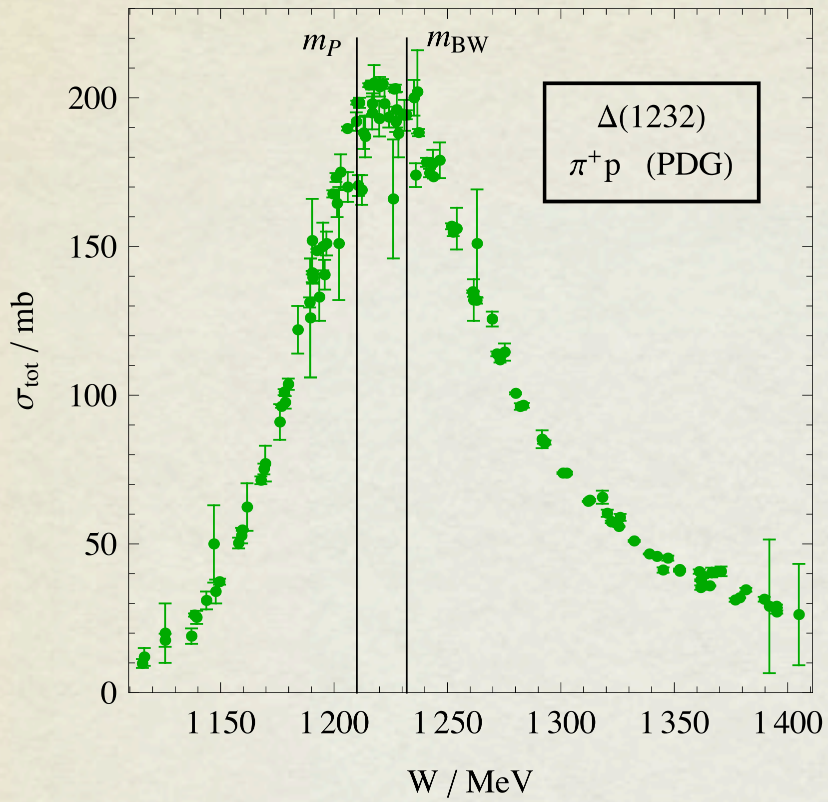
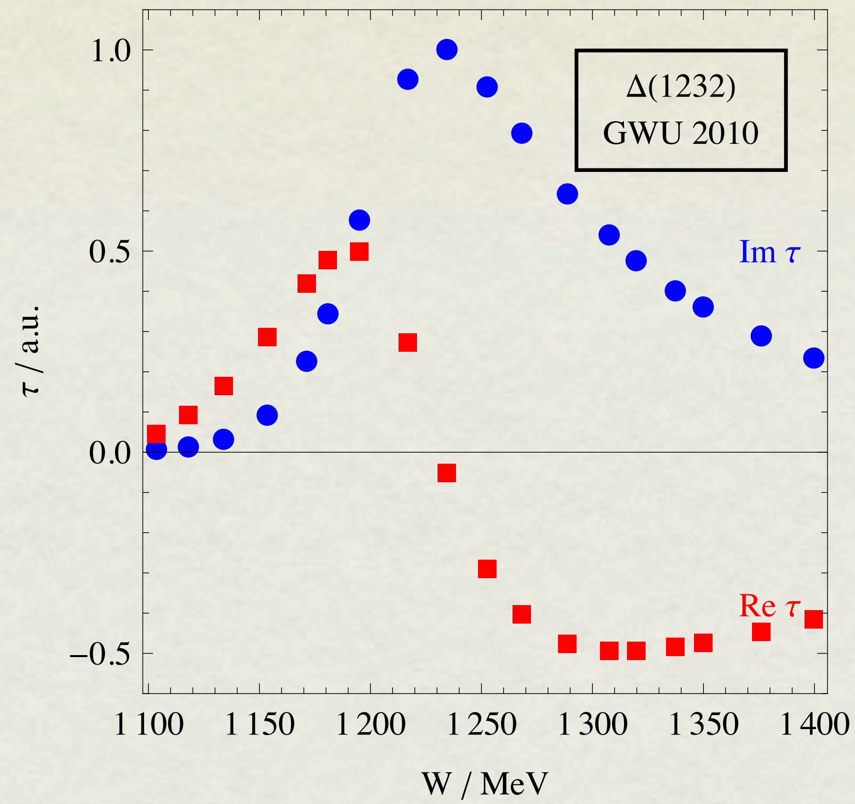
WHAT IS IT ALL ABOUT?

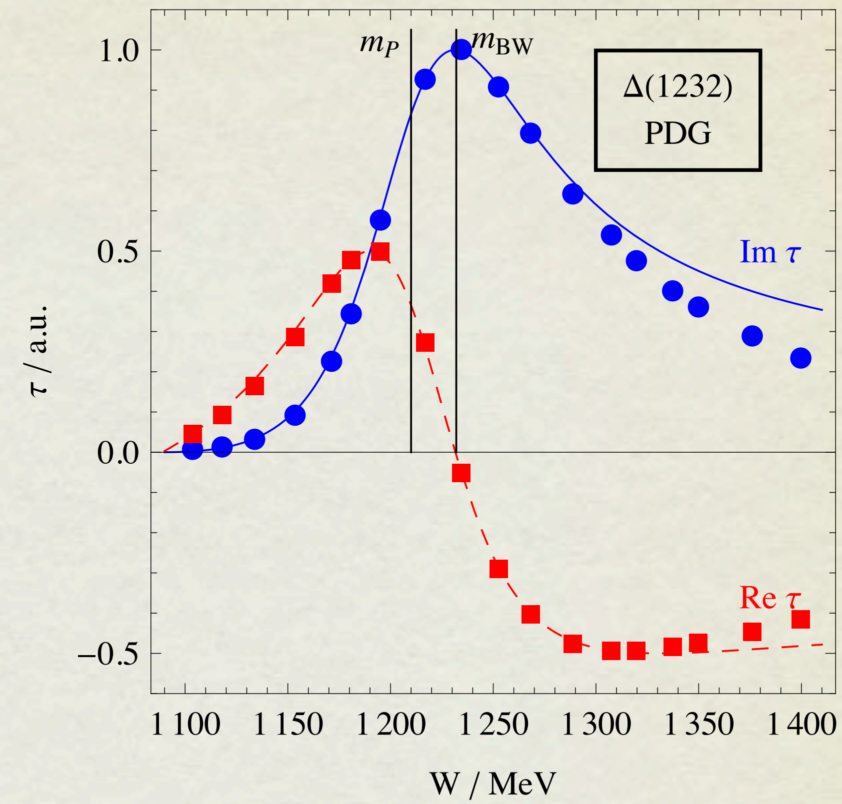
- We want to match theory and experiment
- The matching point has to be uniquely defined, physical, and measurable
- In excited nucleon physics, we match resonance parameters

RESONANCE PARAMETERS

DEFINITIONS







$$\sigma_{\rm tot}(s) \sim |\tau(s)|^2$$

$$\tau(s) \sim \frac{1}{D(s)}$$

$$D(s) = s - m_0^2 + \Sigma(s)$$

Bare poles

$$m_B = m_0$$

Breit − Wigner

$$\mathrm{Re}\, D(x_0) = 0$$

$$m_{BW} = \sqrt{x_0}$$

$$\Gamma_{BW} = \mathrm{Im}\, \Sigma(x_0)/\sqrt{x_0}$$

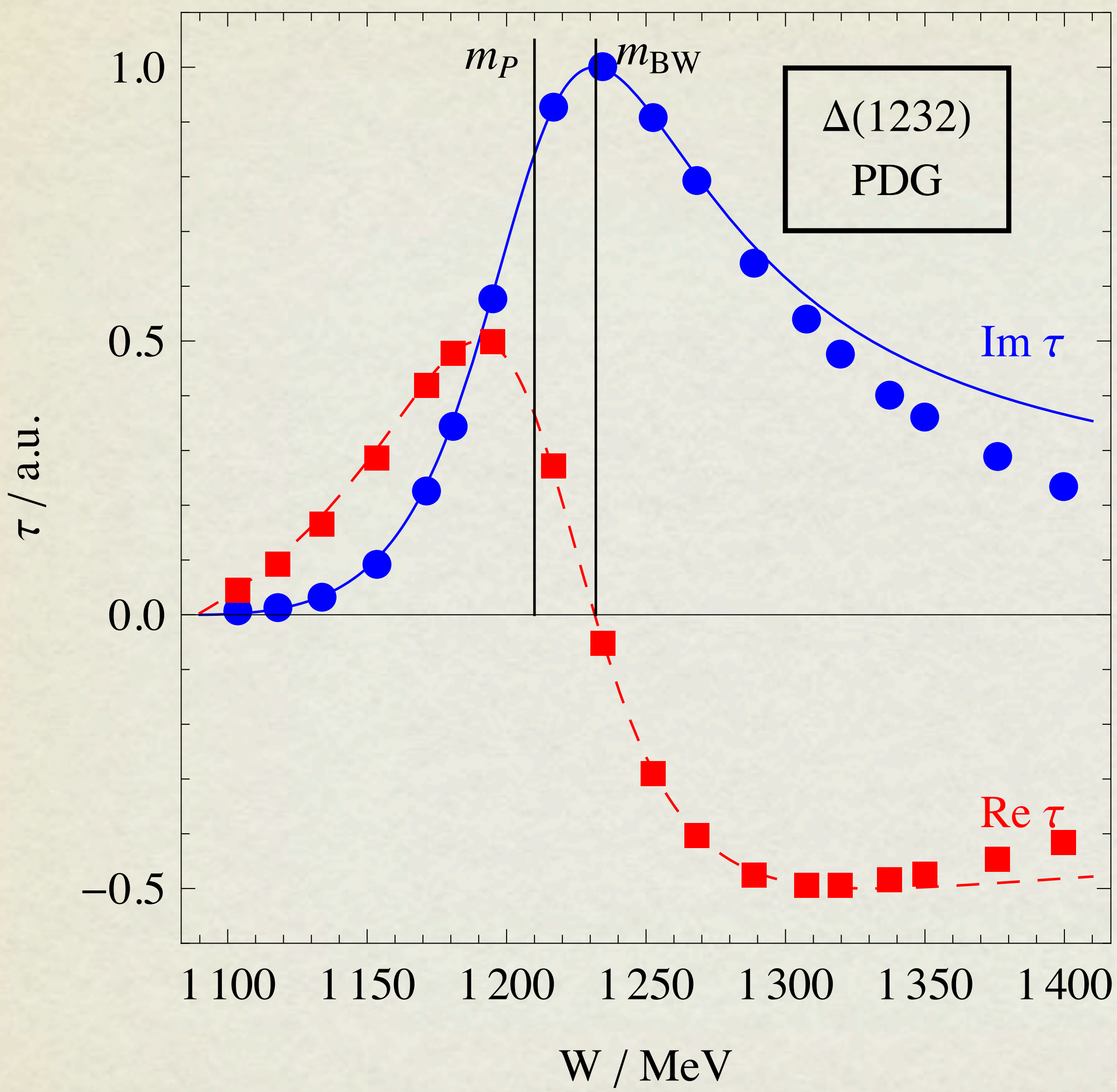
S − matrix poles

$$D(s_0) = 0$$

$$m_P = \mathrm{Re}\sqrt{s_0}$$

$$\Gamma_P = -2\, \mathrm{Im}\sqrt{s_0}$$

RESONANCE PARAMETERS
WHERE DID THE CURVE COME FROM?



PDG N* convention ($W = \sqrt{s}$)

$$\tau(W) = \frac{|r_P|\; e^{i\theta_P}}{m_P - W - i\,\Gamma_P/2} + \tau_b(W)$$

PDG

$m_P = 1210\,\mathrm{MeV}$

$\Gamma_P = 100\,\mathrm{MeV}$

$|r_P| = 50\,\mathrm{MeV}$

$\theta_P = -47^\circ$

Unitarity

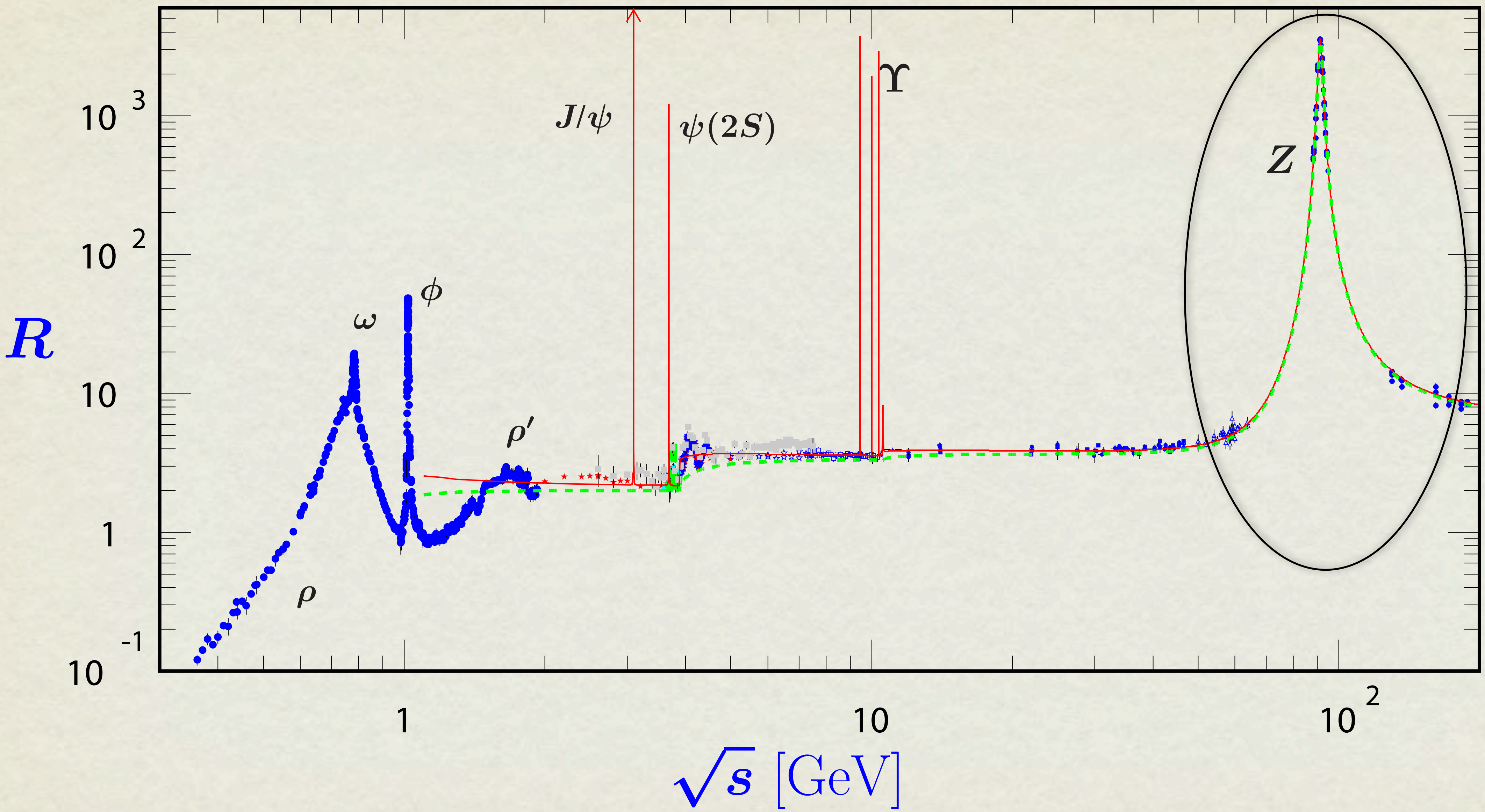
$$\mathrm{Im}\,\tau(W) = \tau(W)^\dagger\,\tau(W)$$

$$|r_P| = \Gamma_P/2$$

$$\tau_b(W) = e^{i\theta_P/2}\sin\theta_P/2$$

RESONANCE PARAMETERS

S-MATRIX POLES ARE (UN)MEASURABLE?



PDG online

RESONANCE PARAMETERS

S-MATRIX POLES ARE (UN)MEASURABLE?

Statistics of data interval fits

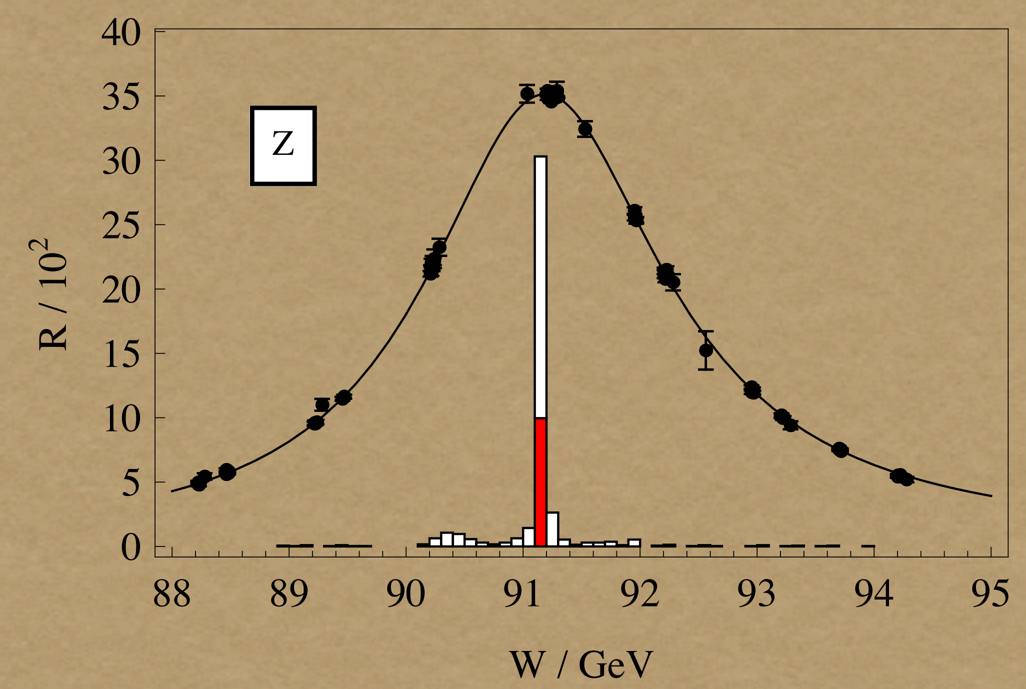
$$\tau(W) \approx \frac{|r_P|\, e^{i\theta_P}}{m_P - W - i\,\Gamma_P/2} + \tau_b$$

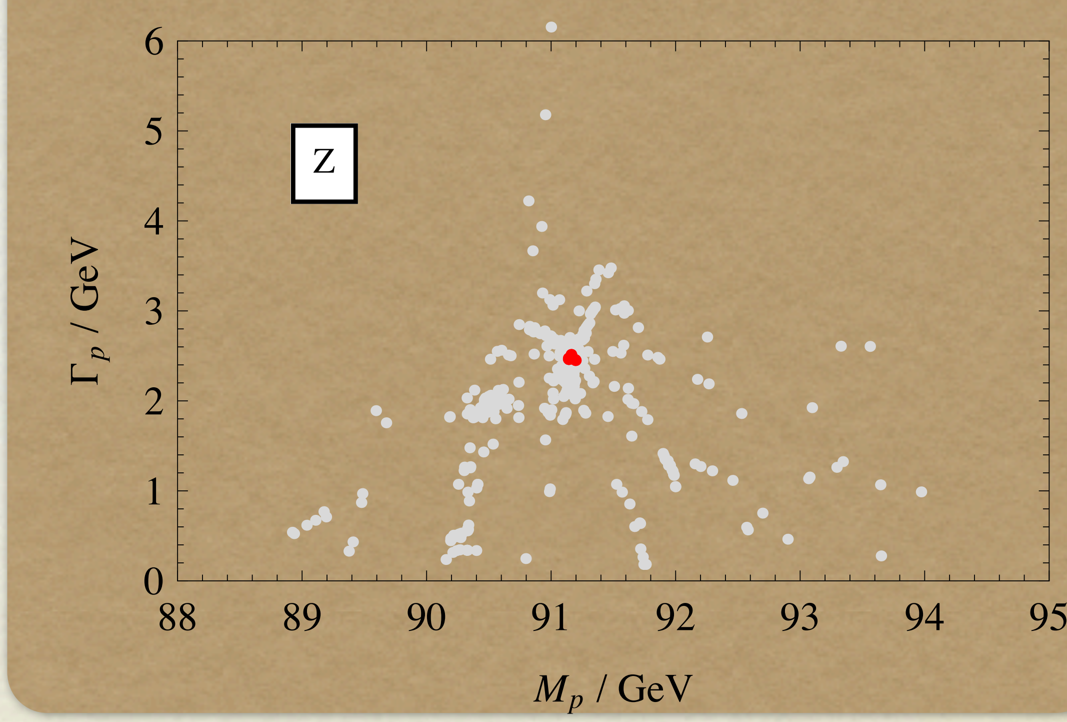
$$\sigma(W) \approx \text{const.} \times |\tau(W)|^2$$

$$\sigma(W) \approx \sigma_\infty \frac{(m_0 - W)^2 + \Gamma_0^2/4}{(m_P - W)^2 + \Gamma_P^2/4}$$

TABLE I: Pole parameters of Z obtained in this work. PDG values of pole and BW parameters are given for comparison.

Z	Pole	Pole PDG [1]	BW PDG [1]
M/MeV	91159 ± 8	91162 ± 2	91188 ± 2
Γ/MeV	2484 ± 10	2494 ± 2	2495 ± 2





RESONANCE PARAMETERS

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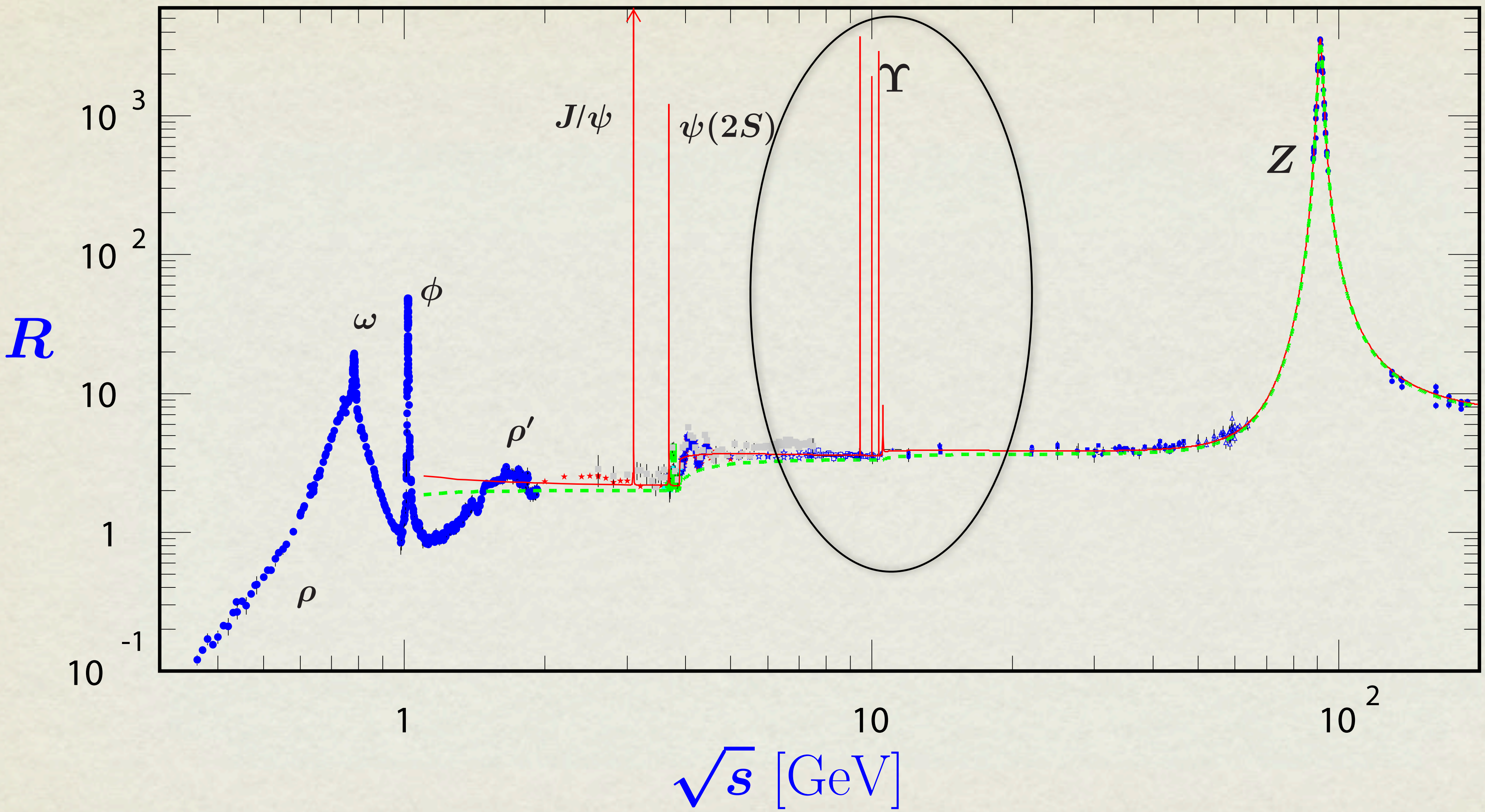


PDG online

RESONANCE PARAMETERS

S-MATRIX POLES ARE (UN)MEASURABLE?

Citation: K. Nakamura *et al.* (Particle Data Group), JPG **37**, 075021 (2010) (URL: http://pdg.lbl.gov)

Υ(11020) $I^G(J^{PC}) = 0^-(1^{--})$

Υ(11020) MASS

VALUE (GeV)	DOCUMENT ID		TECN	COMMENT
11.019±0.008 OUR AVERAGE				
11.019±0.005±0.007	BESSON	85	CLEO	$e^+e^- \to$ hadrons
11.020±0.030	LOVELOCK	85	CUSB	$e^+e^- \to$ hadrons
• • • We do not use the following data for averages, fits, limits, etc. • • •				
10.996±0.002	[1] AUBERT	09E	BABR	$e^+e^- \to$ hadrons

[1] In a model where a flat non-resonant $b\bar{b}$-continuum is incoherently added to a second flat component interfering with two Breit-Wigner resonances. Systematic uncertainties not estimated.

TABLE II: Parameters of Υ(11020) meson. Pole parameters are results of this work.

Υ(11020)	Pole	BaBar [1, 8]	PDG [1]
M/MeV	10999 ± 1	10996 ± 2	11019 ± 8
Γ/MeV	38 ± 1	37 ± 3	79 ± 16

D. M. Manley, Phys. Rev. D **51**, 4837 (1995); D. B. Lichtenberg, Phys. Rev. D **10**, 3865 (1974).

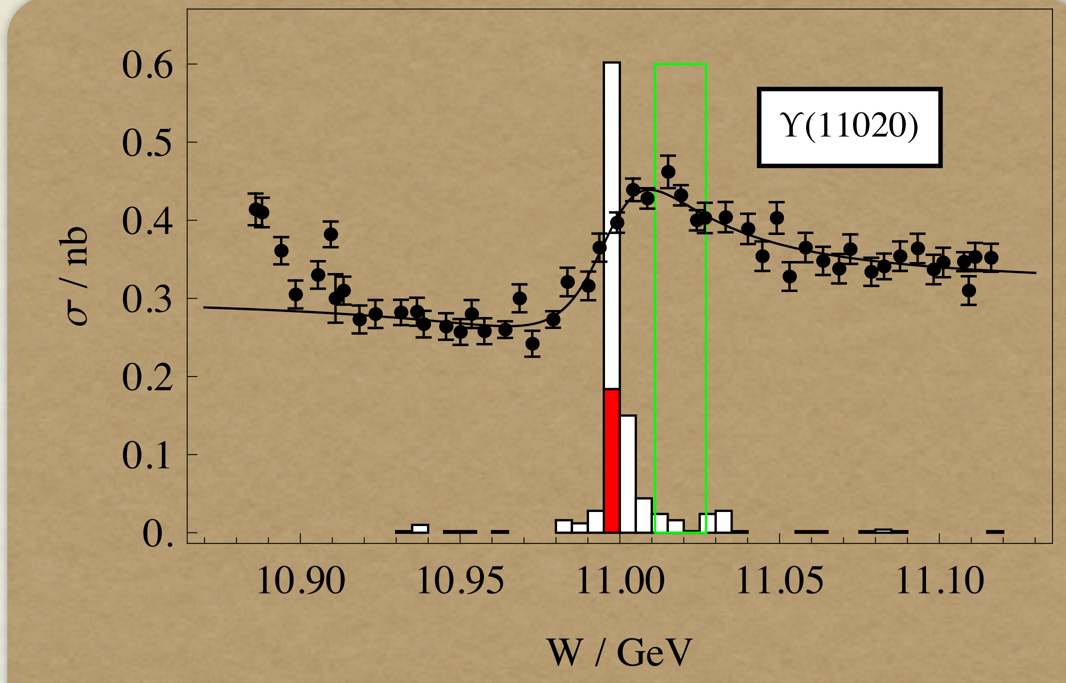
$$M_p = M_b + \sin\theta\,\cos\theta\,\Gamma_b/2,$$

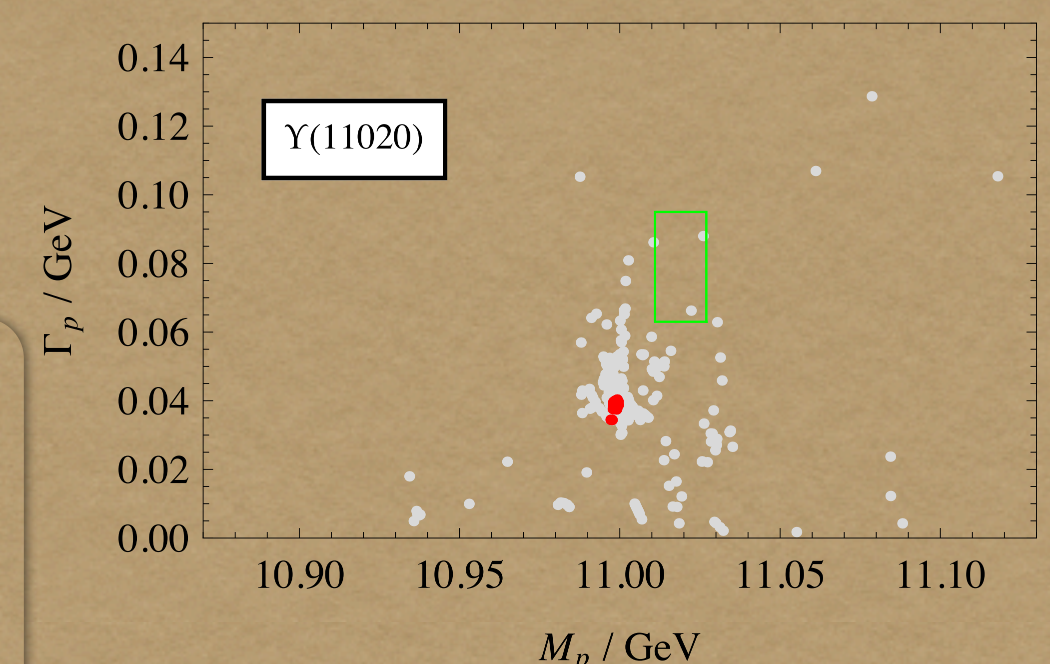
$$\Gamma_p = \cos^2\theta\,\Gamma_b.$$

TABLE IV: The connection between S-matrix pole and Breit-Wigner parameters using only the PDG values.

	$\theta/°$	M_p/MeV PDG[1]	M_p/MeV Eq. (5)
Δ(1232)	-23.0	1210 ± 1	1210
N(1440)	-37.3	1365 ± 15	1368
Υ(11020)	-46.8	10996[a] ± 2	10999
Z	-1.26	91162 ± 2	91161

[a] BaBar value.





RESONANCE PARAMETERS

S-MATRIX POLES ARE (UN)MEASURABLE?

In order to pinpoint the statistical strategy to be used, we did a substantial number of simulations with the data sets that had known poles and zeros. It turned out that **the most successful strategy** was to make an ordered list of all fit results, from best to worst, and then to **drop the worst three quarters** using the following **goodness-of-fit measures**:

- **Akaike information criterion** [11],
- **Schwartz (Bayesian information) criterion** [12],
- **P-values of the extracted fit parameters** (in particular, **Mp** and **Γp**).

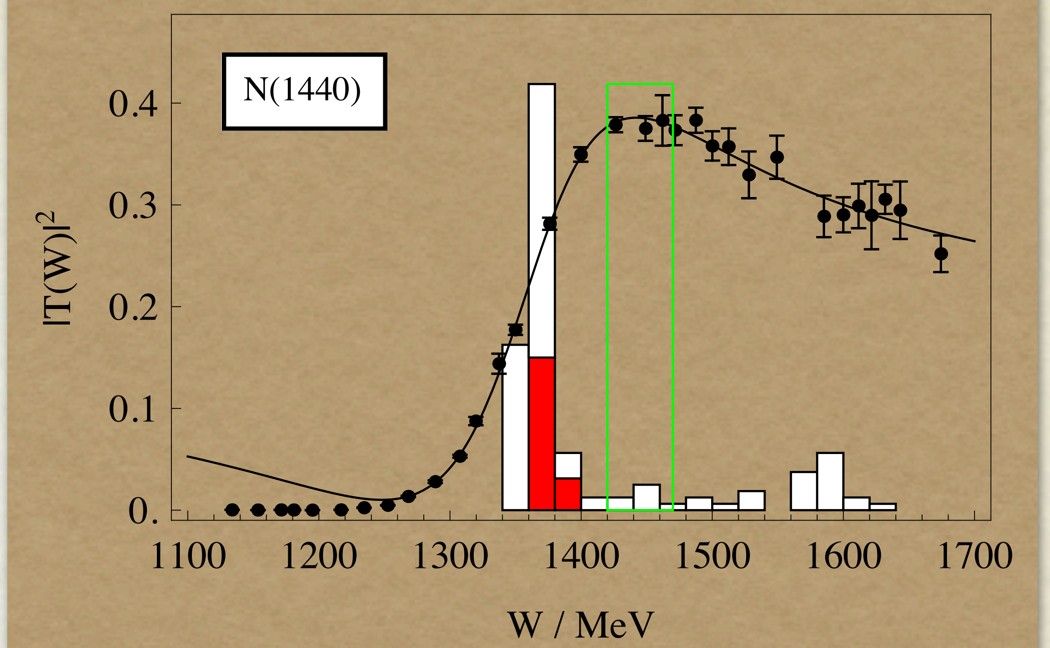
Eventually, we **kept the intersection** of the fits that satisfied **all** criteria.

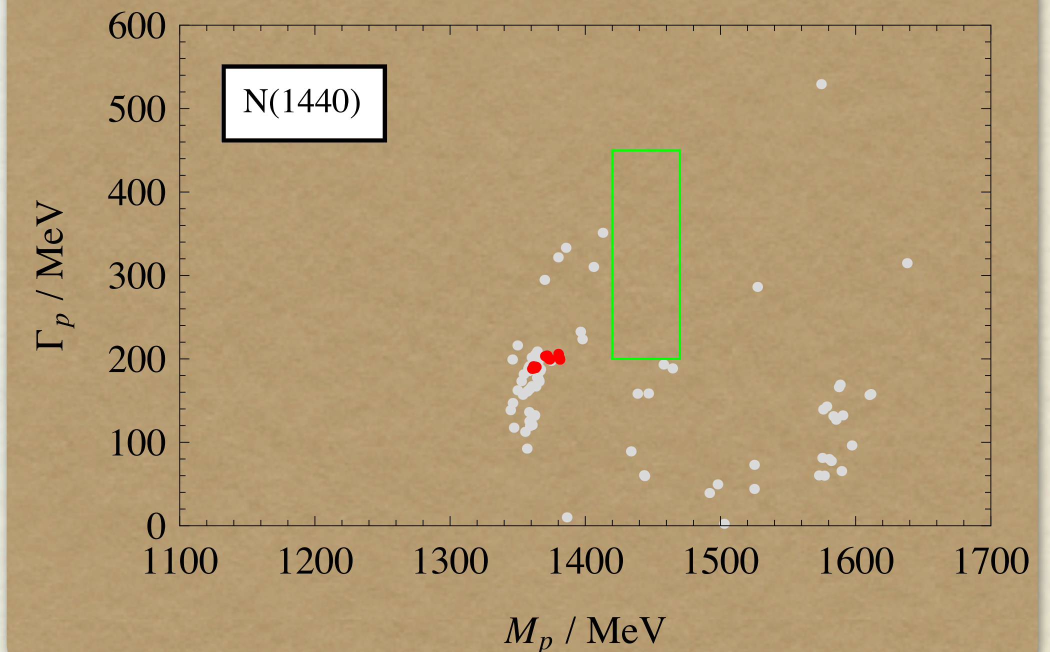
Results closest to the original poles were produced by **averaging the obtained pole** positions of all good fits. The standard deviation turned out to be a good estimate for errors of obtained parameters.

All other approaches we tested, such as keeping only a handful of the best fits, or keeping just those whose values of reduced χ2 were close to one, **failed to accurately reproduce** the original pole parameters.

TABLE III: N(1440) resonance parameters.

$N(1440)$	Pole	Pole PDG [1]	BW PDG [1]
M/MeV	1370 ± 6	1365 ± 15	$1440 \pm^{30}_{20}$
Γ/MeV	197 ± 6	190 ± 30	$300 \pm^{150}_{100}$





Tuesday, May 24, 2011

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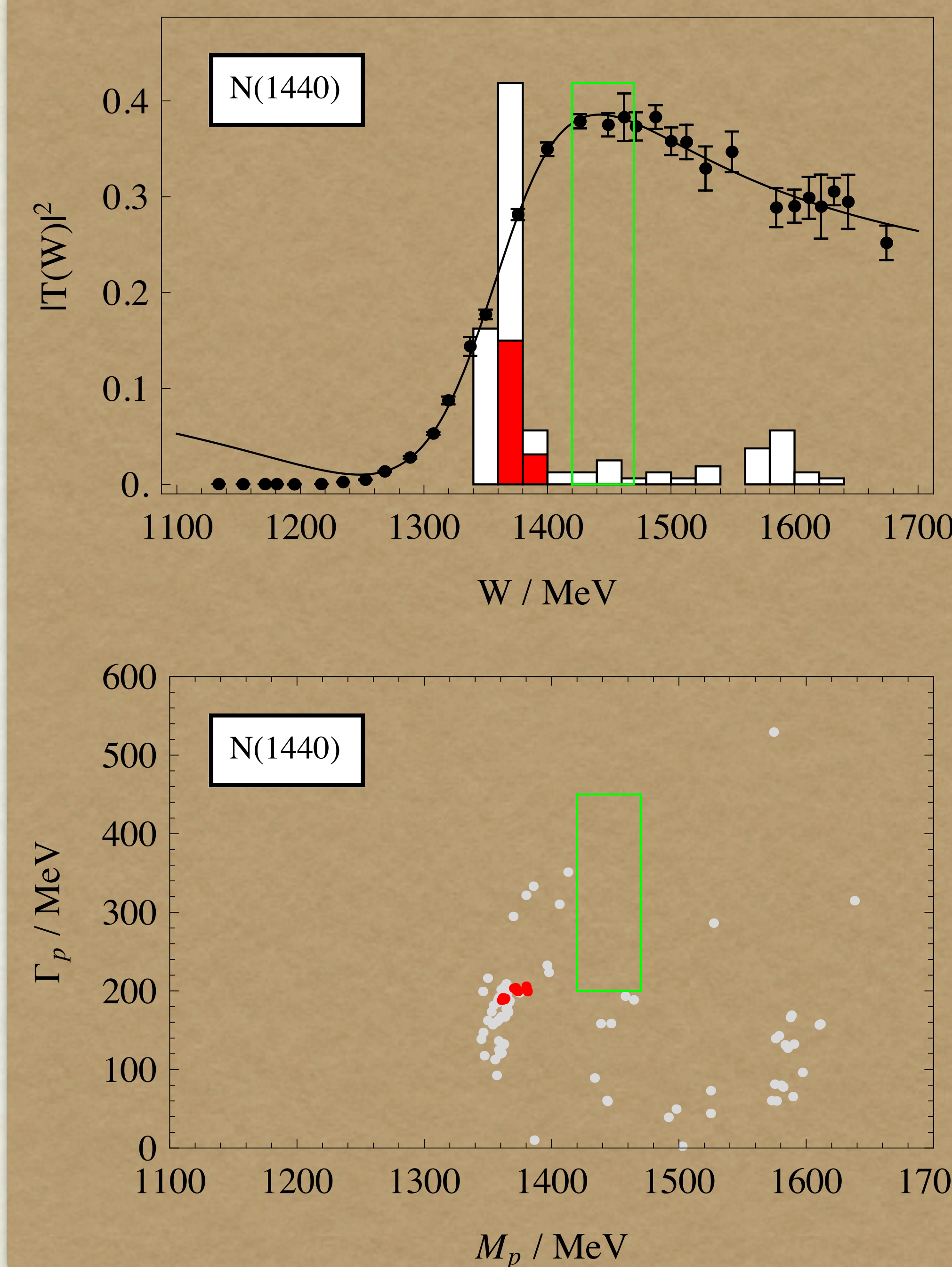


Ceci, Korolija, Zauner; arXiv:1007.4207

(A NOTE ON) THE BREIT-WIGNER PARAMETERS

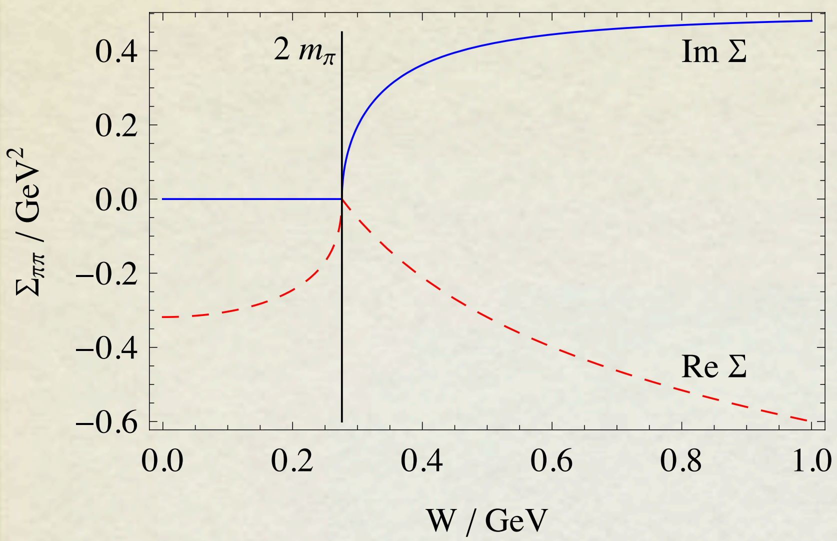
- We could not find a simple (nor unique) parameterization of the amplitude that would result in the model independent Breit-Wigner parameters
- For narrow resonances with small background, BW parameters are similar to the S-matrix pole parameters
- Closest match, depending exclusively on the full amplitude in a model independent way, were K-matrix poles / *PLB 659 (2008) 228*

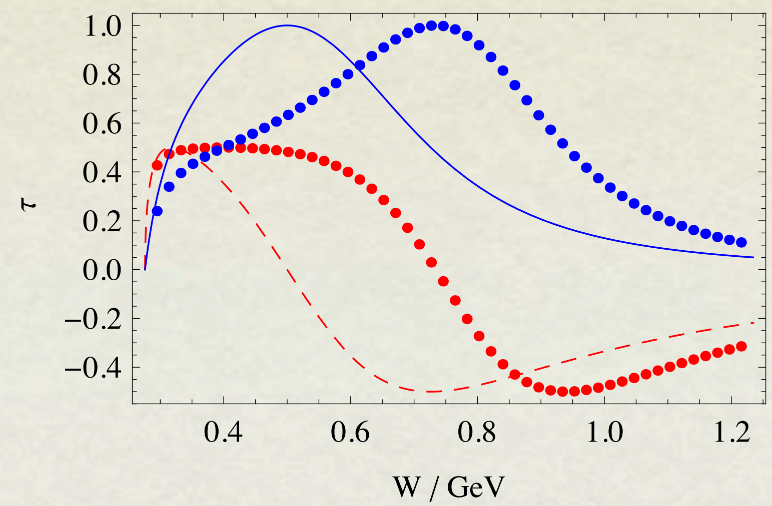
(A NOTE ON) ANALYTICITY

NECESSARY CONDITION FOR EXTRACTION?

- Analyticity is assumed to be necessary model/parameterization feature for the proper S-matrix pole extraction
- We just showed that the S-matrix pole mass can (sometimes) be extracted without assuming analyticity
- Is this the only such exception?

A CRAZY NON-ANALYTICITY EXAMPLE





$$D(s) = s - m_0^2 - \Sigma(s)$$

$$\mathrm{Im}\,\Sigma(s) \sim \frac{q_{\pi\pi}(s)}{\sqrt{s}}$$

$$\mathrm{Im}\,\Sigma(s) = m_0\,\Gamma_0\,\frac{\sqrt{m_0^2}}{q_{\pi\pi}(m_0^2)}\;\frac{q_{\pi\pi}(s)}{\sqrt{s}}$$

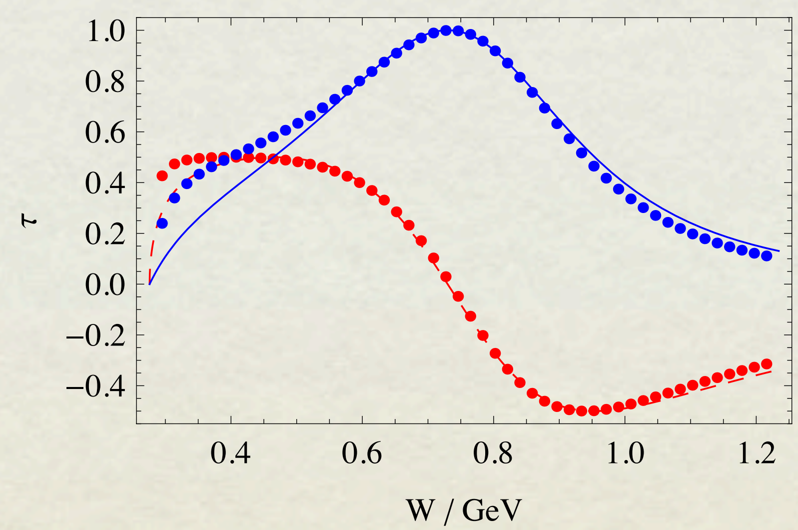
$$\mathrm{Re}\,\Sigma(s) = \mathrm{Disp\,Rel}\ldots$$

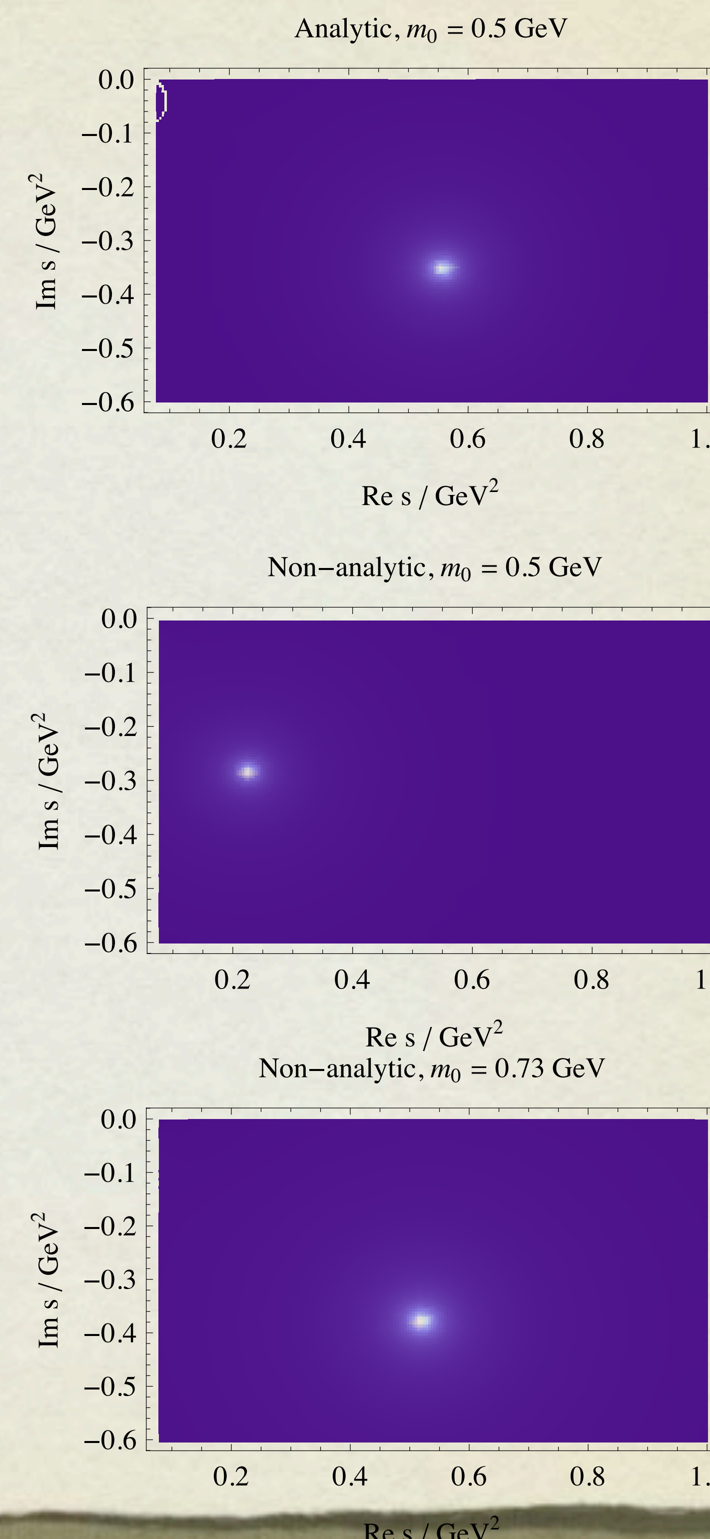
$$m_0 = 500\,\mathrm{MeV}$$

$$\Gamma_0 = 500\,\mathrm{MeV}$$

How do we fix this?

$$m_0 \to 730\,\mathrm{MeV}$$





S-MATRIX POLES
CONCLUDING REMARKS

- Can we say now that **S-matrix mass** may be **measured** directly?
- **If not**, what about the **Breit-Wigner mass**? Can it be measured?
- In both cases we need a particular (mathematical) parameterization or some (physical) model
- All in all, by using **simple parameterization** and **local sequential** fitting excellent estimate of the **S-matrix pole mass** can be obtained
- Current parameterization works all right for the S-matrix pole widths (we are improving it!)
- **The question:** should we really abandon what we have learned just because the approach was not unitary, and had no (proper!) analyticity?

Thank you for your attention!