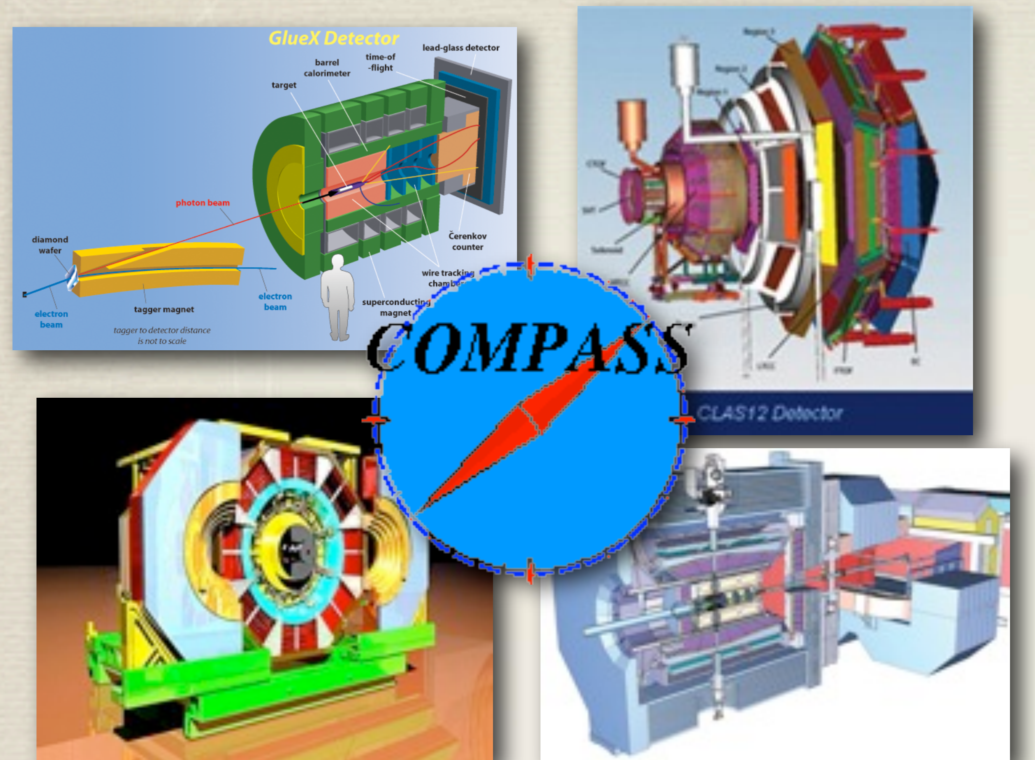
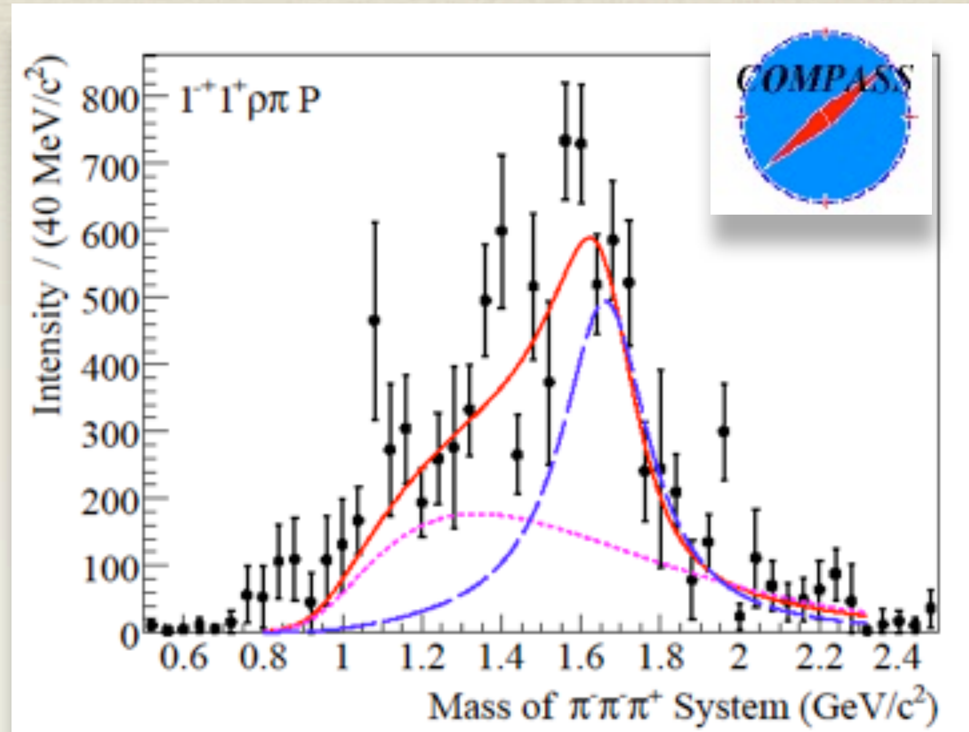
Key issues:

I Amplitudes for data analysis

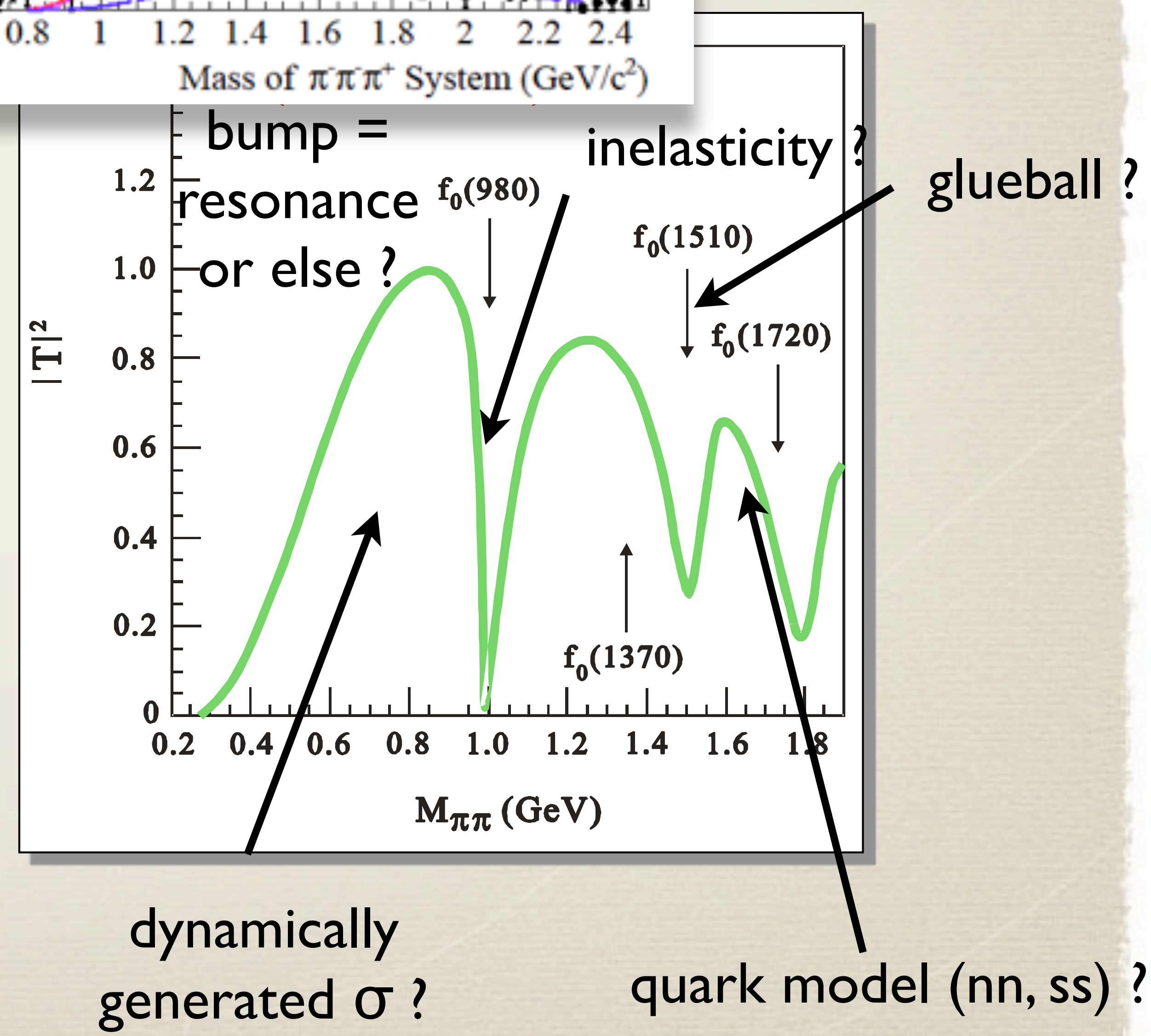




2. Connecting amplitudes (real world) and resonances ("unphysical sheets")

3. What is the connection between resonances and QCD ?

>> amplitude analysis <<
(analytic properties, dispersion relations, QCD and model input)



Outline:

* Aspects of partial wave dispersion relations

* isovector P-wave

* things to do: example forces vs particles

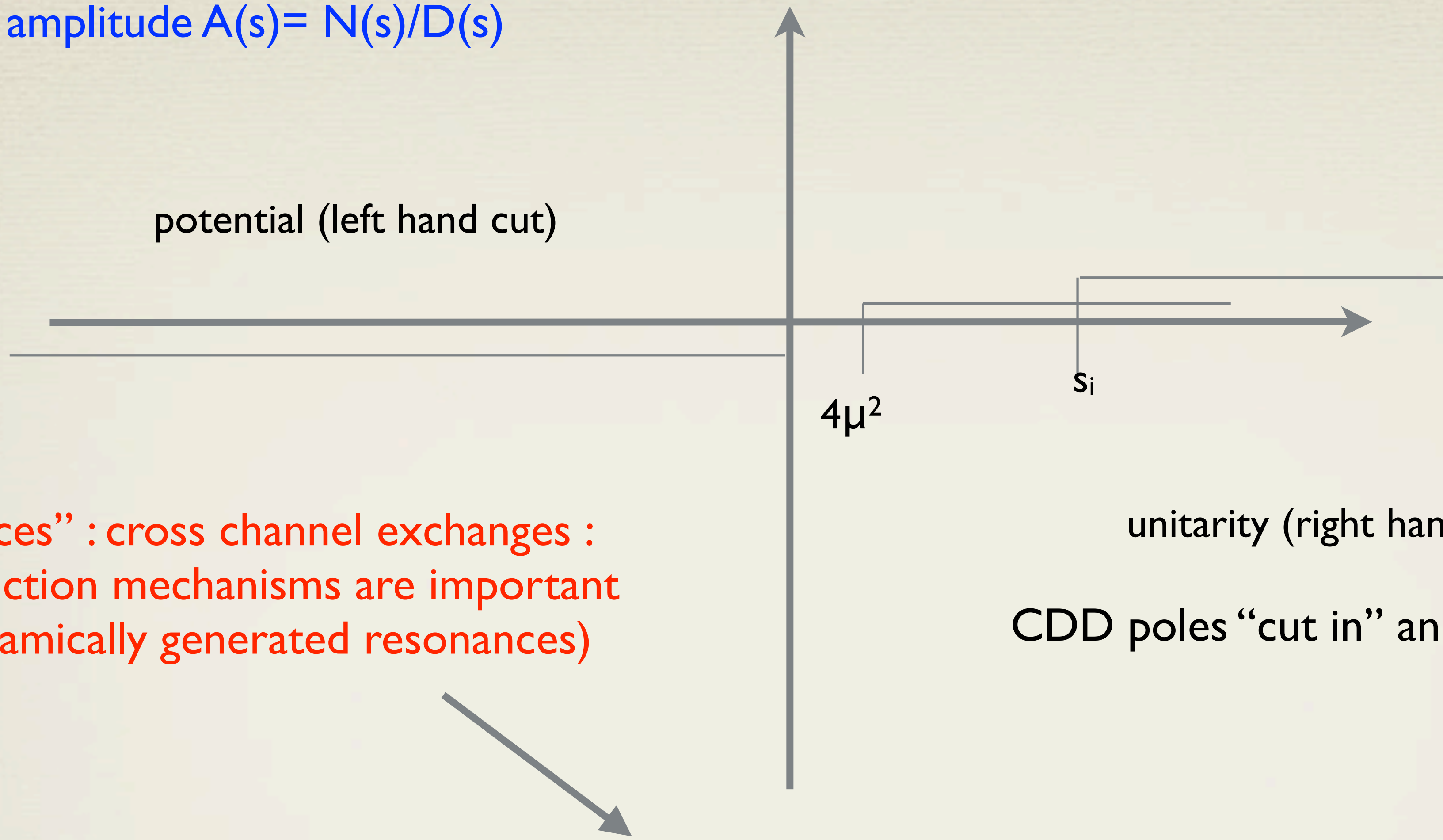
in collaboration with
Peng Guo, Marco Battaglieri, Raffaela De Vita,
Matt Shepherd, Ryan Mitchel

Towards a connection between data and resonances

$\pi\pi \rightarrow \pi\pi$

partial amplitude $A(s) = N(s)/D(s)$

potential (left hand cut)



"forces" : cross channel exchanges : production mechanisms are important (dynamically generated resonances)

unitarity (right hand cut)

CDD poles "cut in" and produce bumps

resonances: poles on unphysical sheets

Towards a connection between data and resonances

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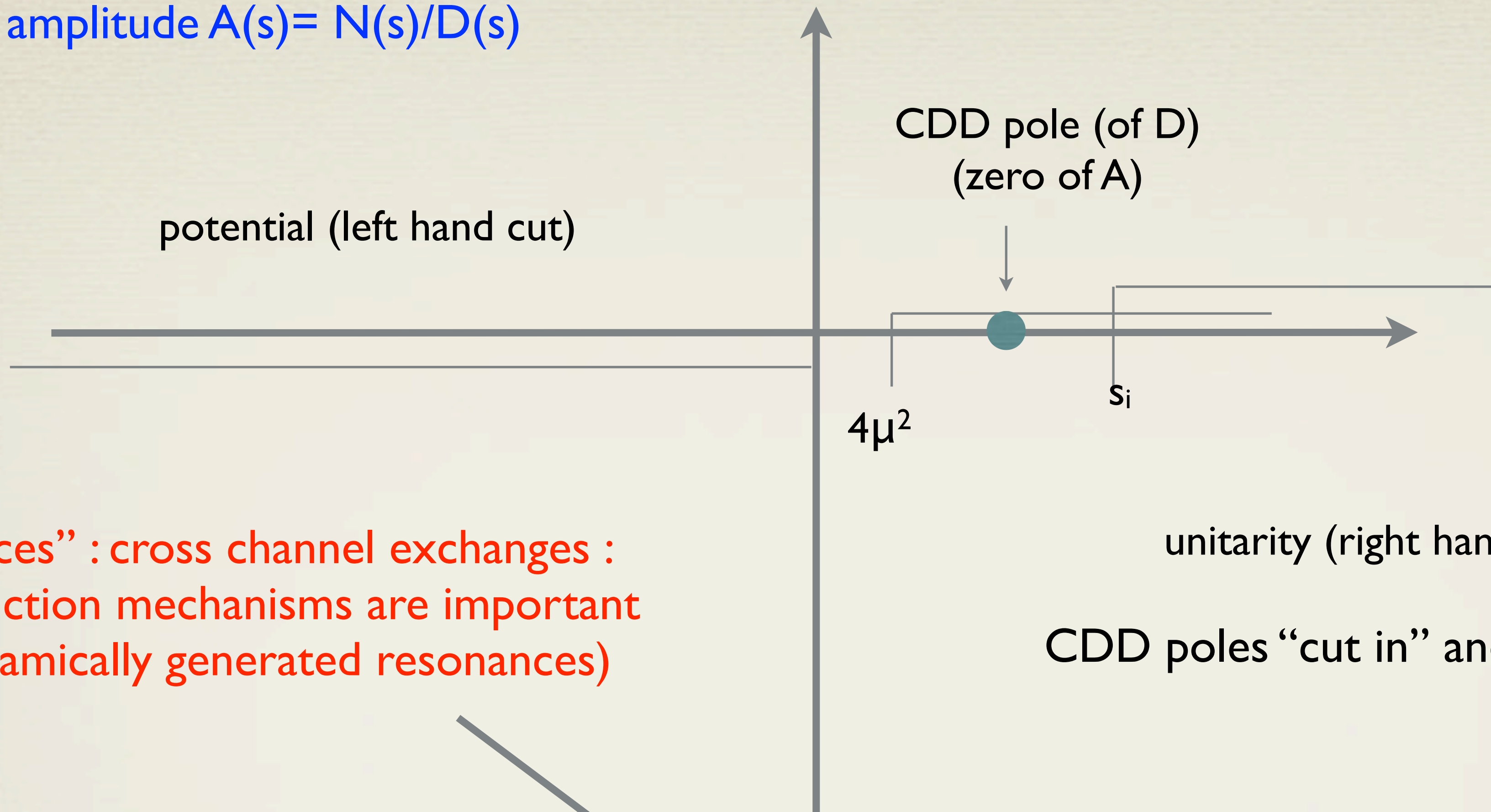

CDD poles "cut in" and produce bumps

"forces" : cross channel exchanges : production mechanisms are important (dynamically generated resonances)

resonances: poles on unphysical sheets

CDD pole corresponds to an elementary particle (move out from inelastic cut when coupling is decreased)

"Schrodinger" equation for the scattering amplitude

$$ImA(s) = R(s)\rho(s)|A(s)|^2$$

$$A(s) = \frac{1}{\pi}\int_{-\infty}^{0} ds' \frac{ImA(s')}{s'-s} + \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{ImA(s')}{s'-s}$$

input ("potential") : through crossing lhc is related to other physical amplitudes

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- Dispersion relations ca 1970
- potential not known everywhere
- in principle many (∞) channels contribute
- x-sections known over limited energy range
- solutions are not unique (CDD)
- analyticity in all channels: complex angular momentum

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* modern developments

* QCD: interpretation of the ambiguities (CDD pols)
* chiral symmetry: low energy constraints

if single hadron states exist: lattice is the place to find them

J.Dudek at al.

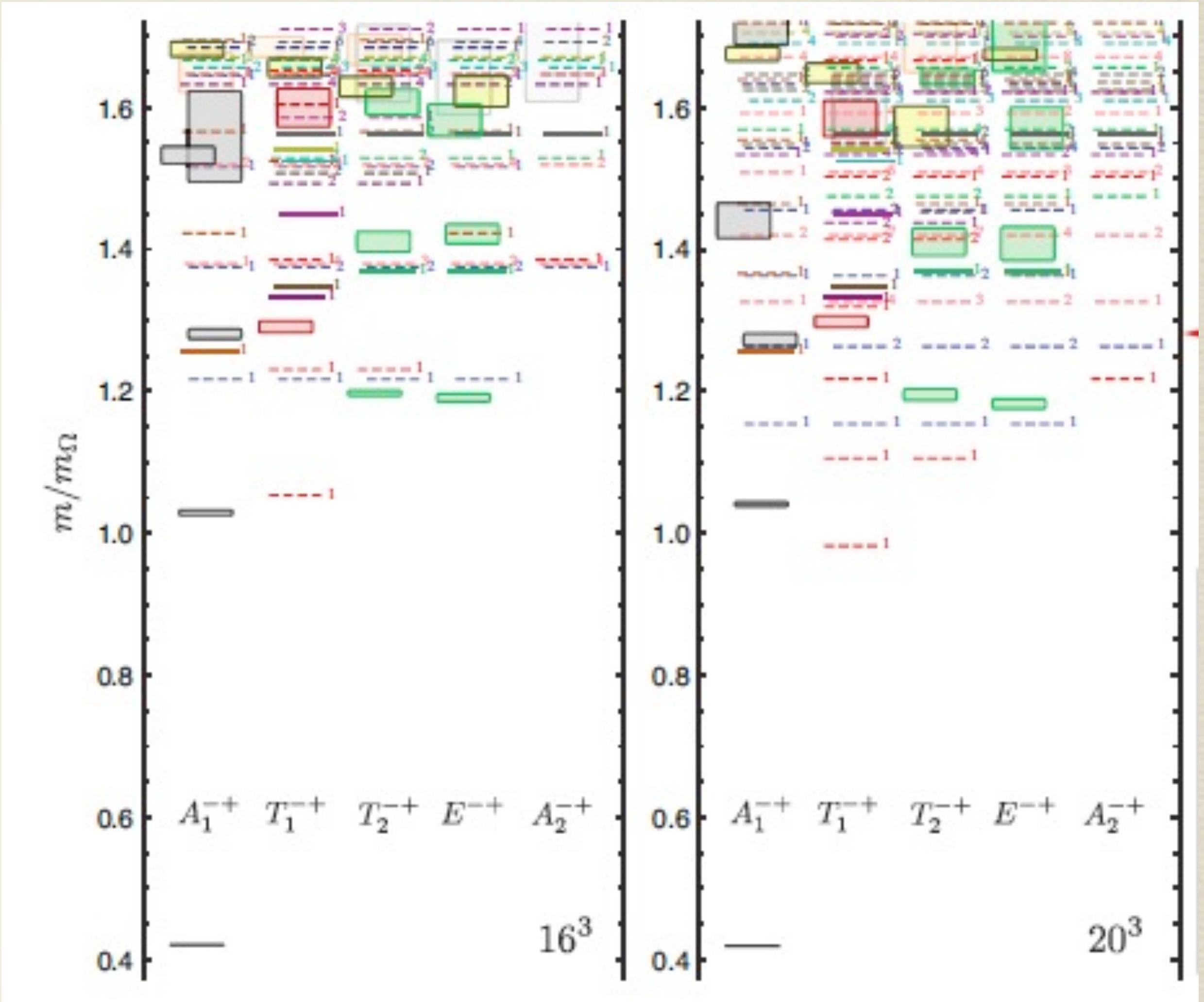
* On finite volume multi-meson state and single hadron states are discrete.

* If there are single hadron states, use volume dependence to disentangle

* Continuum states can have any J,P,C but not single hadron states

* The choice of operators minimizes overlap with multi-meson states

* In the continuum these these states should disappear through cuts onto unphysical sheets (as CDD poles)



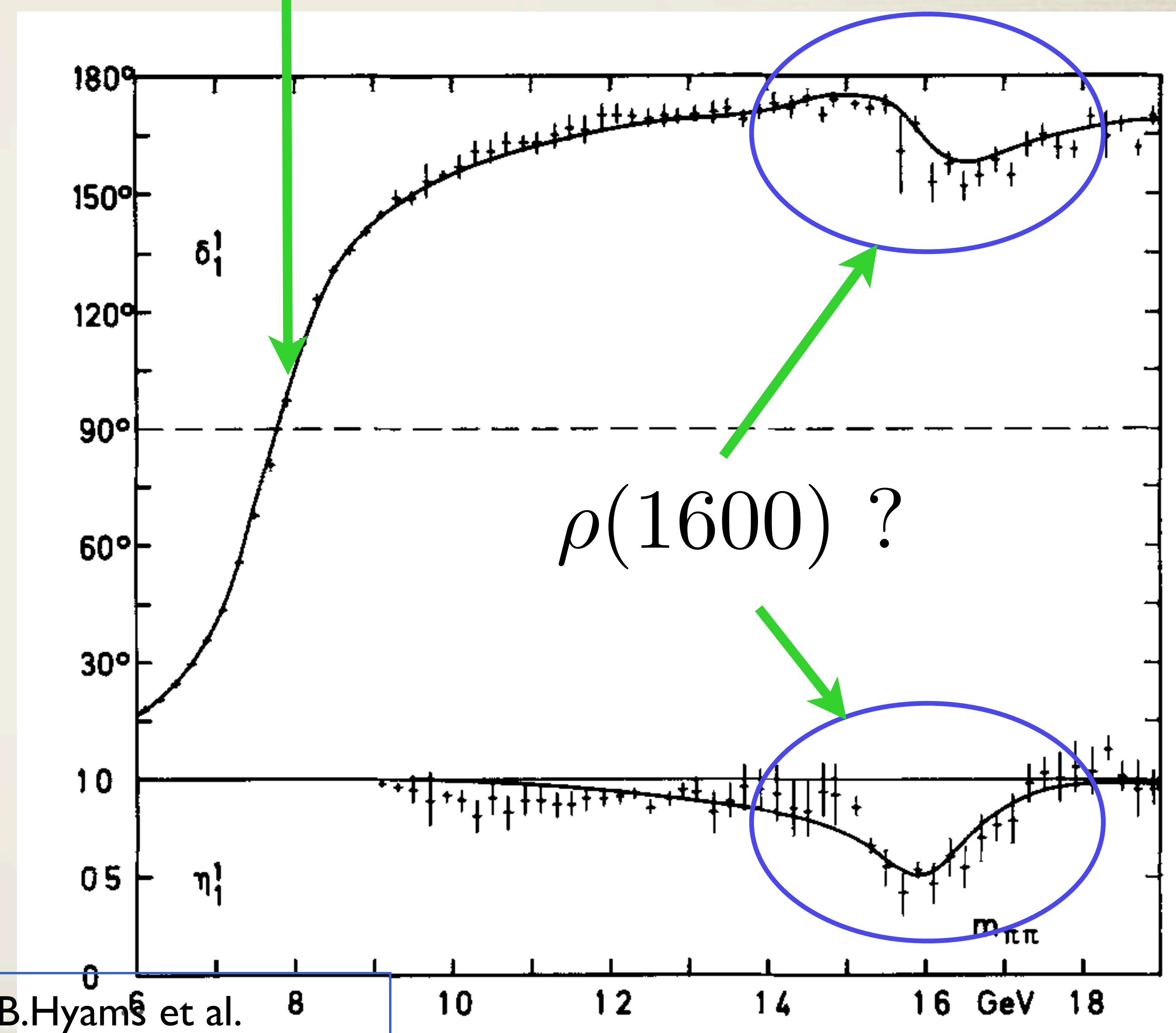
>> there is evidence for single hadron states <<
(no surprising, quark model, CDD poles, etc.)

We will focus on the I=1, P-wave

PDG (before 1988) lists two resonances:
rho(770) and rho(1600)

$\rho(770)$

P-wave $\pi\pi \rightarrow \pi\pi$ scattering data: phase shift and inelasticity



B.Hyams et al.
Nucl.Phys.B64(1973) 134

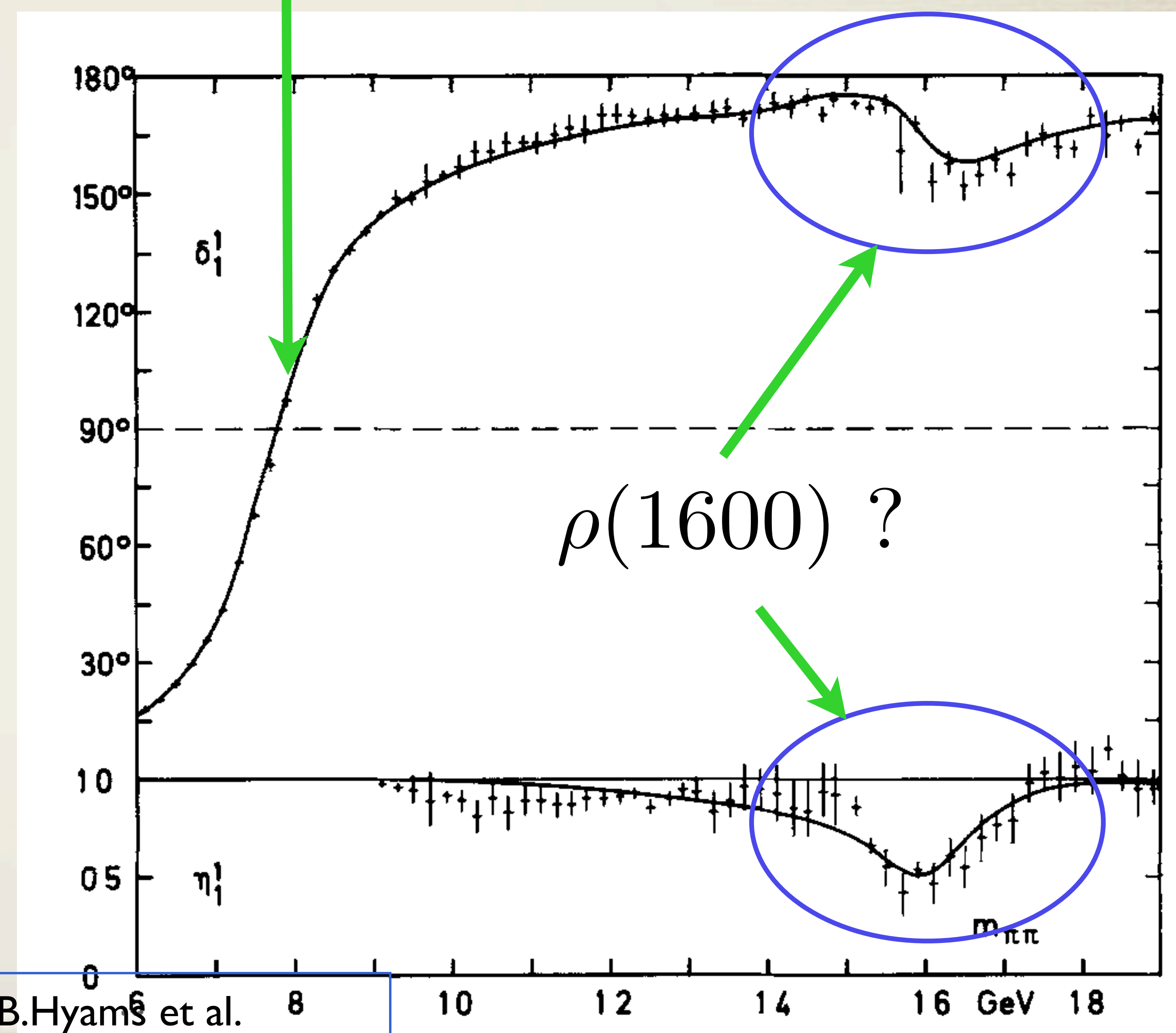
We will focus on the I=1, P-wave

PDG (before 1988) lists two resonances:
rho(770) and rho(1600)

PDG (after 1988) replaces rho(1600) by
rho(1450) and rho(1700)
analysis based on a coherent sum of three BW's parametrization to explain both photoproduction (2pi,4pi) and pion form factor

$\rho(770)$

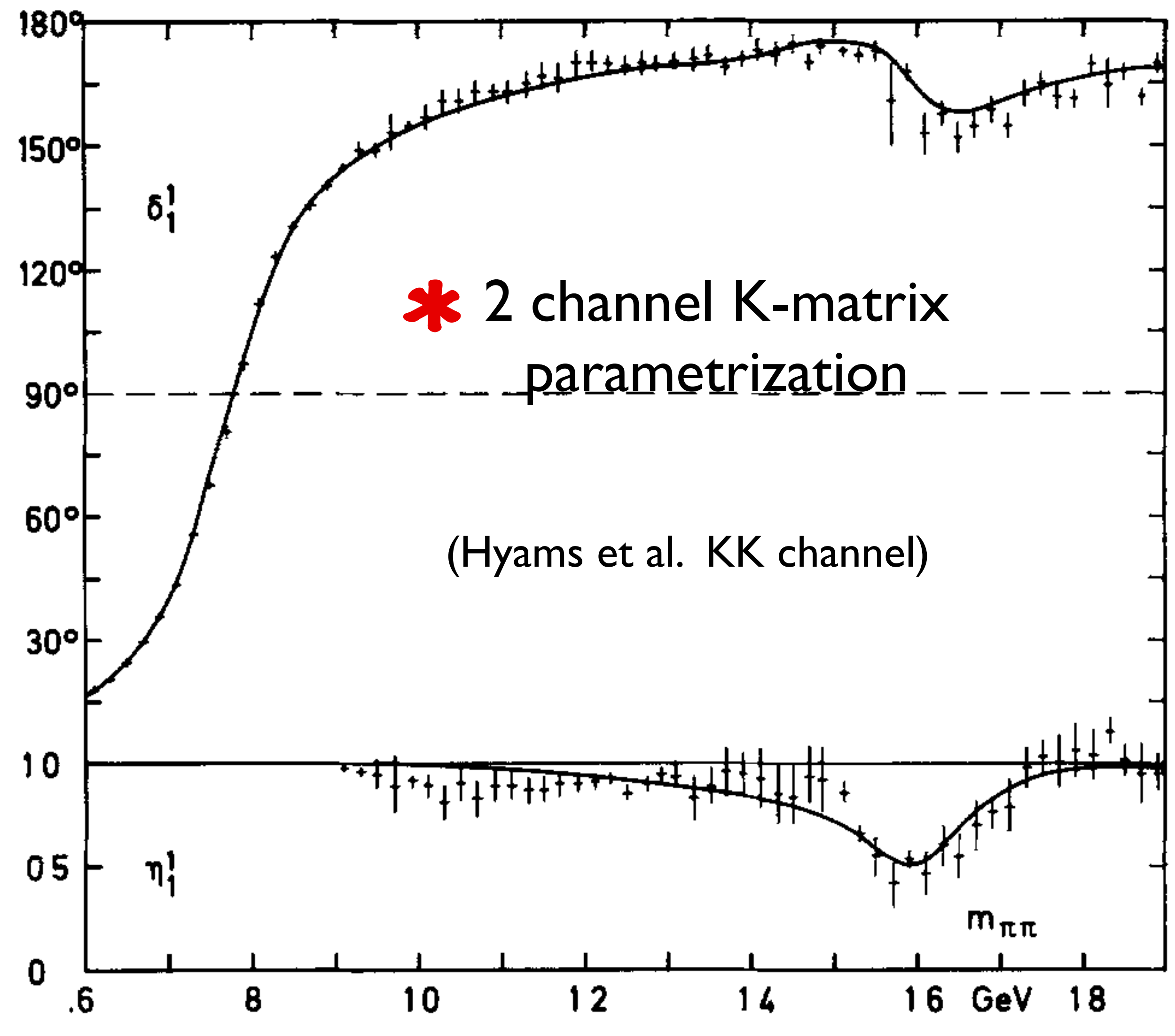
P-wave $\pi\pi \rightarrow \pi\pi$ scattering data: phase shift and inelasticity



$\rho(1600)$?

B.Hyams et al.
Nucl.Phys.B64(1973) 134

PDG 2010
rho(770) and rho(1600)



✱ 2 channel K-matrix parametrization

(Hyams et al. KK channel)

ρ(1700) DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	4π	
Γ_2	$2(\pi^+\pi^-)$	large
Γ_3	$\rho\pi\pi$	dominant
Γ_4	$\rho^0\pi^+\pi^-$	large
Γ_5	$\rho^0\pi^0\pi^0$	
Γ_6	$\rho^\pm\pi^\mp\pi^0$	large
Γ_7	$a_1(1260)\pi$	seen
Γ_8	$h_1(1170)\pi$	seen
Γ_9	$\pi(1300)\pi$	seen
Γ_{10}	$\rho\rho$	seen
Γ_{11}	$\pi^+\pi^-$	seen
Γ_{12}	$\pi\pi$	seen
Γ_{13}	$K\overline{K}^*(892)$+ c.c.	seen
Γ_{14}	$\eta\rho$	seen
Γ_{15}	$a_2(1320)\pi$	not seen
Γ_{16}	$K\overline{K}$	seen
Γ_{17}	e^+e^-	seen
Γ_{18}	$\pi^0\omega$	seen

$\rho(1450)$

$I^G(J^{PC}) = 1^+(1^{--})$

See our mini-review under the $\rho(1700)$.

$\Gamma(\pi\pi)/\Gamma(\omega\pi)$ Γ_1/Γ_3

VALUE	DOCUMENT ID		TECN
• • • We do not use the following data for averages, fits, limits, etc. • • •			
~ 0.32	CLEGG	94	RVUE

$\Gamma(\omega\pi)/\Gamma(4\pi)$ Γ_3/Γ_2

VALUE	DOCUMENT ID		TECN
• • • We do not use the following data for averages, fits, limits, etc. • • •			
<0.14	CLEGG	88	RVUE

$\Gamma(\pi\pi)/\Gamma(4\pi)$ Γ_1/Γ_2

VALUE	DOCUMENT ID		TECN	COMMENT
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.37±0.10	[37,38] ABELE	01B	CBAR	0.0 $\bar{p}n \to 5\pi$

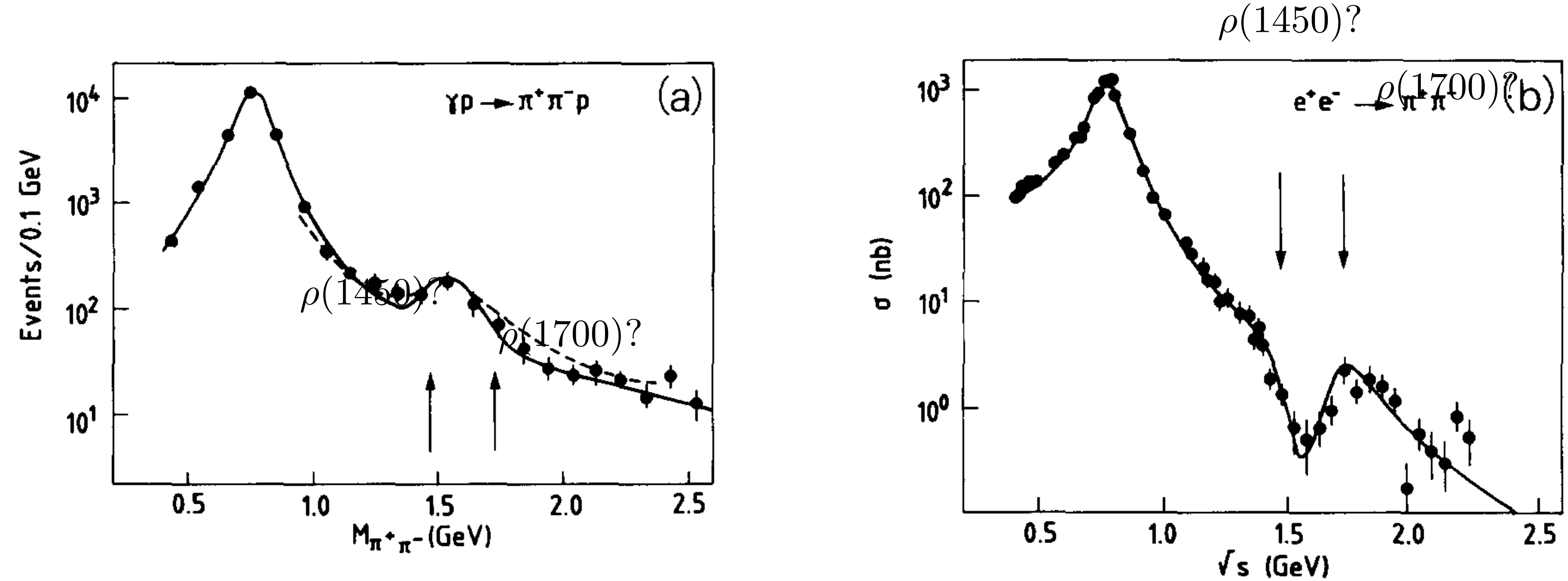
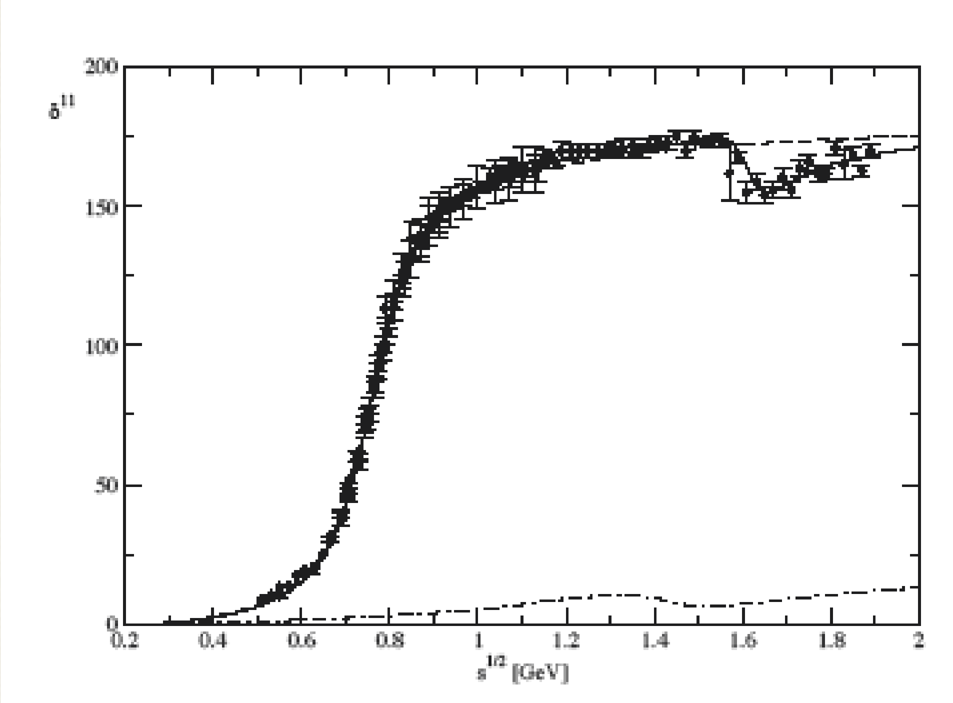
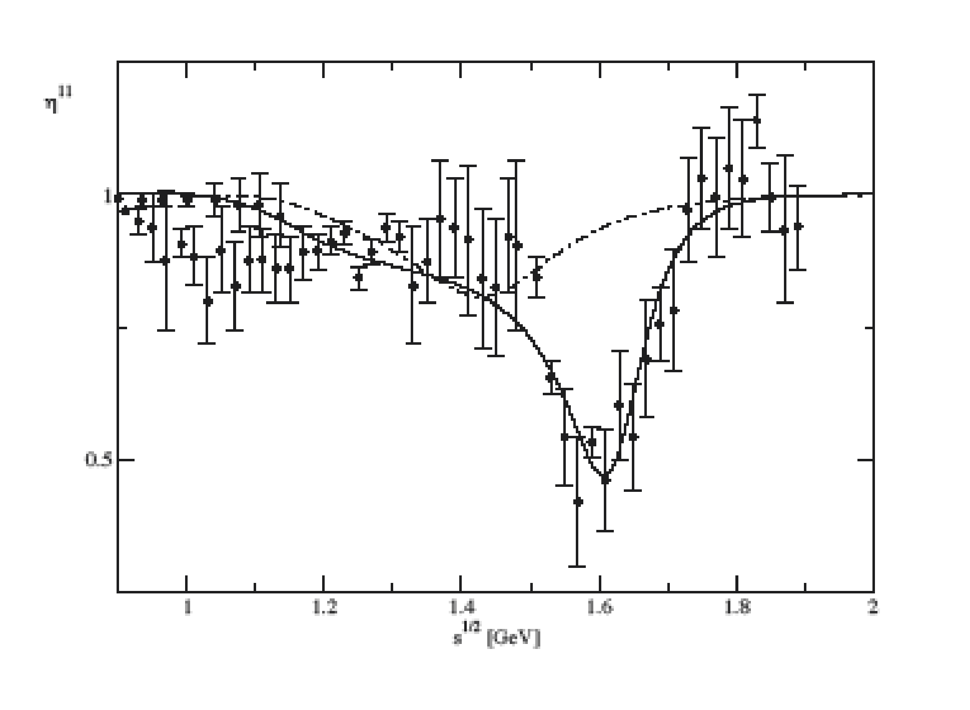


Fig. 8. (a) $m_{\pi^+\pi^-}$ from $\gamma p \to \pi^+\pi^- p$. Data points from ref. [42] corrected for a contribution from the ρ_3. Dashed line fits the data from ref. [43]. Full line is explained in the text. (b) $e^+e^- \to \pi^+\pi^-$, $\sqrt{s} < 1.4$ GeV, ref. [44]; $1.4 \leqslant \sqrt{s} \leqslant 2.1$ GeV, ref. [45].

B. Diekmann
Phys.Rep.159(1988) 99

Amplitude construction I





* 2 channel K-matrix parametrization

K-matrix: use "many" uncontrolled CDD poles and left hand poles

(Hyams et al. used an "approximation") $\rho(s) \to \sqrt{s}\rho(s)$

and the K-matrix representation becomes

$$[\hat{t}^{-1}(s)]_{\alpha\beta} = [K^{-1}(s)]_{\alpha\beta} + \delta_{\alpha\beta}(s - s_\alpha)\sqrt{s_\alpha - s}.$$

* The "standard" K-matrix approximation

$$Im t^{-1} = -\rho$$
$$t^{-1}(s) = -i\rho(s)$$

while what is should be is

$$t^{-1}(s) = \frac{1}{\pi}\int ds' \frac{\rho(s')}{s' - s}$$

$$K_{\pi\pi} = \frac{\alpha_\pi^2}{M_\rho^2 - s} + \frac{\beta_\pi^2}{s_2 - s} + \gamma_{\pi\pi},\ K_{KK} = \frac{\beta_K^2}{s_2 - s} + \gamma_{KK}$$
$$K_{\pi K} = K_{K\pi} = \frac{\beta_\pi \beta_K}{s_2 - s} + \gamma_{\pi K}, \quad (32)$$

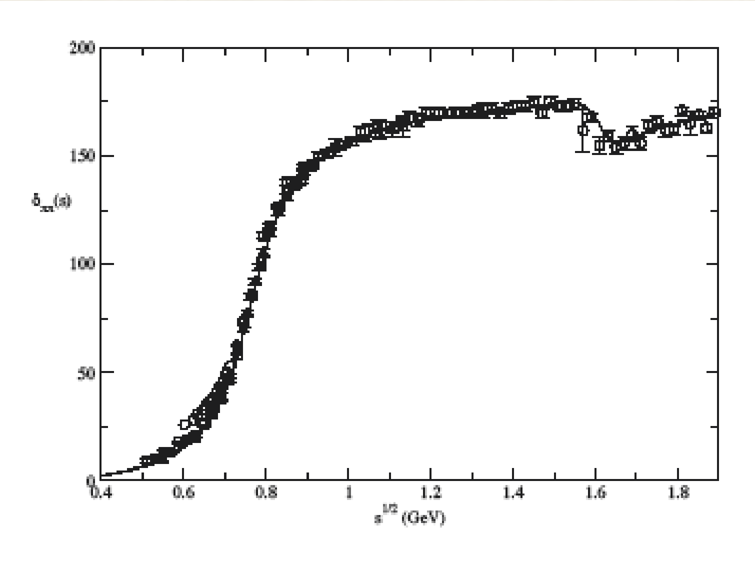
$$t_{\alpha\beta}(s) = \frac{N_{\alpha\beta}(s)}{D_{\alpha\beta}(s)}$$

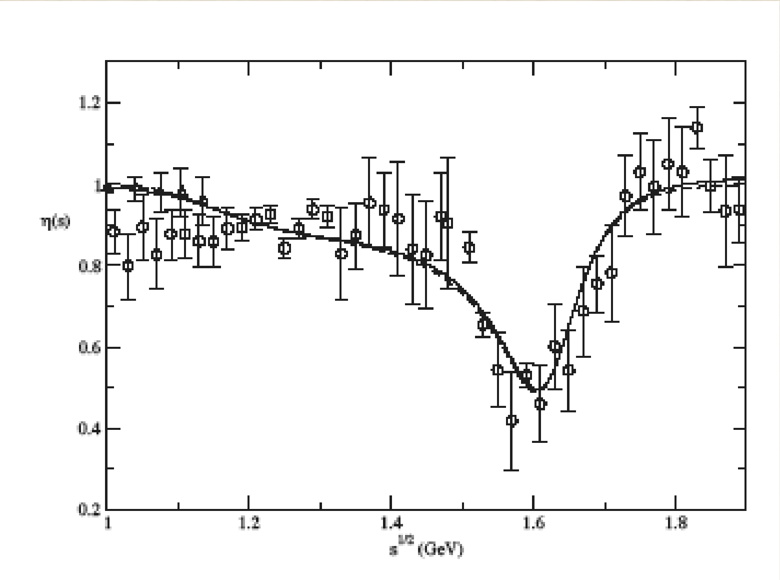
Analytical structure on first Riemann sheet

$$\begin{aligned}
t_{\pi\pi}(s) &= \lambda_{\pi\pi}\frac{(s-4m_\pi^2)(s-z_{\pi\pi})}{(s-s_{L,1})(s-s_{L,2})}e^{\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds'\,\frac{\varphi_{\pi\pi}(s')}{s'(s'-s-i0)}}, \\
t_{\pi K}(s) &= (q_\pi q_K)\lambda_{\pi K}\frac{(s-m_\rho^2)(s-z_{\pi K})}{(s-s_{L,1})(s-s_{L,2})}e^{\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds'\,\frac{\varphi_{\pi K}(s')}{s'(s'-s-i0)}}, \\
t_{KK}(s) &= \lambda_{KK}\frac{(s-4m_K^2)(s-z_{KK})}{(s-s_{L,1})(s-s_{L,2})}e^{\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds'\,\frac{\varphi_{KK}(s')}{s'(s'-s-i0)}}.
\end{aligned}$$

$$\varphi_{\alpha\beta}(s) = \tan^{-1}\frac{Im[t_{\alpha\beta}(s)]}{Re[t_{\alpha\beta}(s)]}$$

2 channel K-matrix fit   looking good but...





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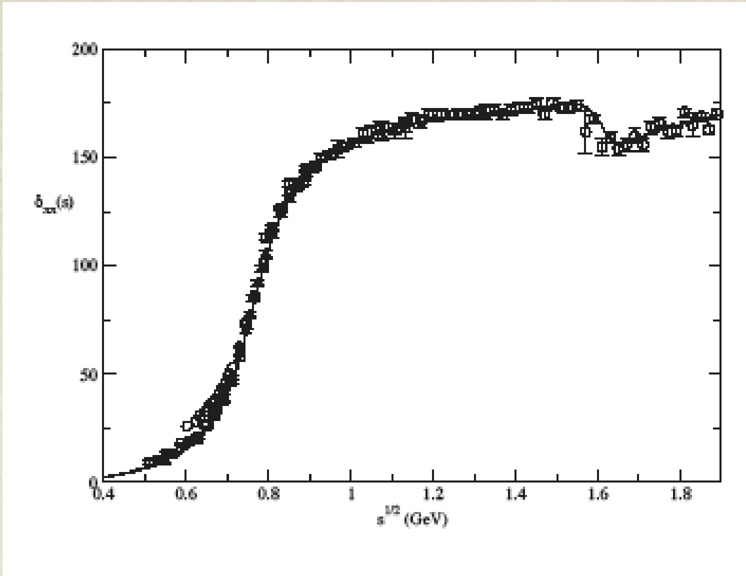

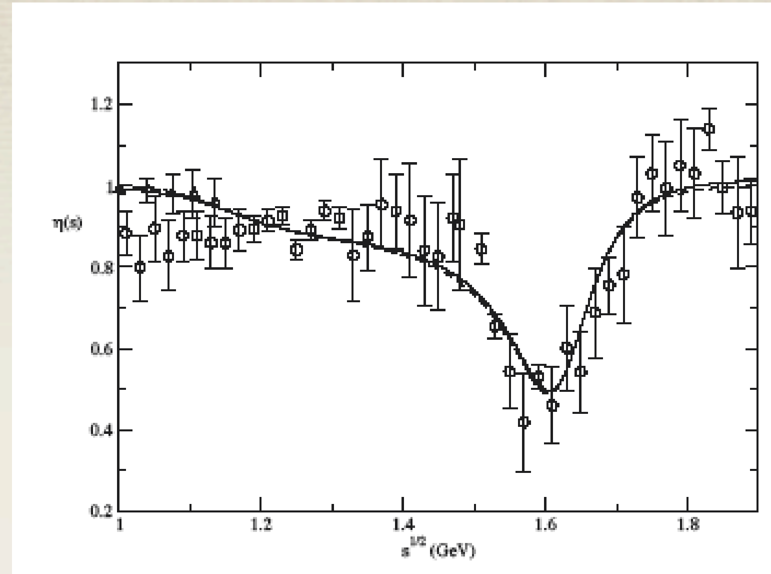

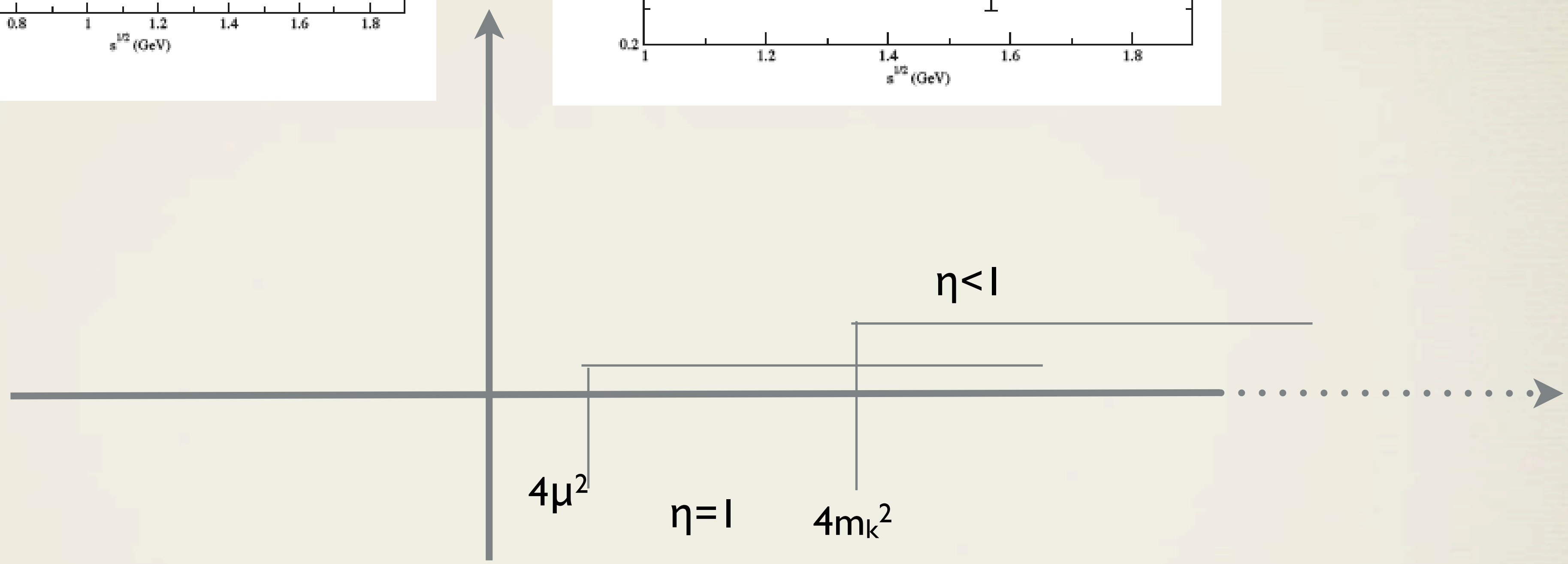

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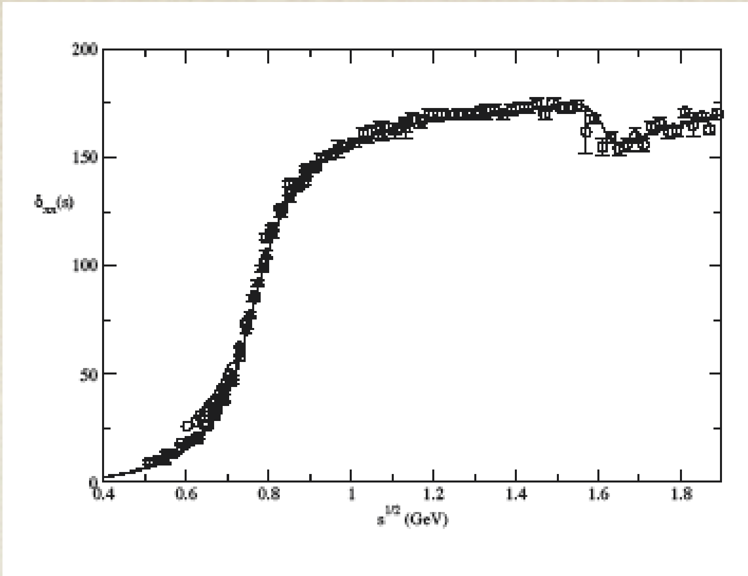

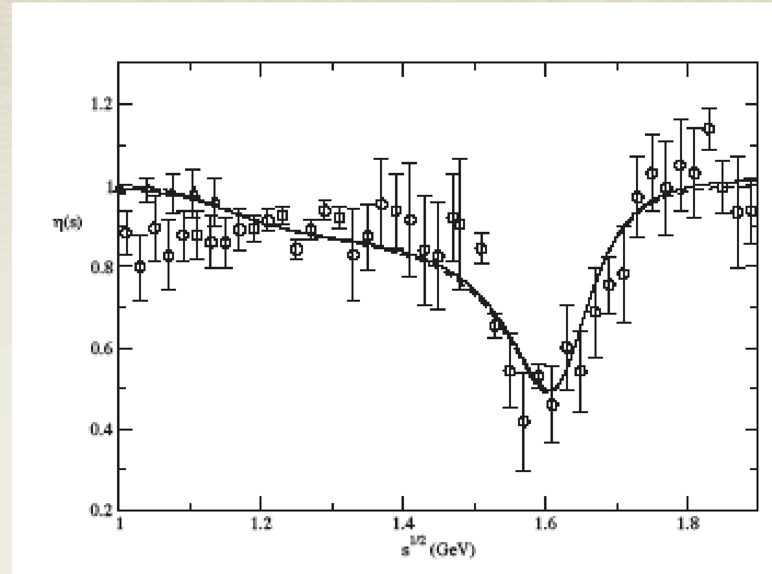

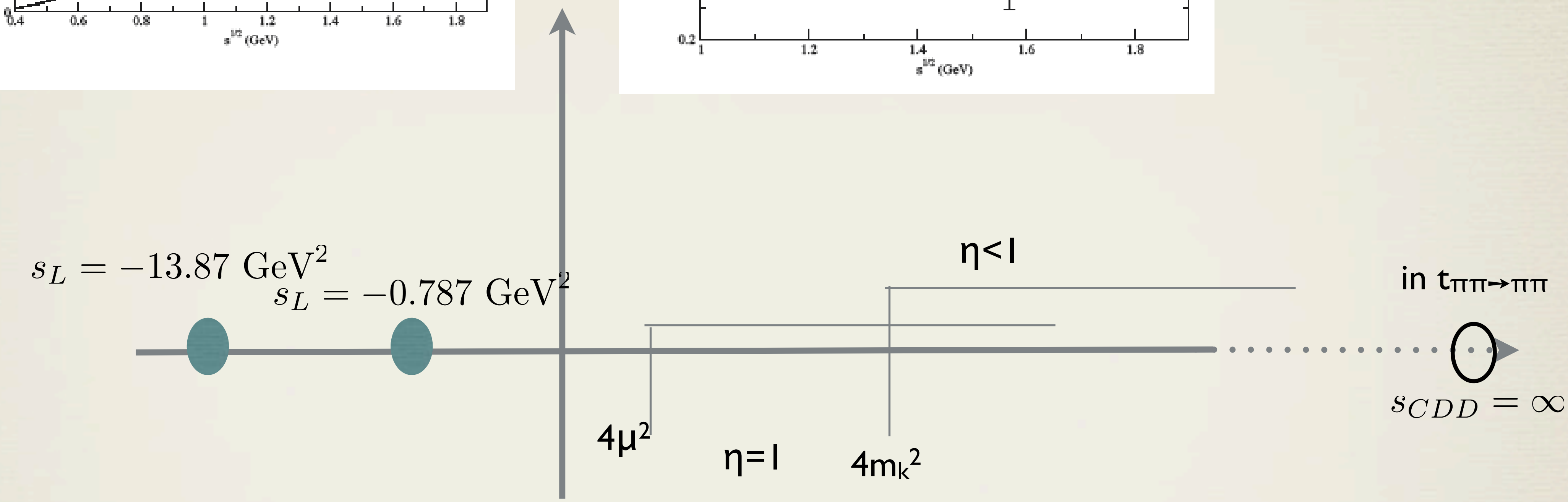

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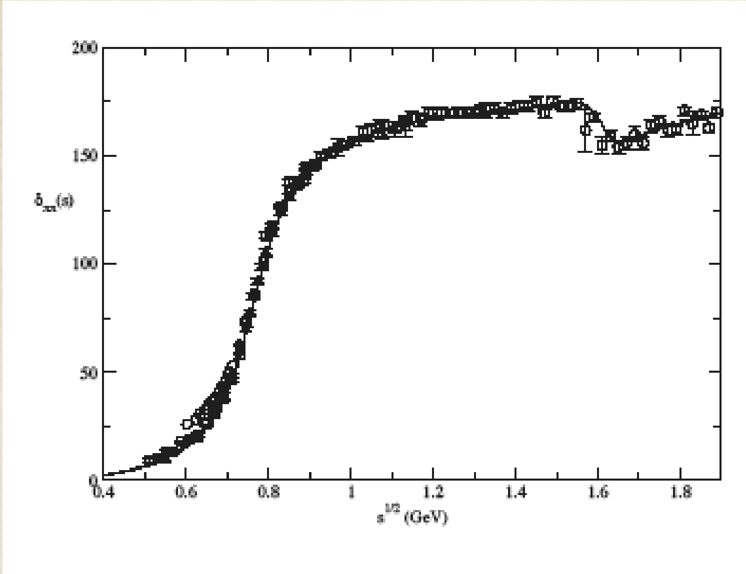
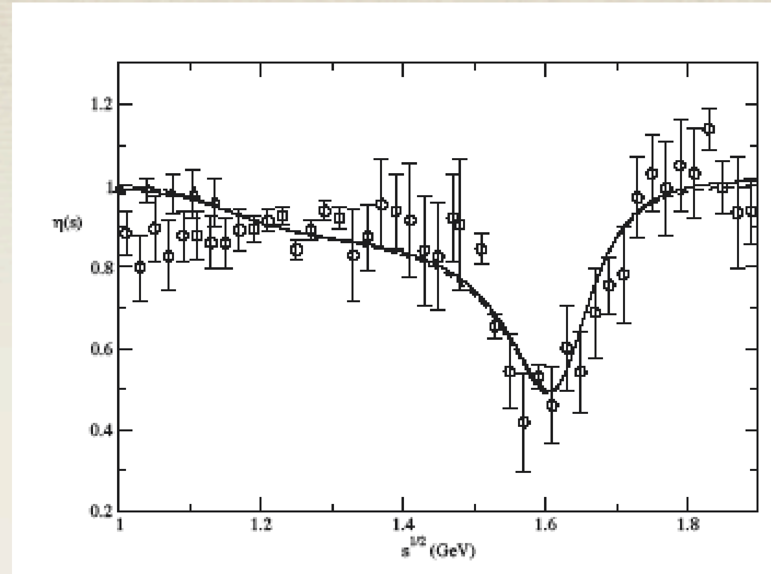
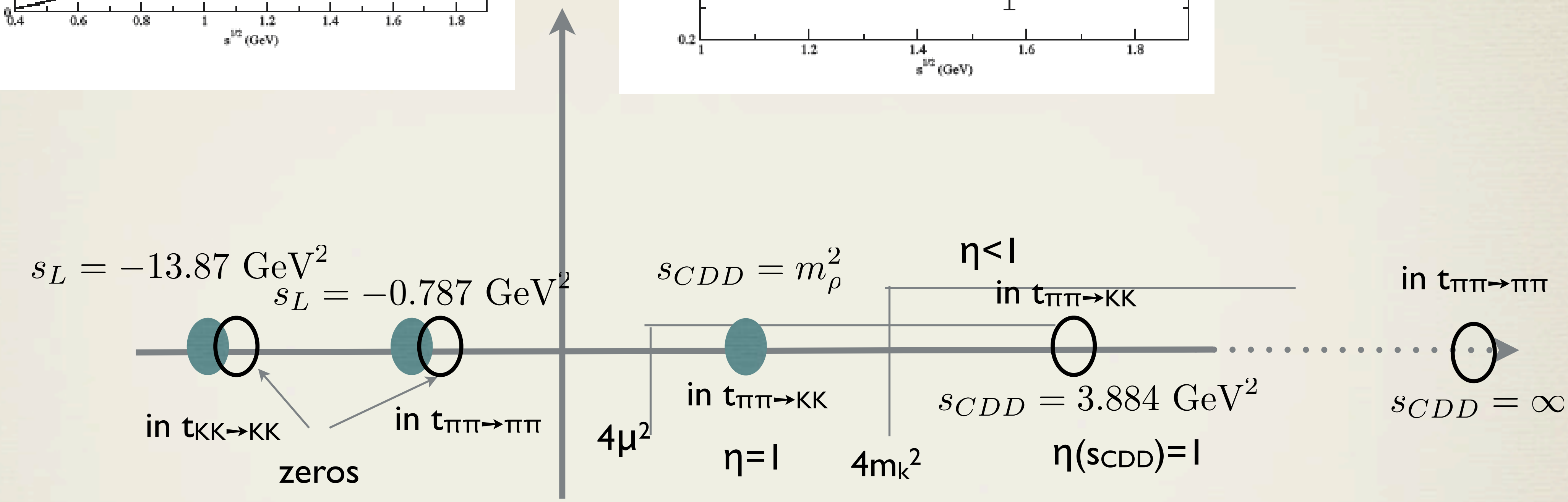
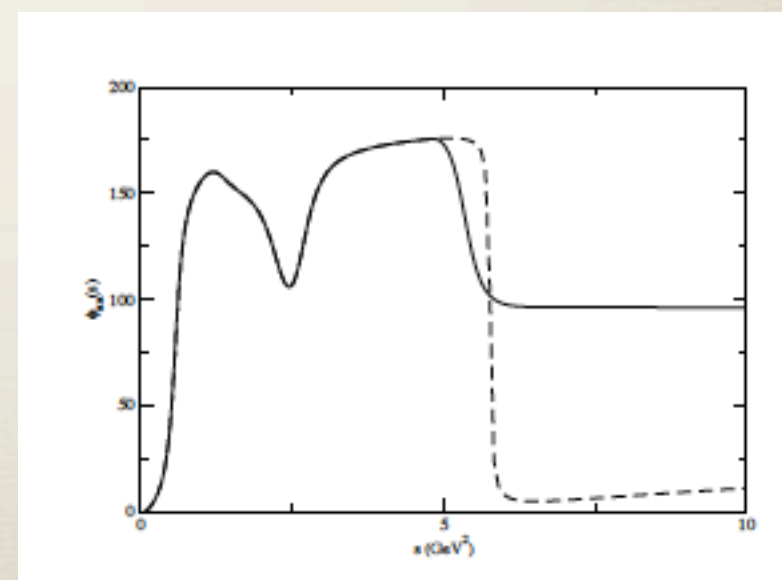


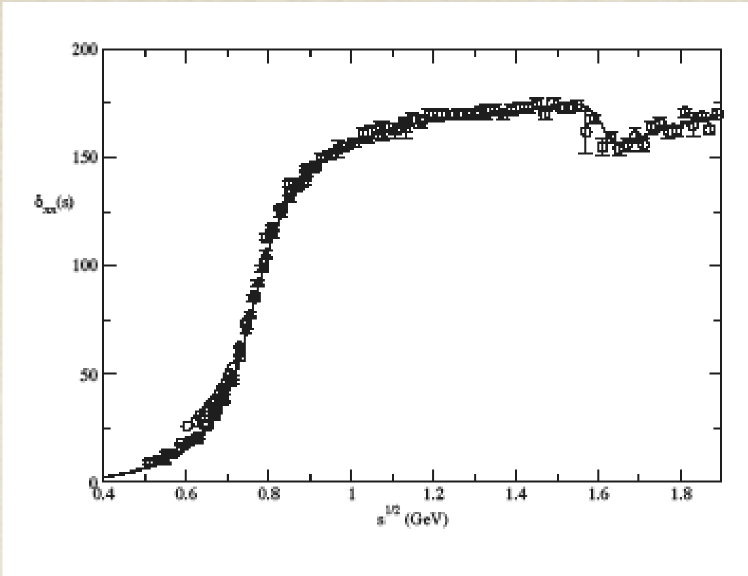




and another zero in KK -> KK



2 channel K-matrix fit



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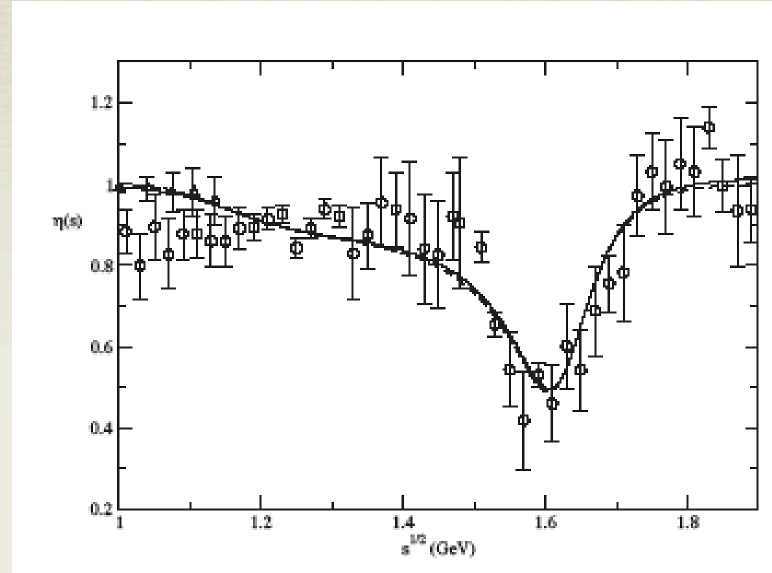

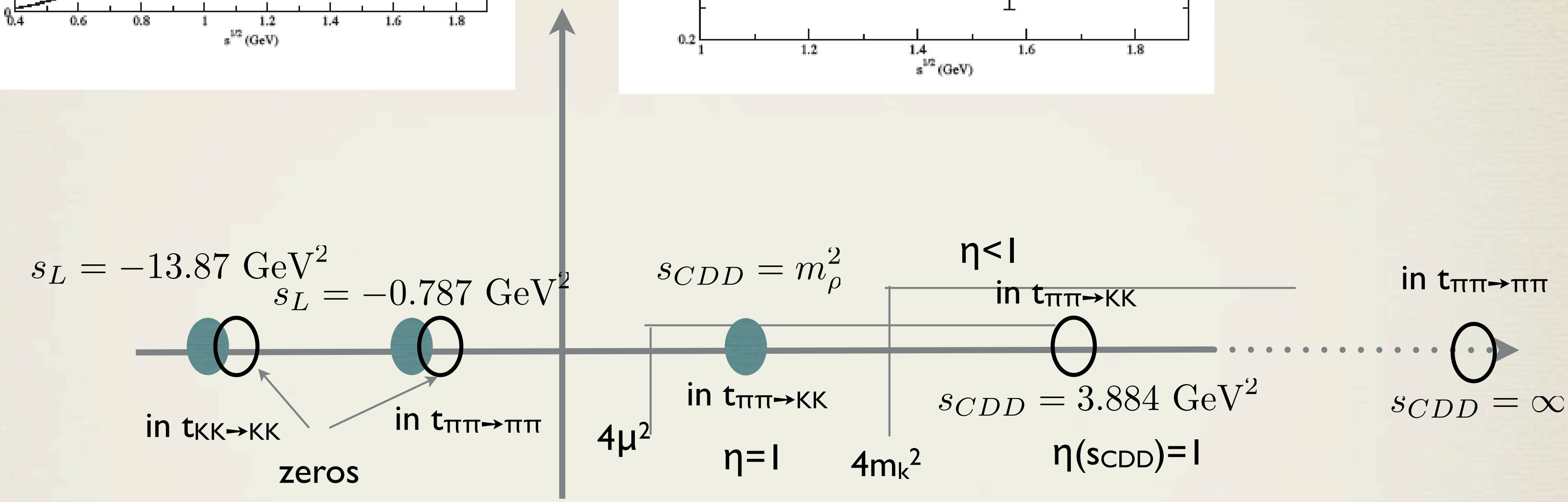

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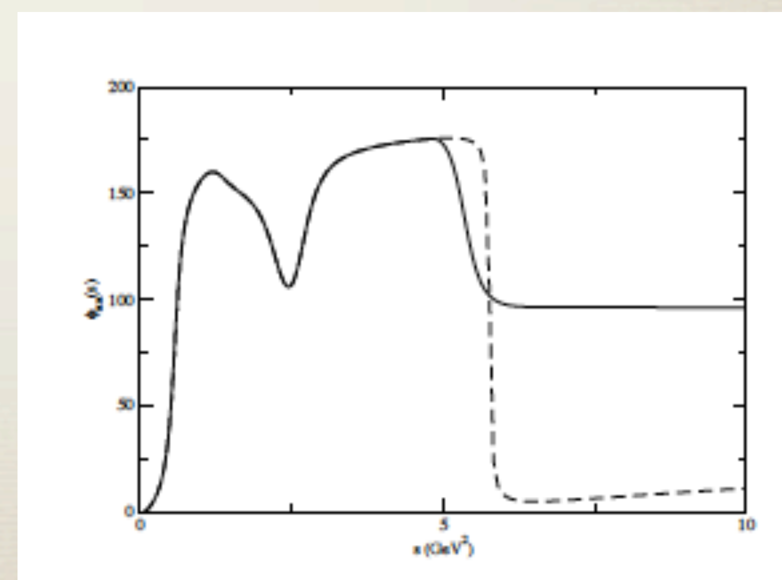
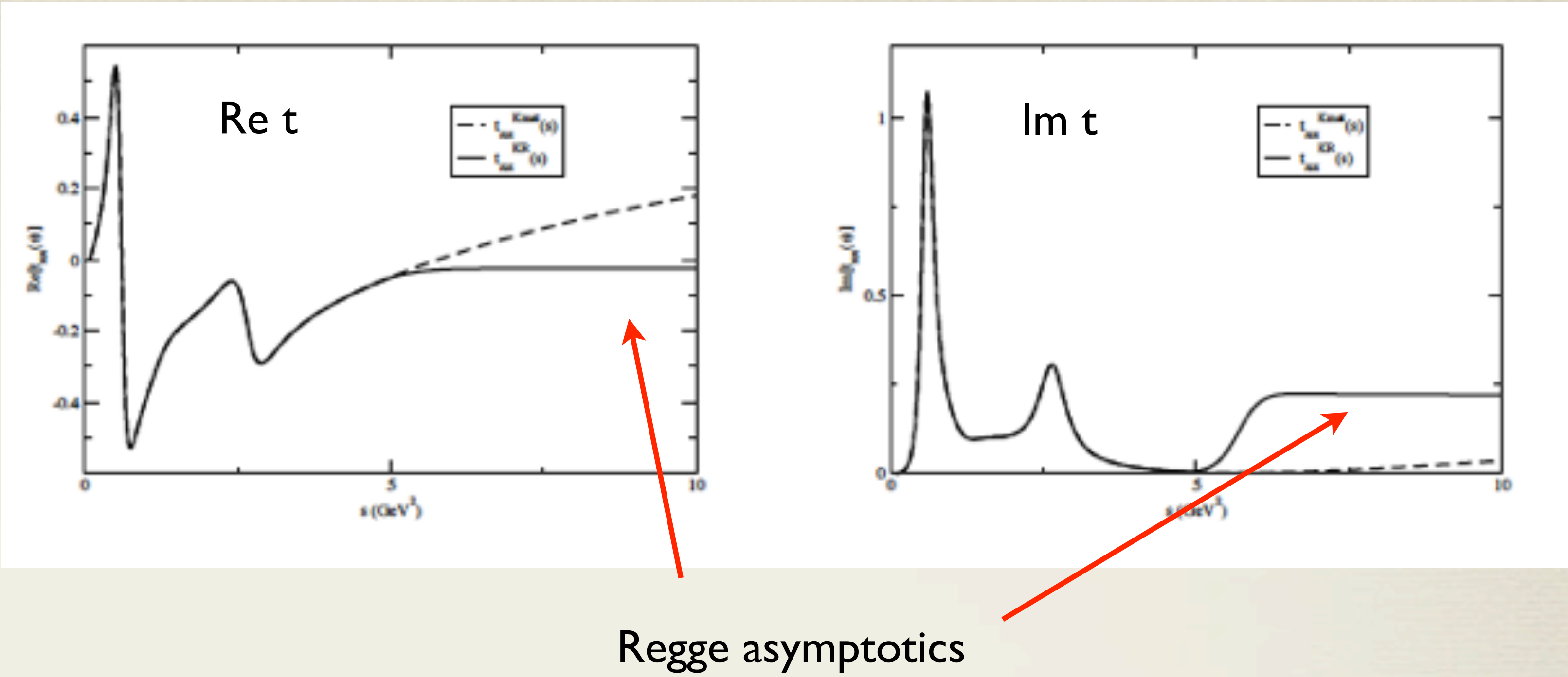


* K-matrix in general unreasonable (the "reality" of the near poles and zeros can be checked)

Amplitude construction II

- use K-matrix in the data region
- extrapolate using Regge asymptotic



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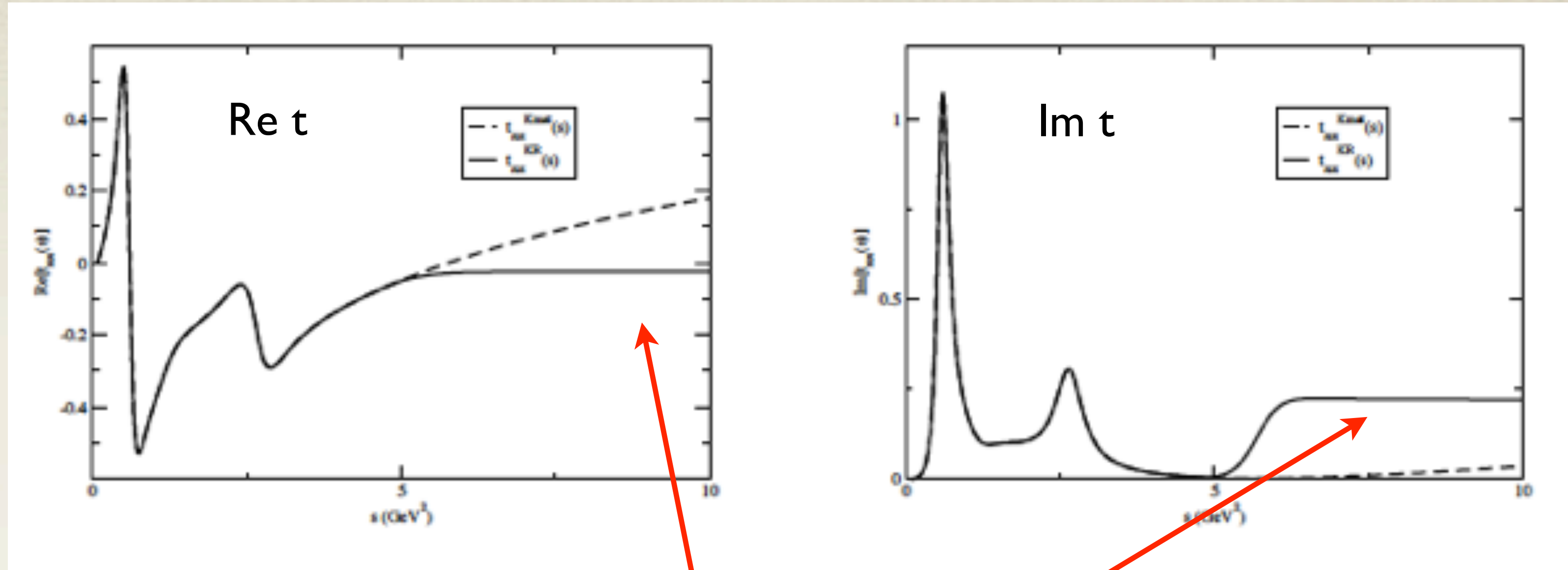


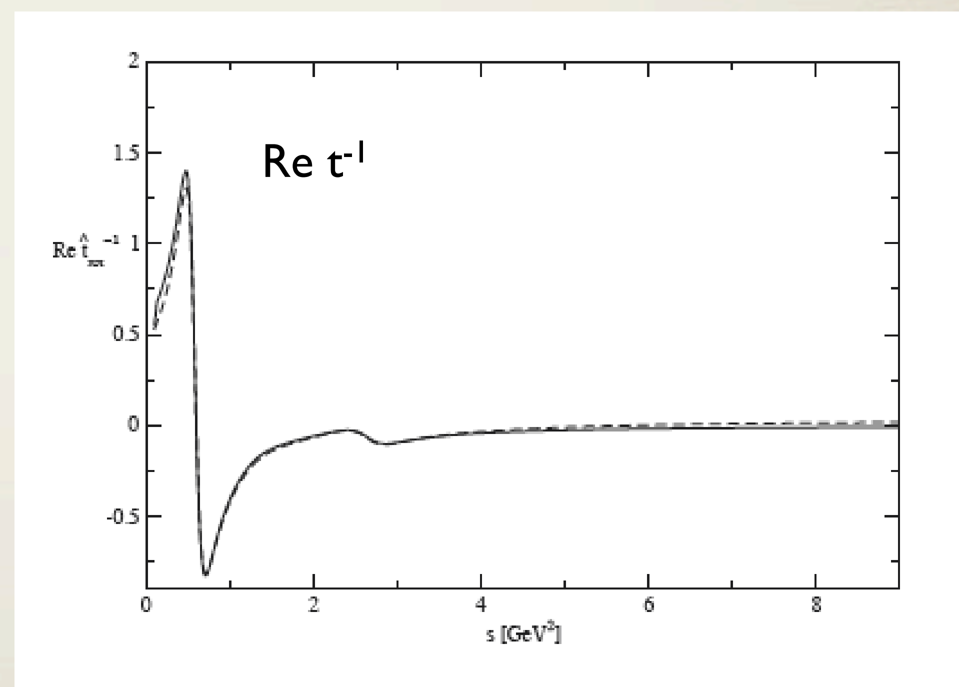
Regge asymptotics

Amplitude construction III

recompute phase $\varphi_{\alpha\beta}(s) = \tan^{-1}\frac{Im[t_{\alpha\beta}(s)]}{Re[t_{\alpha\beta}(s)]}$

and D(s) $t_{\alpha\beta}(s) = \frac{N_{\alpha\beta}(s)}{D_{\alpha\beta}(s)}$ via Omnes-Muskhelishvili integral (right hand cut)

fit a simple N to reproduce data $N_{\alpha\beta}(s) = \frac{\lambda_{\alpha\beta}}{s - s_L}$



Comparison with dispersion relation

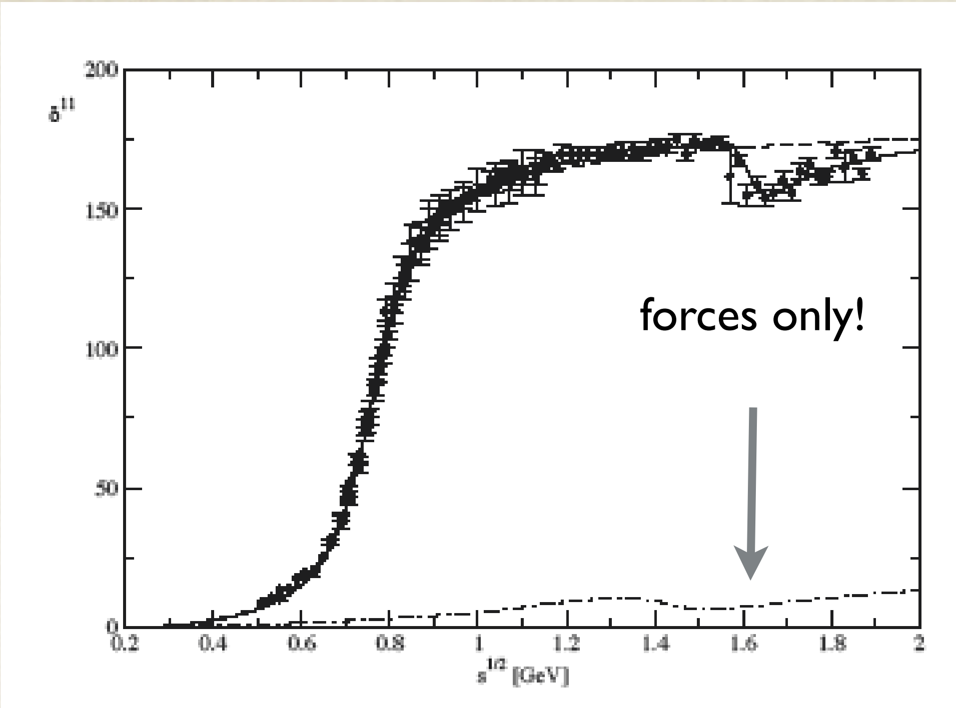
$$A(s) = \frac{1}{\pi}\int_{-\infty}^{0} ds' \frac{ImA(s')}{s'-s} + \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{ImA(s')}{s'-s}$$

crossing symmetry (low energy),
Regge limit (high energy)

$$ImA(s) = \rho(s)|A(s)|^2$$

assume elastic unitarity

Comparison with dispersion relation



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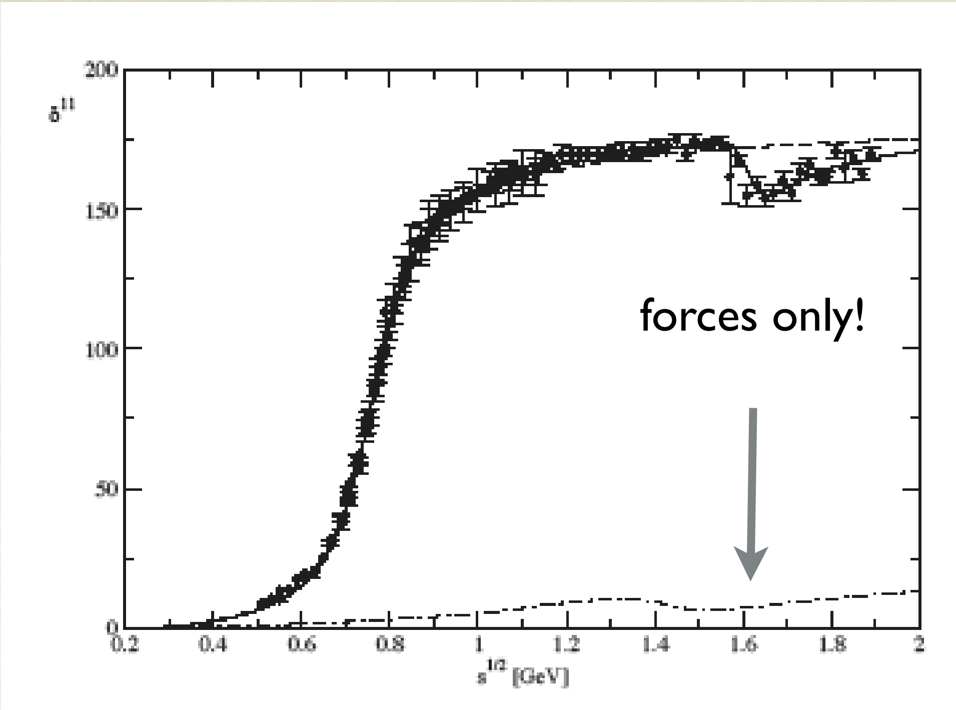
* CDD pole required !

Comparison with dispersion relation



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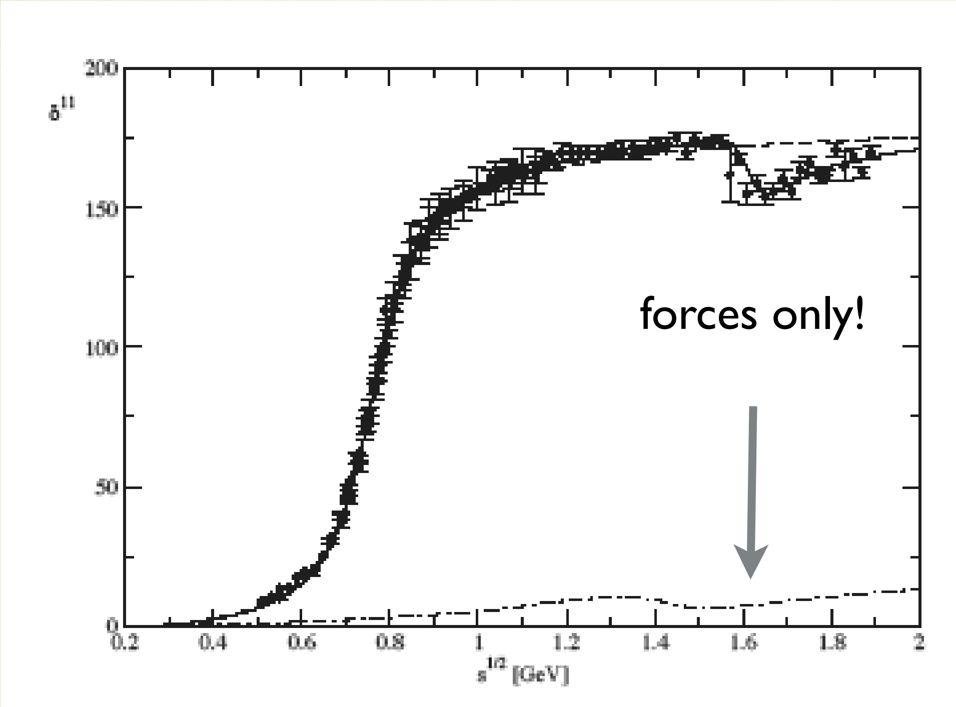

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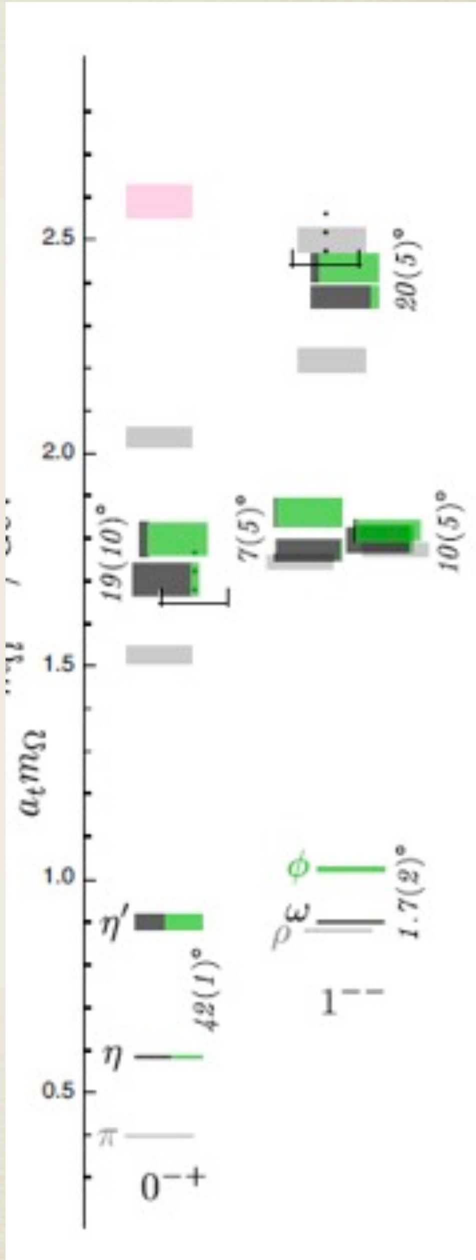
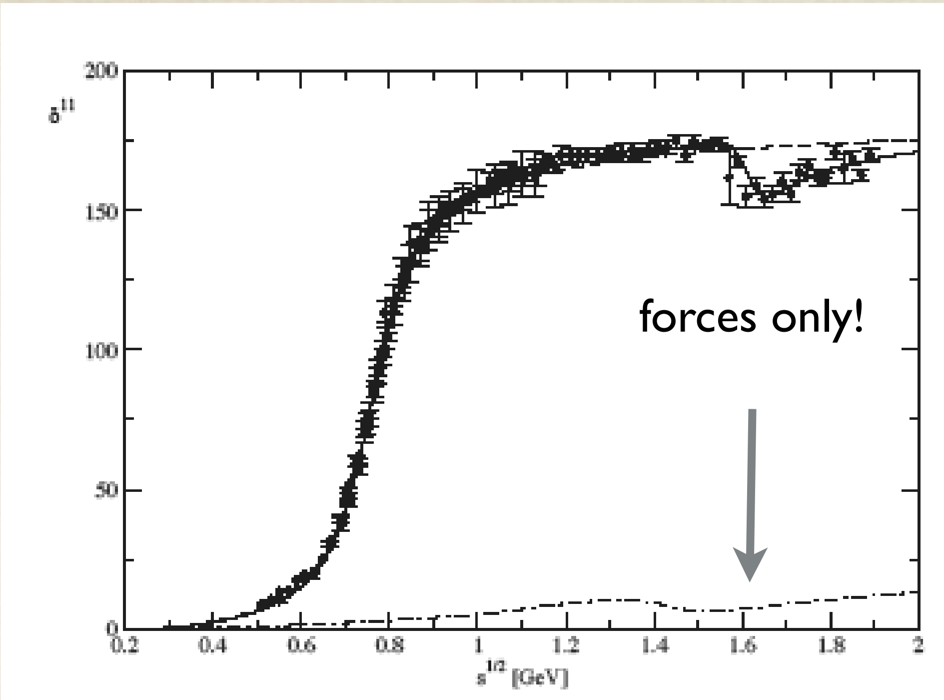


J.Dudek et al. 2011

Comparison with dispersion relation



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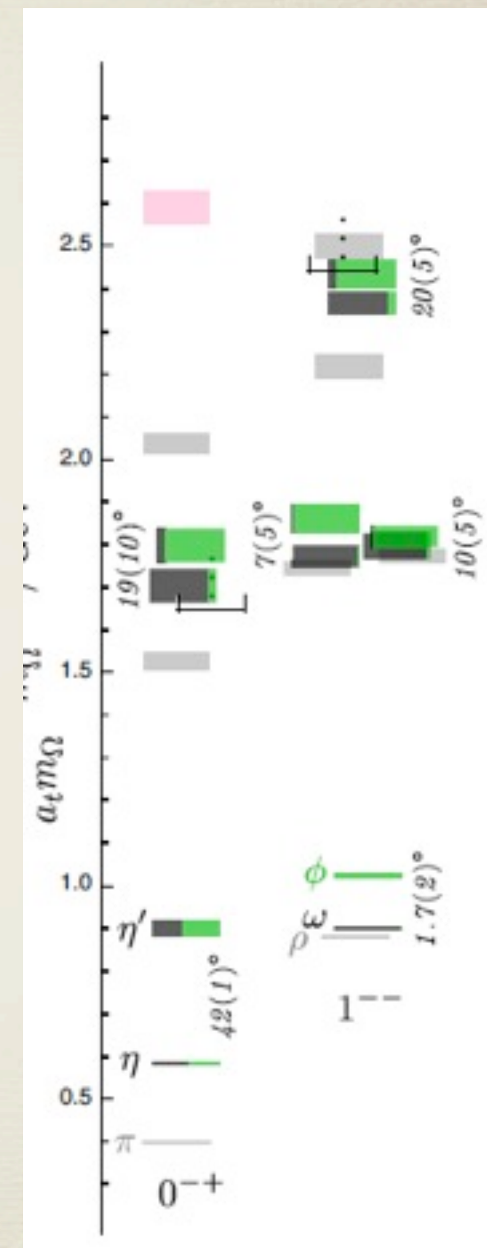
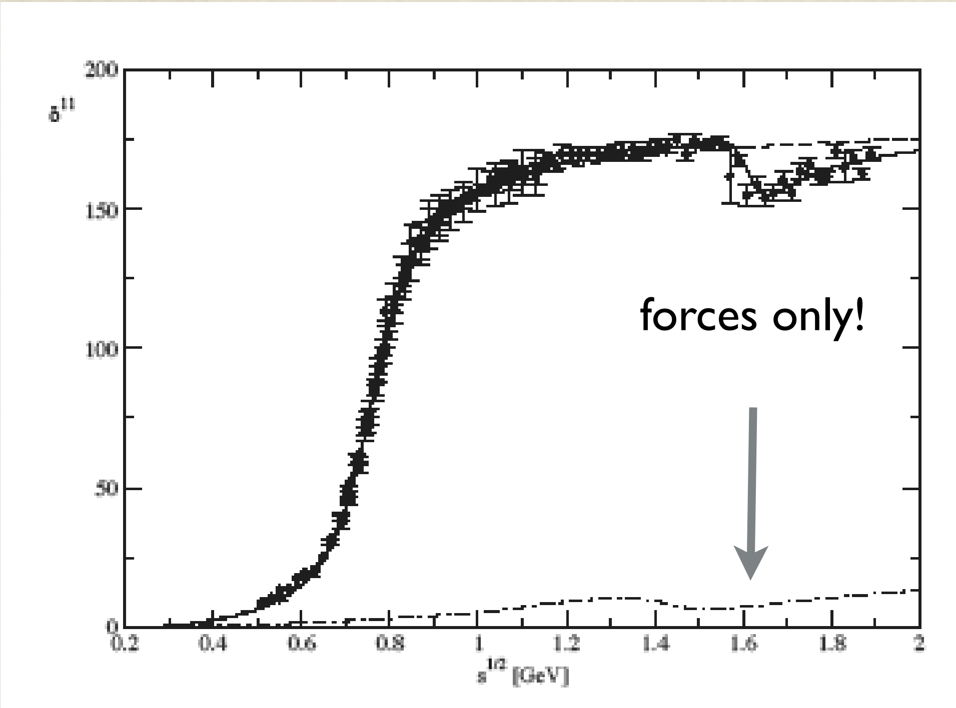
most resonances do not originate from meson-meson interactions but from the underlying QCD dynamics.



J.Dudek et al. 2011

Comparison with dispersion relation



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crossing symmetry (low energy), Regge limit (high energy)

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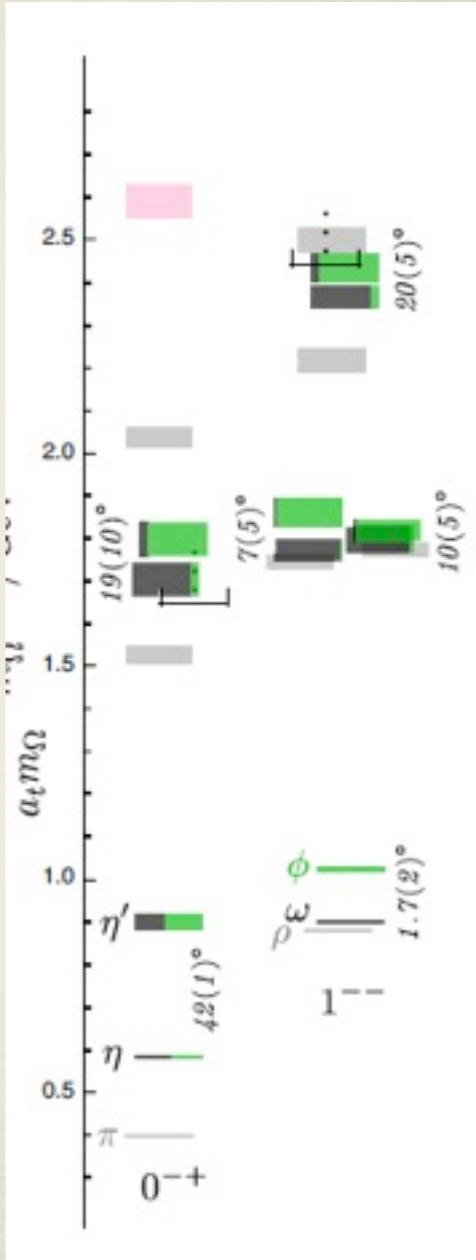
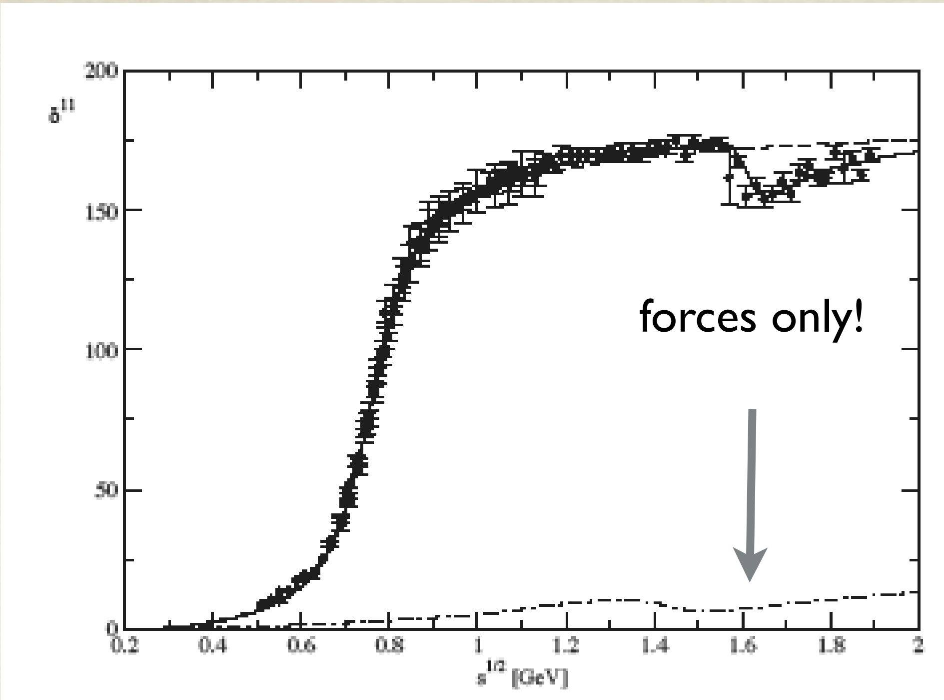


J.Dudek et al. 2011

most resonances do not originate from meson-meson interactions but from the underlying QCD dynamics.

resonances are not generated dynamically from interactions between other resonances

Comparison with dispersion relation



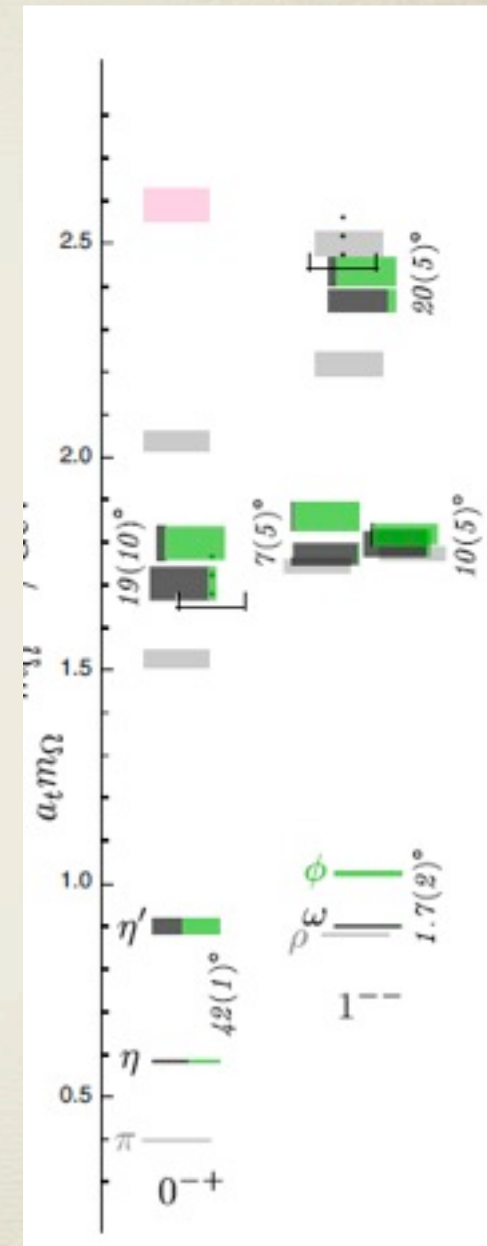
$$A(s) = \frac{1}{\pi}\int_{-\infty}^{0} ds' \frac{ImA(s')}{s'-s} + \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{ImA(s')}{s'-s}$$

crossing symmetry (low energy),
Regge limit (high energy)

$$ImA(s) = \rho(s)|A(s)|^2$$

assume elastic unitarity

* CDD pole required !
* bootstrap failed
* and the quark model was born or as lattice suggests there are single hadron states in the spectrum



J.Dudek et al. 2011

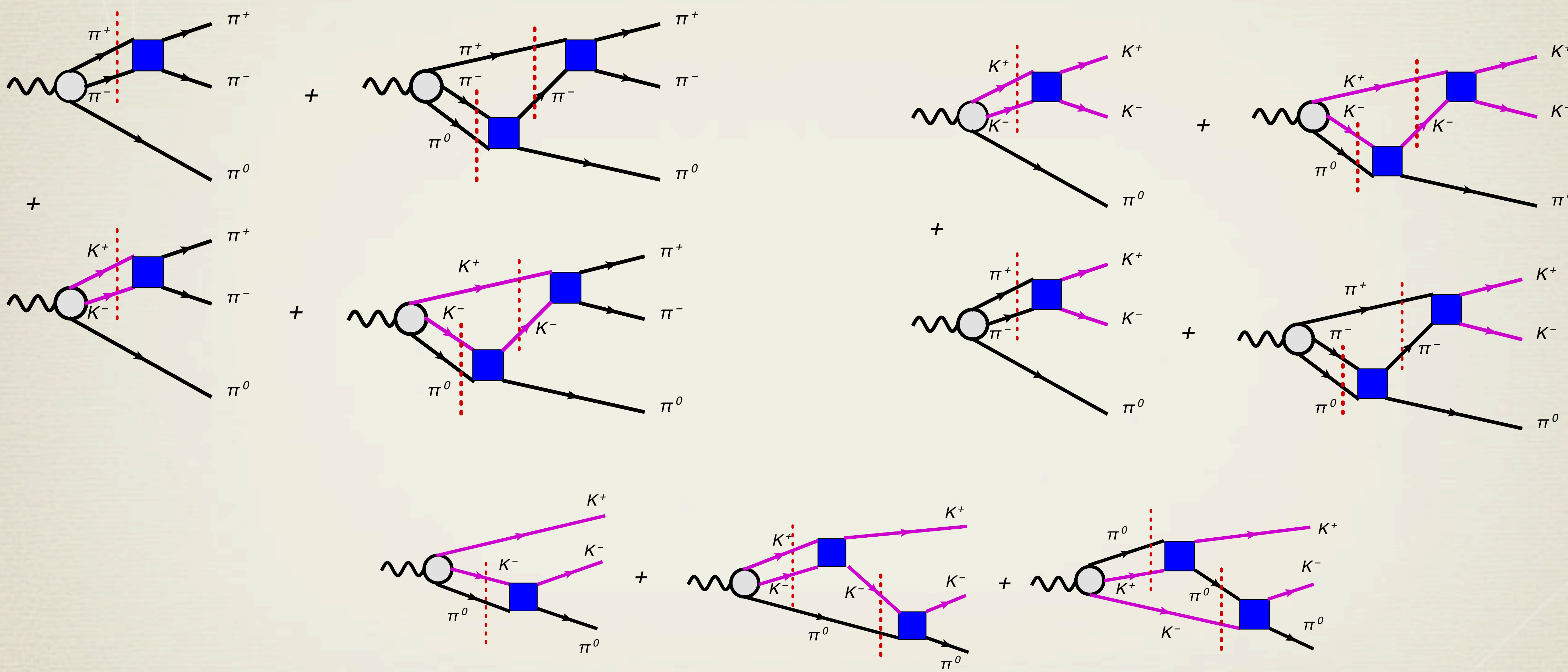
most resonances do not originate from meson-meson interactions but from the underlying QCD dynamics.

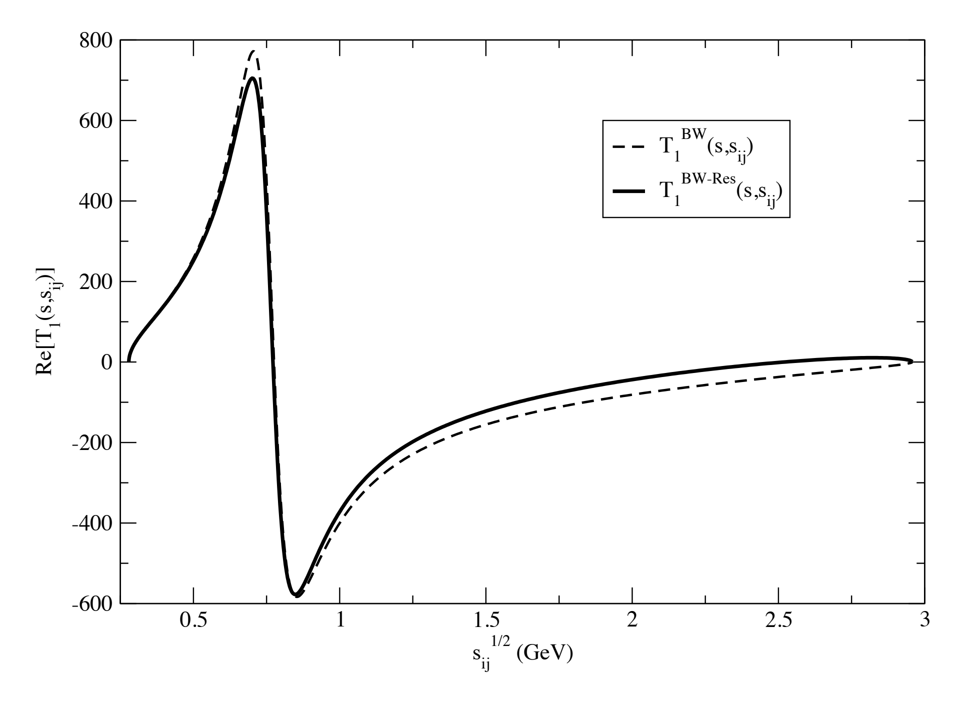
resonances are not generated dynamically from interactions between other resonances

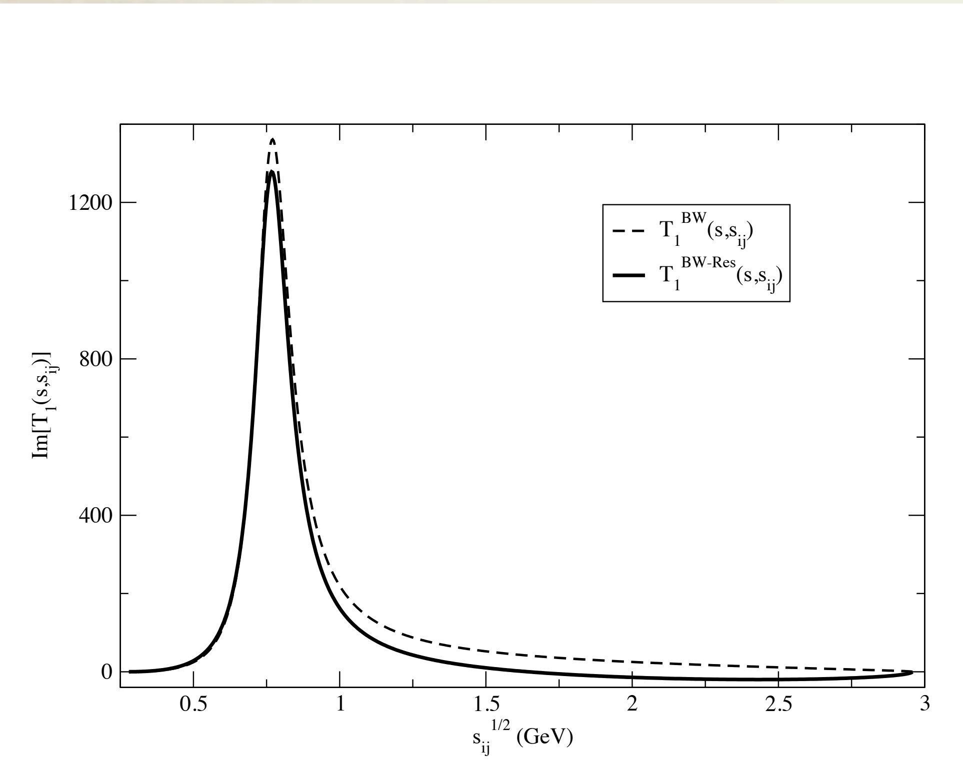
how does it fit in with the success of dynamically generated resonance program from a unitarized chi-PT approach ?
It does in U chi-PT resonances also come form short distance (QCD) physics via subtractions, cut offs, and not meson-meson interactions

Applications J/Ψ → $\pi^+\pi^-\pi^0$, $K^+K^-\pi^0$

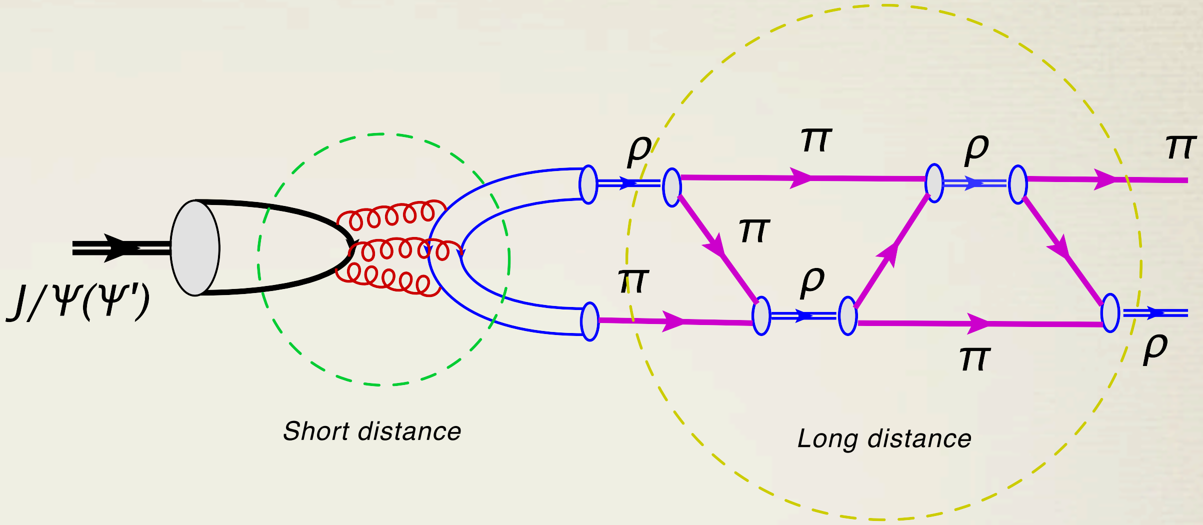
Isobar model interactions (diagonal and channel mixing)
and re-scattering (beyond isobar)







re-scattering corrections are small



Khuri-Treiman (1960)
Pasquier-Pasquier (1968-1970)
Aitchison,Brehm (late 70's)

Application to J/Ψ → $\pi^+\pi^-\pi^0$

$$\langle \pi^0\pi^+\pi^-, out | J/\psi(\lambda), in\rangle = (2\pi)^4\delta^4(\sum_{i=0,\pm} p_i - P) i T_\lambda,$$

- P-wave ππ

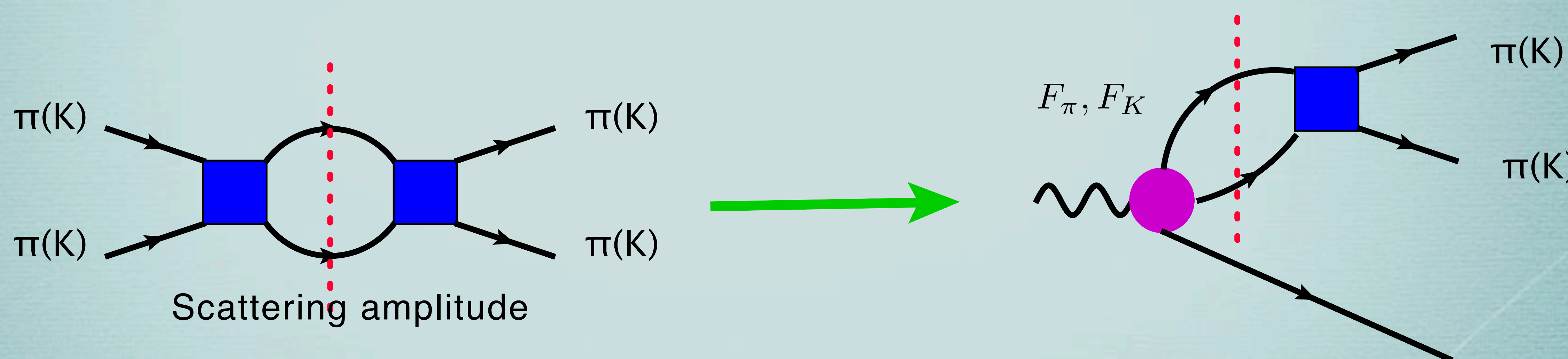
$$T_\lambda = \sum_{i=0,\pm}\sum_{\mu=\pm,0} D^{1*}_{\lambda,\mu}(r_i) d^1_{\mu,0}(\theta_i) F_\mu(s_{jk})$$

$$F_{+1} = F_{-1} = F, F_0 = 0$$

$$\begin{aligned} Im\hat{F}_\pi(s+i\epsilon) &= \hat{t}^*_{\pi\pi}(s)\hat{\rho}_\pi(s)\hat{F}_\pi(s)\theta(s-4m_\pi^2) \\ &+ \hat{t}^*_{\pi K}(s)\hat{\rho}_K(s)\hat{F}_K(s)\theta(s-4m_K^2). \end{aligned}$$

$$\begin{aligned} Im\hat{F}_K(s+i\epsilon) &= \hat{t}^*_{K\pi}(s)\hat{\rho}_\pi(s)\hat{F}_\pi(s)\theta(s-4m_\pi^2) \\ &+ \hat{t}^*_{KK}(s)\hat{\rho}_K(s)\hat{F}_K(s)\theta(s-4m_K^2) \end{aligned}$$

- this unitary relations have simple (algebraic) solution provided $N_{\alpha\beta}(s) = N(s)$



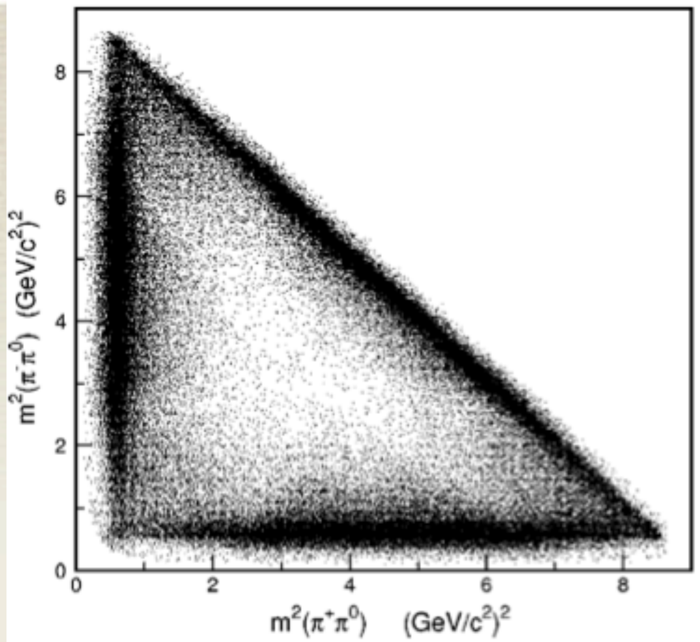
Scattering amplitude

Decay amplitude

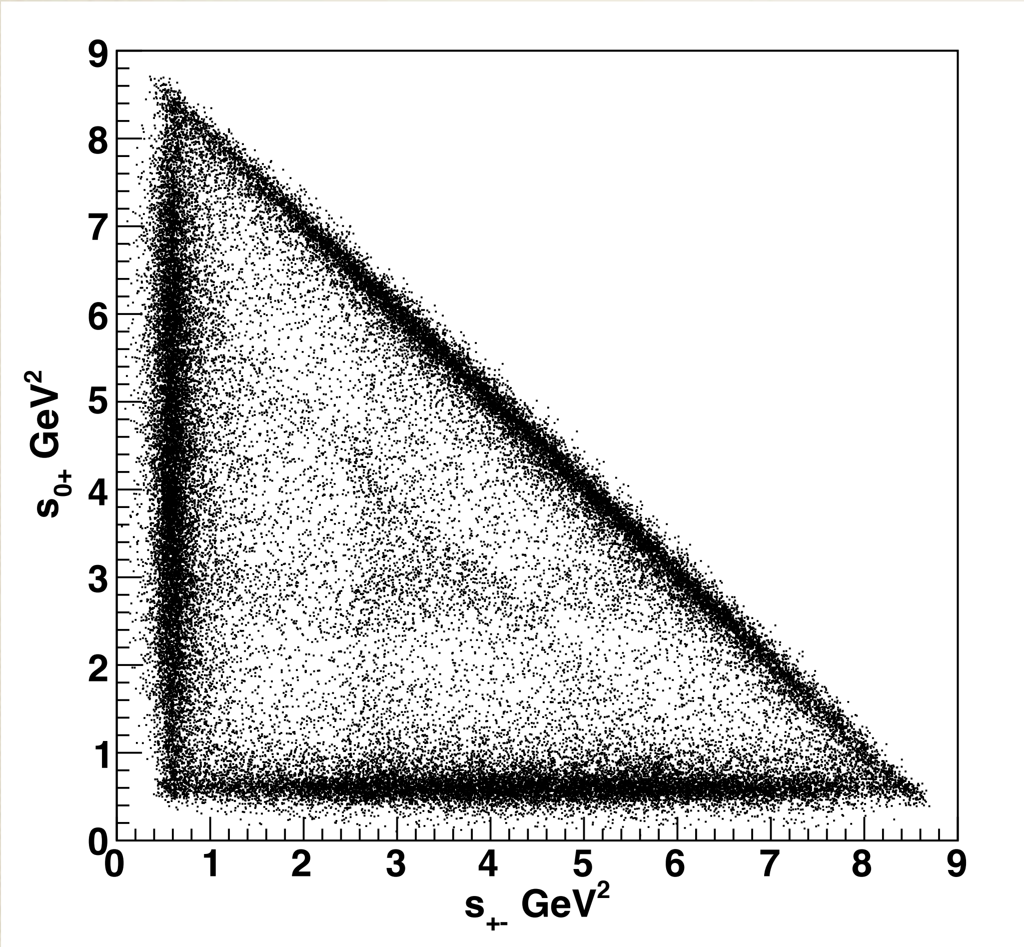
$$\sim \frac{N_{\alpha\beta}(s)}{D_{\alpha\beta}(s)}$$

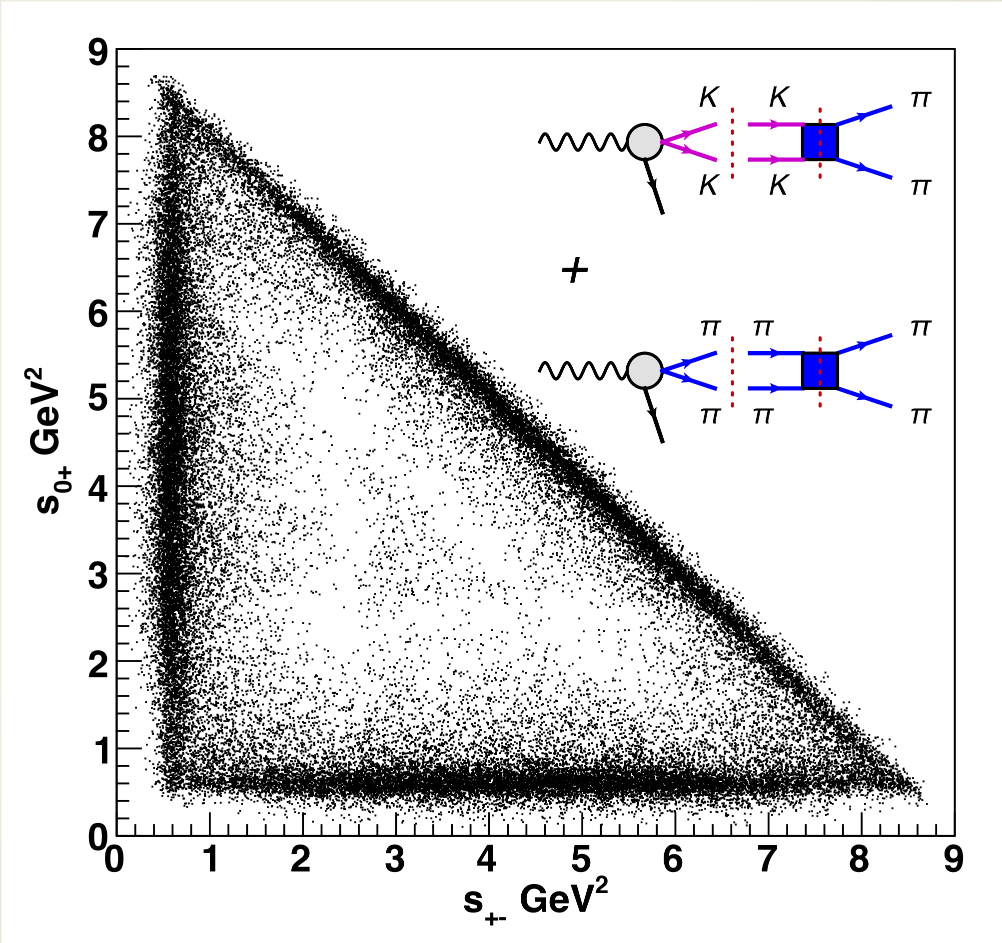
$$\sim \frac{P_{\alpha\beta}(s)}{D_{\alpha\beta}(s)}$$

$$J/\psi \to 3\pi$$



BES Collaboration
Phys.Rev.D70:012005,2004

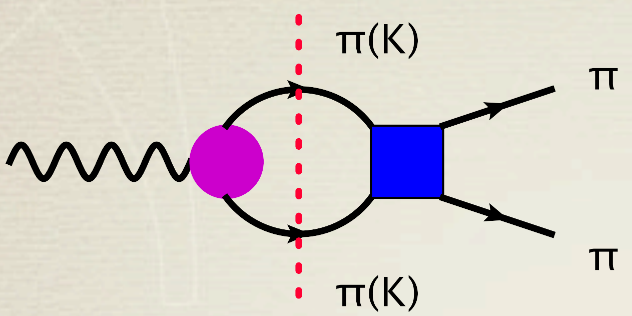


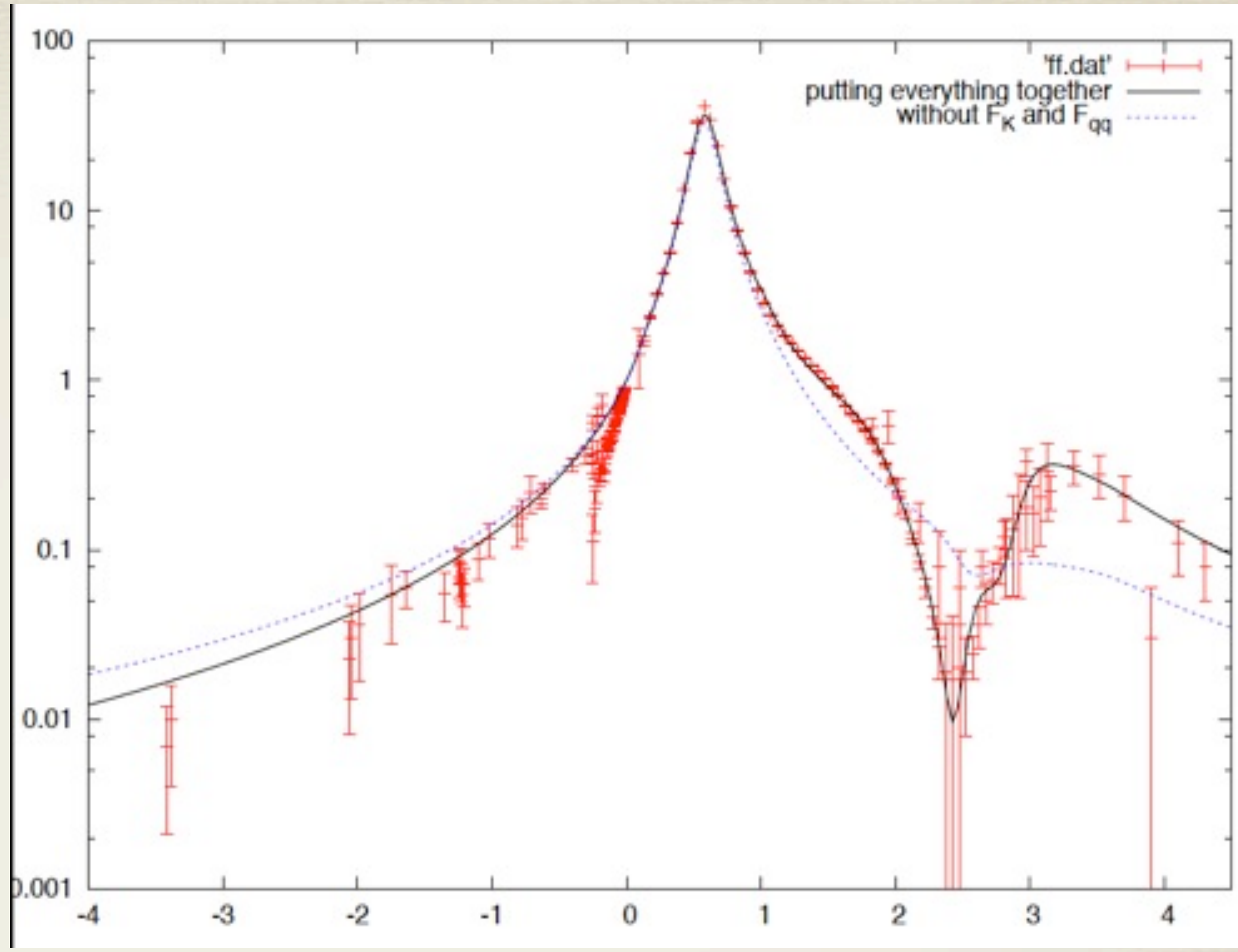


$$F(s) \sim \frac{1}{D_{\pi\pi\to\pi\pi}(s)}$$

$$F(s) \sim \frac{1 + c_1 s}{D_{\pi\pi\to\pi\pi}(s)} + \frac{c_0}{D_{K\bar{K}\to\pi\pi}(s)}$$

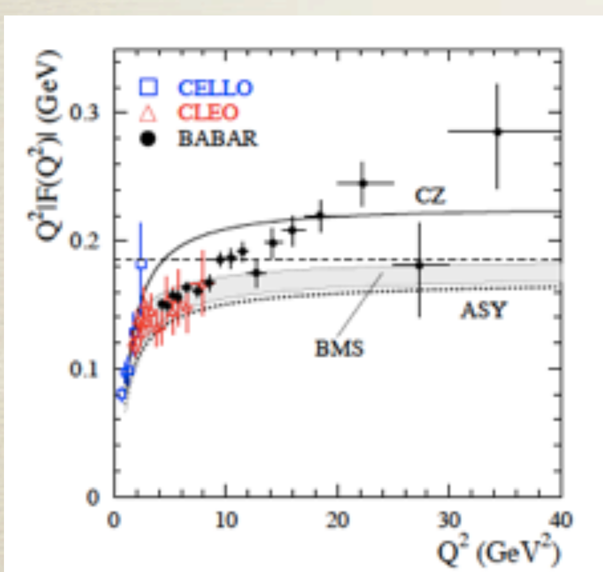
Pion formfactor: $|F_{\pi\pi}(s)|^2$

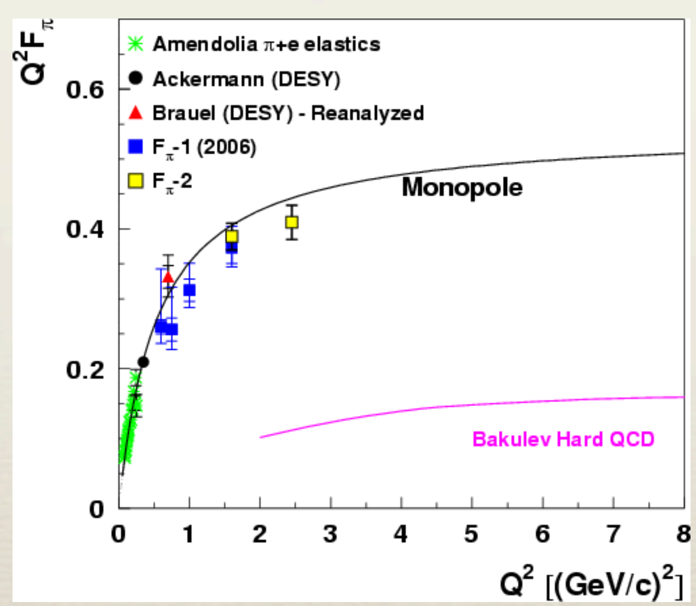


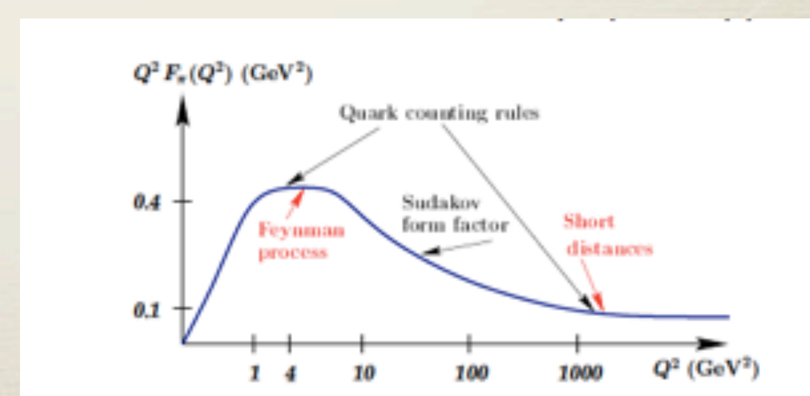


$$F(s) \sim \frac{1 + c_1 s}{D_{\pi\pi\to\pi\pi}(s)} + \frac{c_0}{D_{K\bar{K}\to\pi\pi}(s)}$$

Novel interpretation of asymptotic behavior
(M.Gorshteyn,P.Guos,AS (2011))



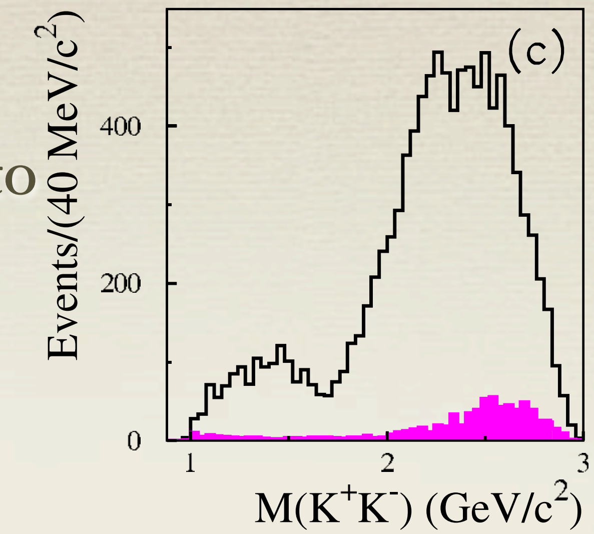


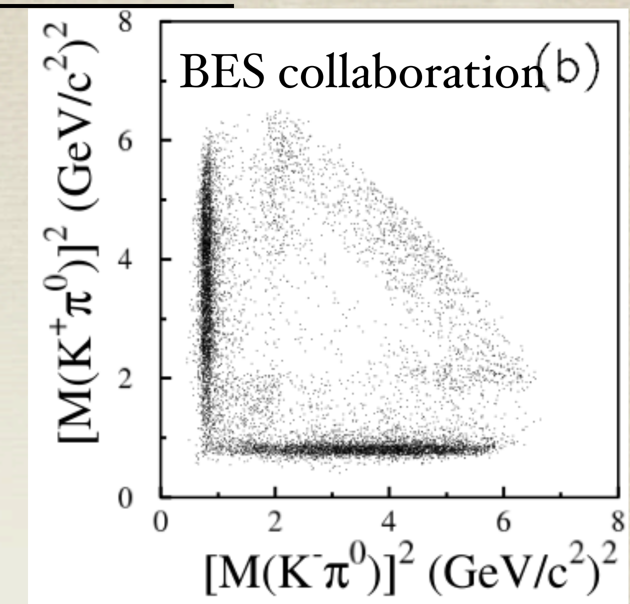


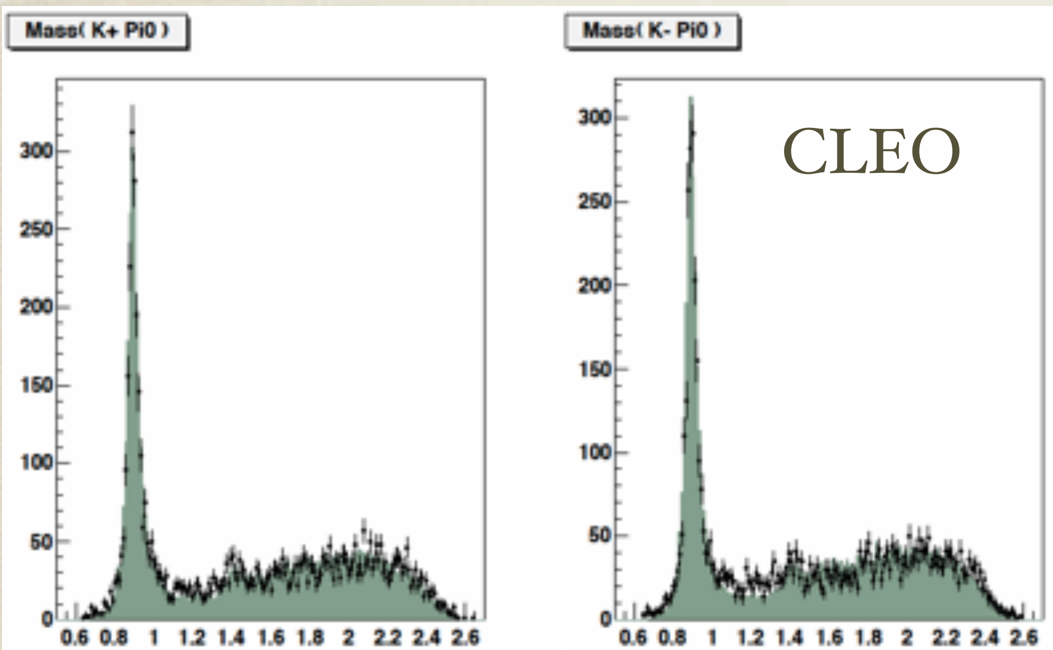
$$J/\psi \to K\bar{K}\pi^0$$

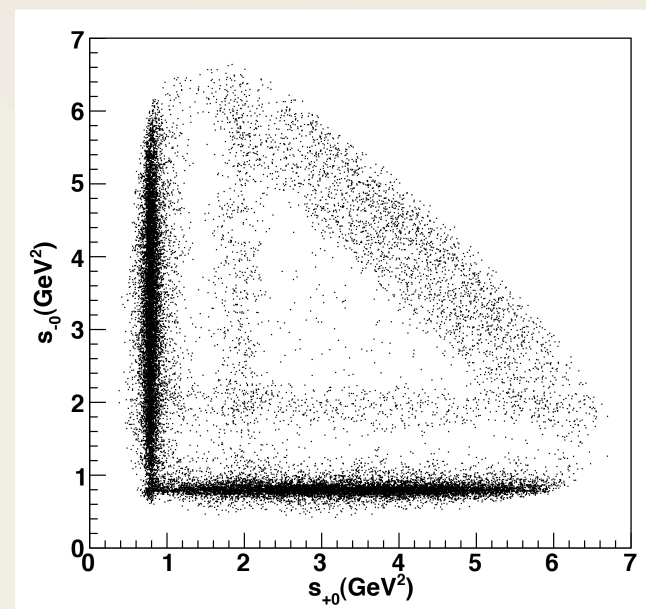
PRL **97**, 142002 (2006)

Broad bump in low mass KK region is difficult to be explained by a single BW.

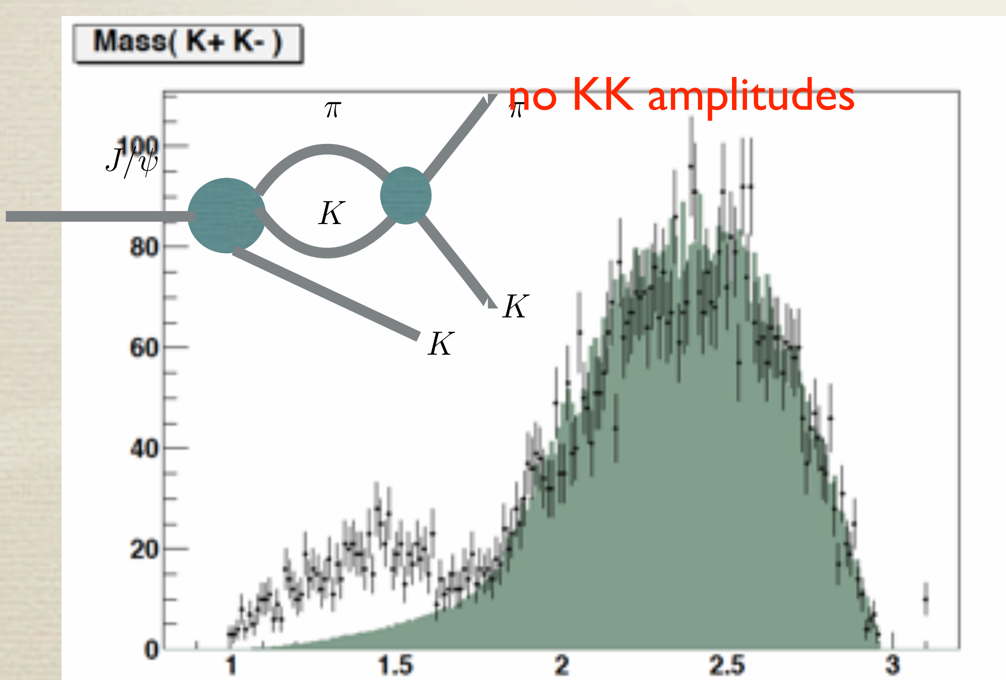


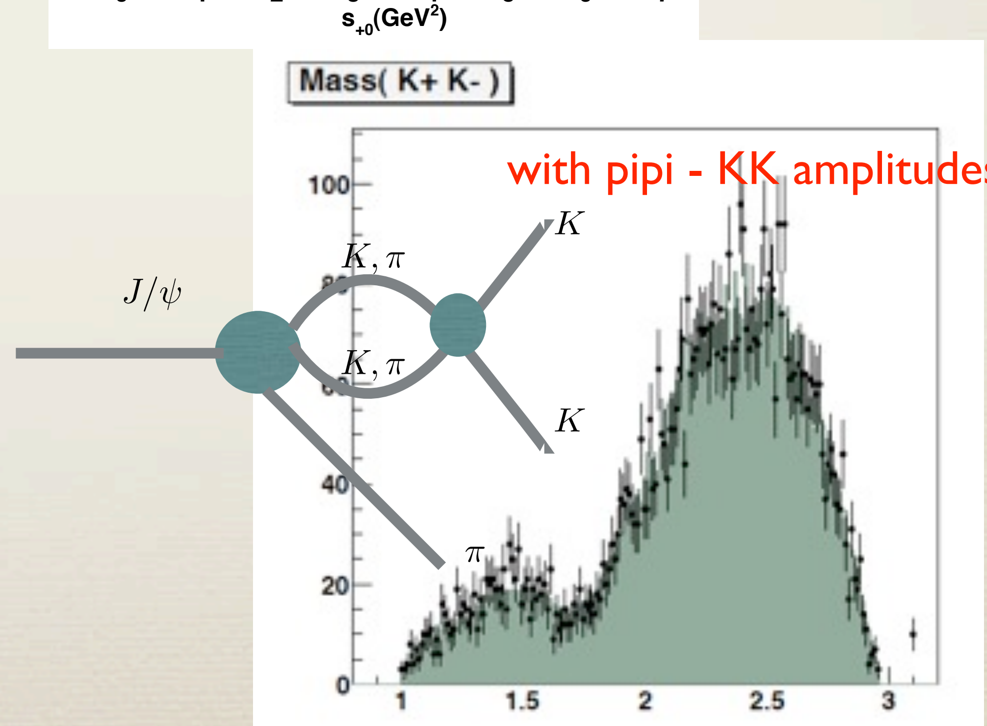




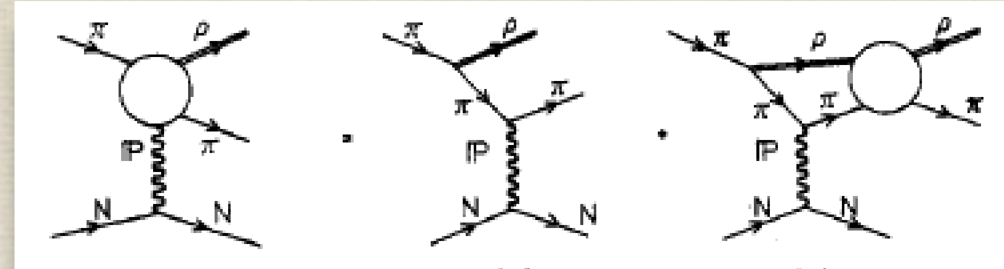


PRELIMINARY

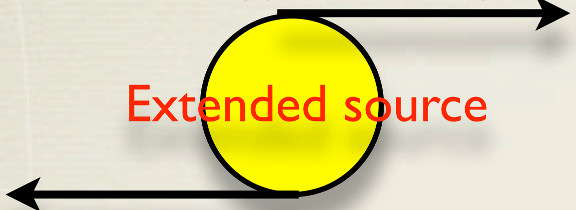




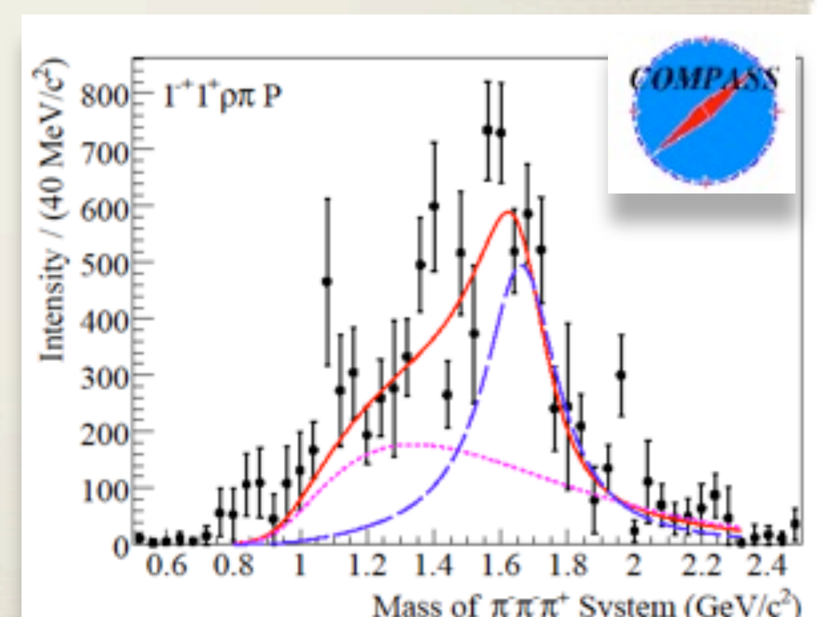
$e^{i\delta(E)} \cos\delta(E)$



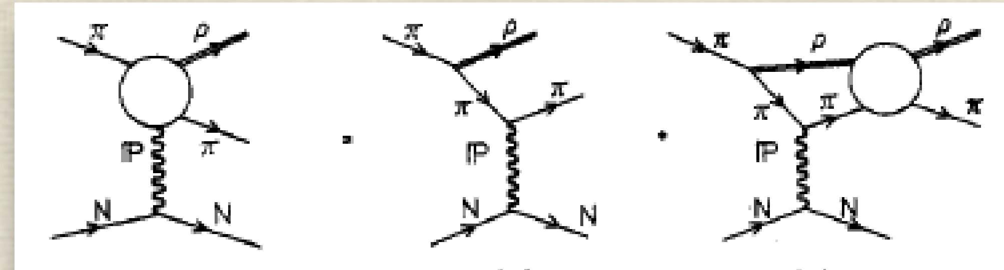
force=Regge exchange



$e^{i\delta(E)} \frac{\sin\delta(E)}{k}$

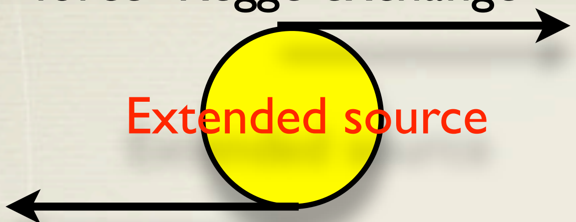


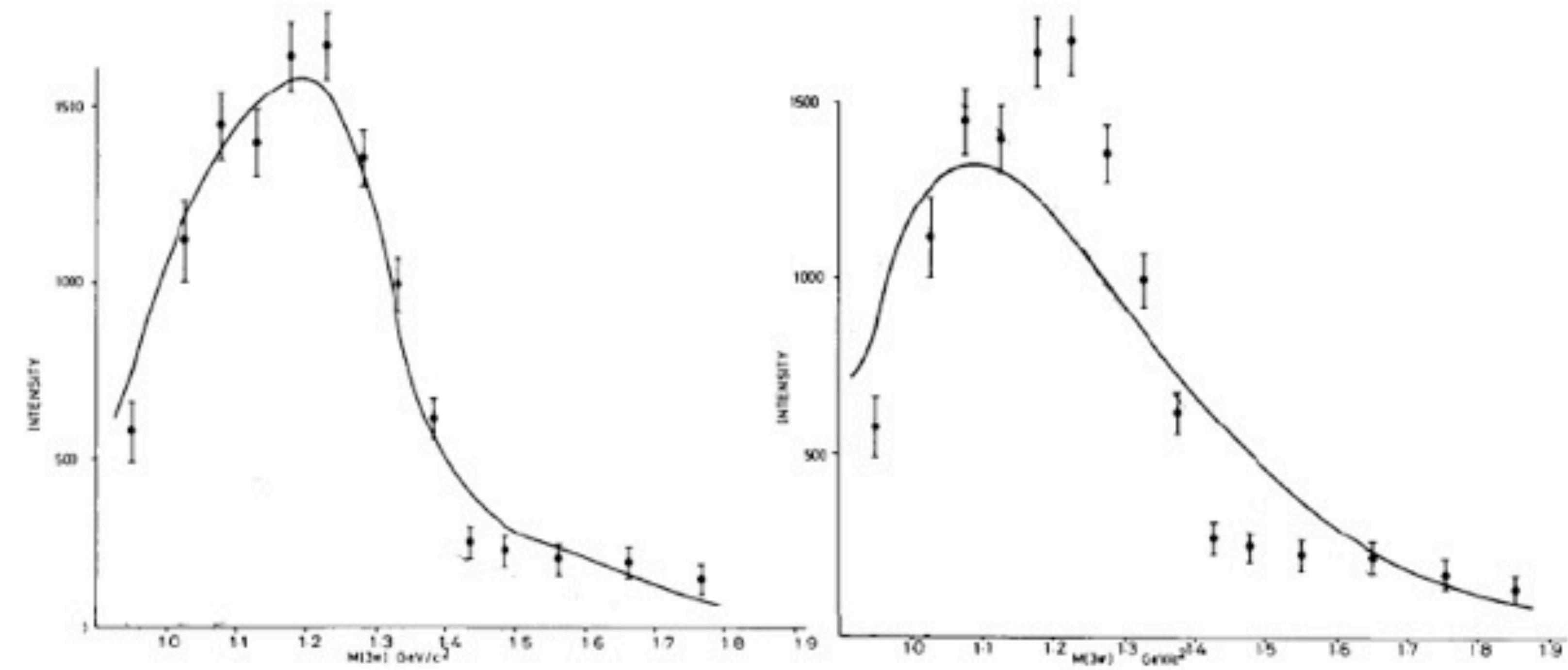
$e^{i\delta(E)} \cos\delta(E)$



force=Regge exchange

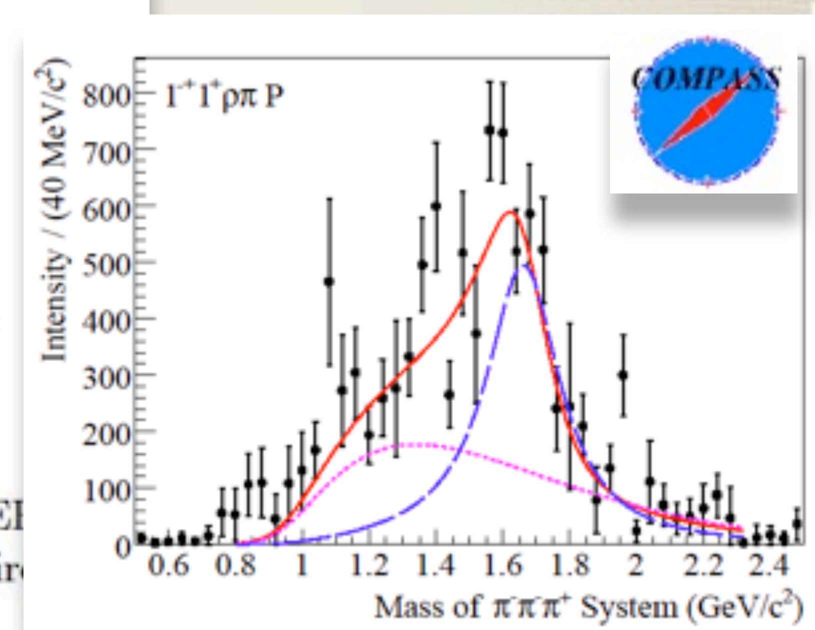
$e^{i\delta(E)} \dfrac{\sin\delta(E)}{k}$



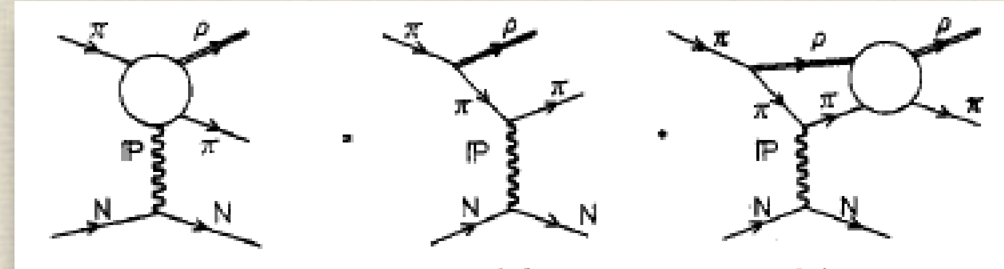


M.G.Bowler,(1975)

Figure 11: Fit to the 1^+ $\rho\pi$ intensity from $\pi^- p \to \pi^-\pi^-\pi^+ p$ at $E_\pi = 25$ and $E_\pi = 40$ GeV, CE[...] data [70], with (left) both long-range production from one pion exchange and short-range dir[...] production and (right) short-range direct production only [63].



$e^{i\delta(E)} \cos\delta(E)$

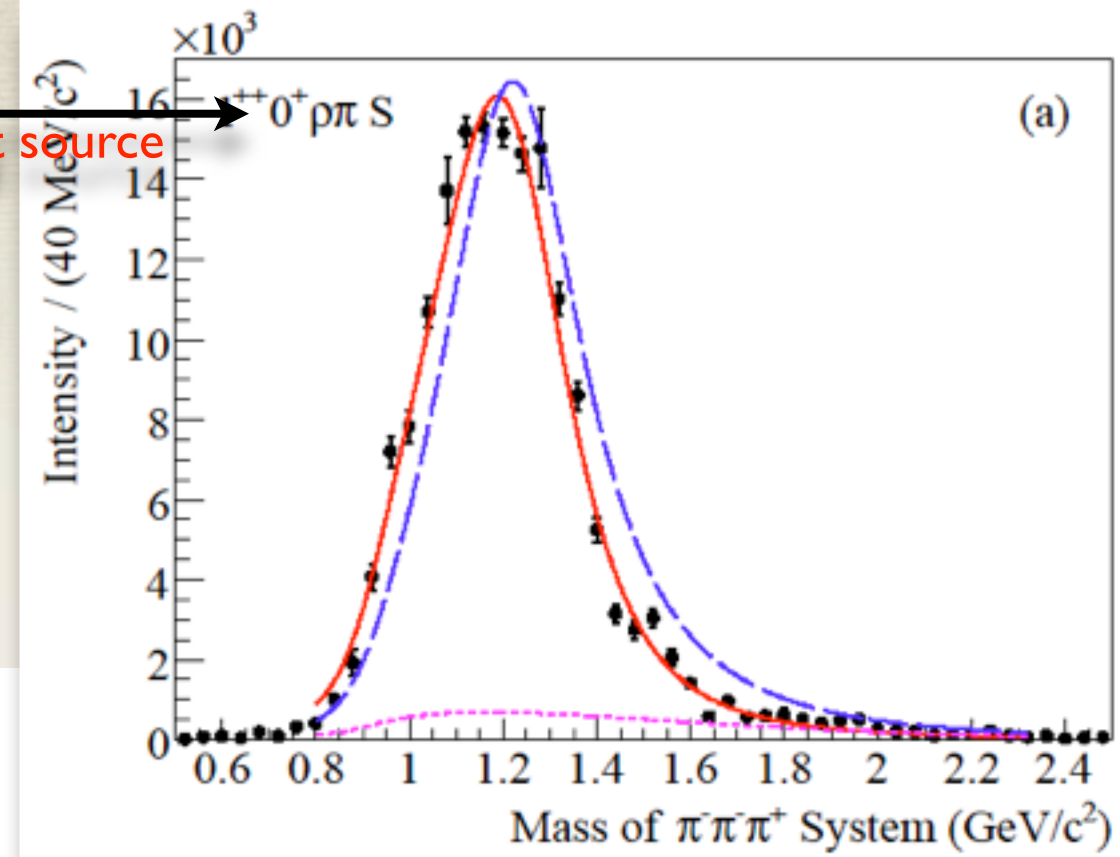


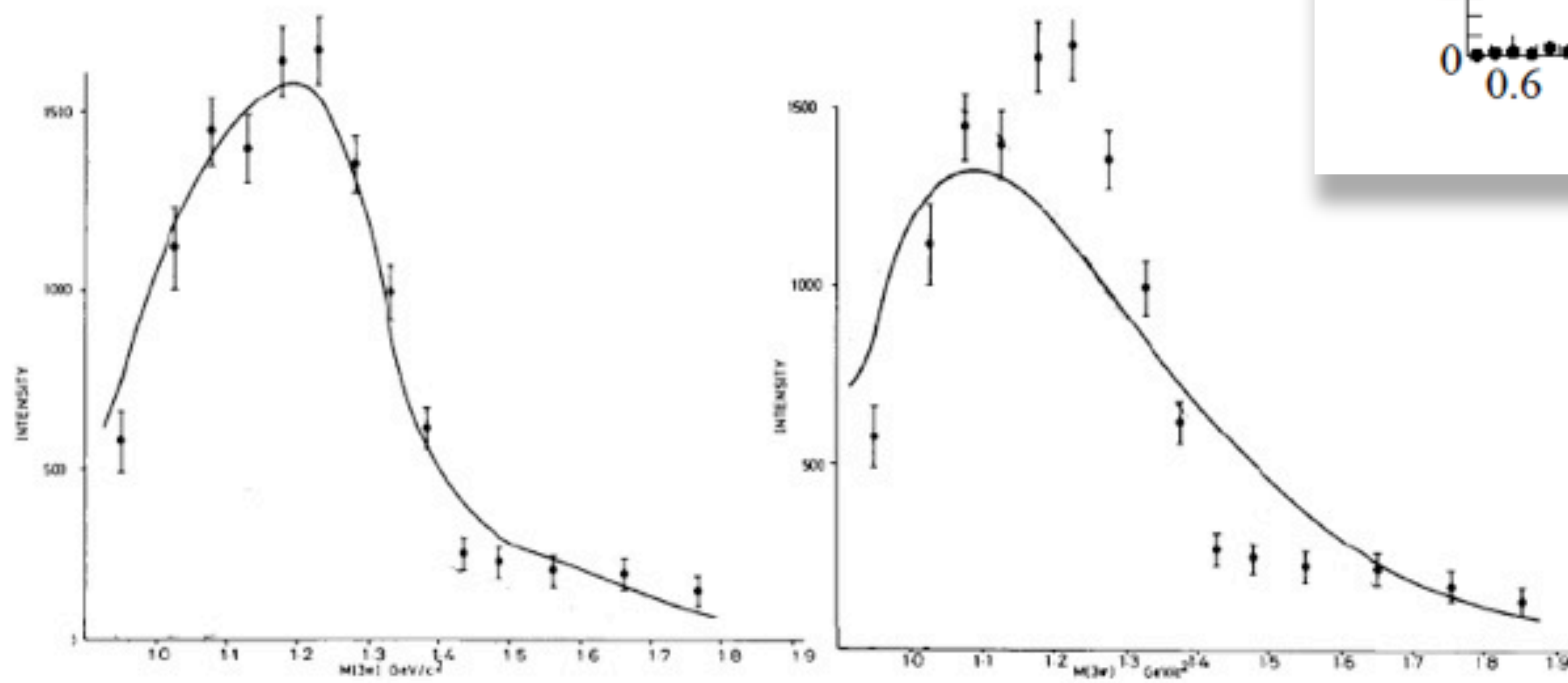
force=Regge exchange

Extended source

Compact source

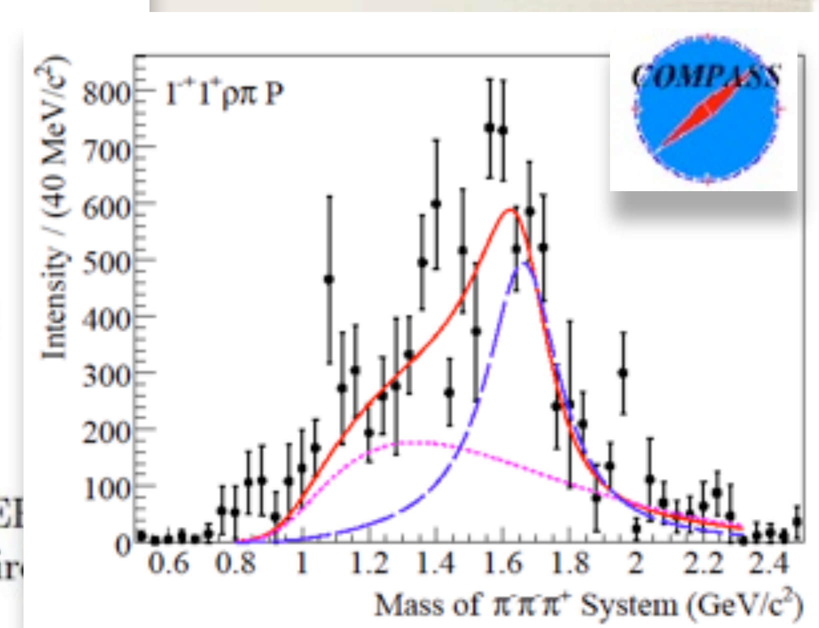
$e^{i\delta(E)} \frac{\sin\delta(E)}{k}$





M.G.Bowler,(1975)

Figure 11: Fit to the 1^+ $\rho\pi$ intensity from $\pi^- p \to \pi^-\pi^-\pi^+ p$ at $E_\pi = 25$ and $E_\pi = 40$ GeV, CE data [70], with (left) both long-range production from one pion exchange and short-range dir production and (right) short-range direct production only [63].



*** PWA work-day ****
*** Saturday June 25th, JLab ****

9:00-10:00 PWA of existing photo-production data (20" each)

9:00 - 9:20 PWA analysis of (old) CLAS data (g6c, 3pi, BNL amplitudes)
Dennis Weygand
9:20 - 9:40 PWA analysis of (new) CLAS data (g12, 3pi or summary of ongoing analyses, BNL amplitudes,
Paul Eugenio
9:40 - 10:00 -PWA analysis of (new) CLAS data (g11, 2pi , moments approach)
Marco Battaglieri/Raffaella Devita

10:00-10:45 Discussion: Amplitude construction Mike Pennington

10:45 - 11:00 Coffee break

11:00-12:00 PWA of future photo-production data (20" each)

11:00 - 11:20 - IU tools Matt Shepherd
11:20 - 11:40 - New tools applied to CLAS/CLAS12 data (g11, 2k , moments approach Derek Glazier
11:40 - 12:00 New tools applied to GLUEX Curtis Meyer
12:00 - 12:20 PWA analysis issues in charmonium Ryan Mitchell

12:20-13:30 Pizza Lunch

13:30-15:00 Discussion: Interfacing theory and experiment Adam Szczepaniak

Summary:

* Dispersion relations constrain partial waves

* CDD ambiguities: use lattice as guidance

* resonances are generated from short distance physics and not from meson-meson rescattering

* explore full analyticity and unitarity constraints from crossed channels (L-plane singularities)