Bootstrap constraints for the πN resonance spectrum from effective scattering theory

K. Semenov-Tian-Shansky, V. Vereshagin

[1] Department of High Energy Physics
University of Saint-Petersburg, Russia

[2] CPHT, École Polytechnique & LPT Université d'Orsay

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Introduction

- Hadron scattering problem can hardly be formulated in terms of fundamental degrees of freedom of QCD.
- Effective theory approach may be useful for description of hadron scattering

Theorem

Most general S-matrix consistent with given symmetry requirements: Lagrangian contains all the monomials consistent with a given symmetry (Weinberg'79).

- Spontaneously broken chiral symmetry $\Rightarrow$ chiral Lagrangian. Systematic expansion in power of momenta: χPT.
- **Good news:** the effective theories are renormalizable (contrary to usual believes). All necessary counterterms are included.
- **Bad news:** problem of the parameters. Infinite number of RPs to fix finite part of counterterms is needed. E.g. NLO $E\chi L$: 10, NNLO > 100!
- Infinite number of parameters $\not\Rightarrow$ no predictive power. χPT: finite number of constants are to be fixed at a given order (power counting at work).

What could be an effective theory for hadron scattering in the resonance region?

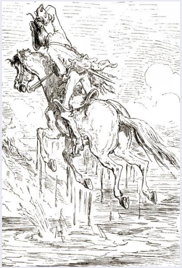
- most general effective theory formulated in terms of fields corresponding to the true asymptotic states (say π, N) consistent with given (linear) symmetry requirements
- we do not assume the existence of any kind of inner cutoff
- rely upon the intrinsically quantum Weinberg's scheme of constructing QFT:
  - ok for any algebraic symmetry
  - problems with nonlinearly realized symmetries. Need to be formulated on the amplitude level (soft pion theorems)
- the approach is adjusted only for S-matrix calculations $\Rightarrow$ effective scattering theory (EST)

Problem of parameters

Theory has no predictive power: need to fix infinite number of parameters!

- Classify parameters as essential (contribute on-shell) and redundant (contribute only off-shell): only essential parameters require formulation of RPs to compute S-matrix

- Major hope: Consistency condition for the perturbative expansion (asymptotic uniformity) together with fundamental QFT requirements (analyticity, unitarity) will dramatically reduce number of independent RPs for essential parameters of EST.
- Incarnation of old bootstrap program but in the QFT framework.



How to include resonances?

- Still no room for resonances: only stable particles survive in the asymptotic states.

We restrict the class of EST and attain the concept of localizability:

- Initial Dyson series for tree-level amplitude converges at a certain small domain of relevant variables.
- Outside this domain it should be considered as formal.
- To get the solution in the wider domain we need to resum it in a special way.
- Analytic continuation of tree-level amplitude to wider domain can be performed by means of extended (Dyson's type) perturbation scheme containing auxiliary fields (resonances).
- Resonances are unstable with respect to decay into true asymptotic states of the theory.
- S-matrix computed in the extended perturbation scheme still acts on the space of true asymptotic states: (Veltman'63) suggests a way to proceed to establish unitarity.

- Example of extension of perturbative scheme: J. Cornwall D. Levin G. Tictopoulos'74 renormalizable theory of week interaction from Cabibbo theory
- The hypothetical localizable effective theory of strong interaction requires an infinite extension of perturbative scheme by introduction of an infinite tower of baryon and meson resonances of arbitrary high spin and mass

Main questions:

1. How to assign meaning to perturbation series? Non-trivial even at tree level: see infinite number of resonance exchanges.
2. How to reduce the number of parameters for which it is necessary to formulate RPs?
3. Practical use from the scheme?

These problems were addressed in:
A. Vereshagin and V. Vereshagin, Phys.Rev. D **68**, 025002 (2004);
K.S., A. Vereshagin, and V. Vereshagin, Phys.Rev. D **73**, 025020 (2006);
K.S., A. Vereshagin, and V. Vereshagin, Phys.Rev. D **77**, 025028, (2008);

The Structure of πN Scattering Amplitude

The amplitude $M_{a\alpha}^{b\beta}$ of the reaction

$$\pi_a(k) + N_\alpha(p,\lambda) \to \pi_b(k') + N_\beta(p',\lambda')$$

can be presented in the following form:

$$M_{a\alpha}^{b\beta} = \left\{\delta_{ba}\delta_{\beta\alpha}M^+ + i\varepsilon_{bac}(\sigma_c)_{\beta\alpha}M^-\right\}.$$

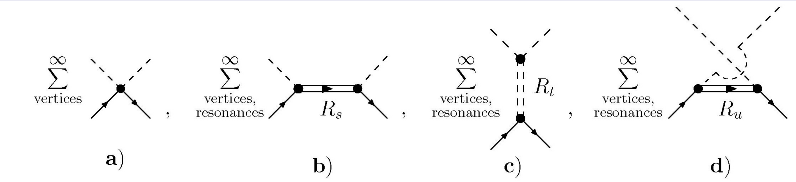
Here

$$M^\pm = \overline{u}(p',\lambda')\left\{A^\pm + \left(\frac{\not{k}+\not{k}'}{2}\right)B^\pm\right\}u(p,\lambda) \quad .$$

The invariant amplitudes $A^\pm,\ B^\pm$ are certain functions of Mandelstam variables. $(s+t+u = 2m^2+2\mu^2)$.

Tree Level Binary Amplitude

Tree level amplitude is the sum of the following graphs:



Two steps to assign meaning to the formal series

- switch to minimal parameters (natural building blocks for essential parametrs)
- summability (maximal analyticity) and asymptotic uniformity principles $\Rightarrow$ uniformly converging series of singular terms defining the tree-level amplitude as the polynomially bounded meromorphic function (Cauchy forms).

Reduction Procedure & Minimal Parametrization

Main ideas of minimal parametrization:

- all theory parameters which contribute only off-shell are redundant
- knowledge of the singularities is enough to reconstruct the whole function

Reduction to minimal parameters

$$H^{\mathcal{H}}(\varphi\pi\pi) = \frac{1}{2} g^{\mathcal{H}}_{\varphi\pi\pi\ 000}\, \varphi\pi\pi + \frac{1}{2} g^{\mathcal{H}}_{\varphi\pi\pi\ 011}\, \varphi\partial_\mu\pi\partial_\mu\pi + \frac{1}{2} g^{\mathcal{H}}_{\varphi\pi\pi\ 020}\, \varphi\partial^2\pi\pi + \dots .$$

t-channel exchange matrix element reads:

$$-\underbrace{\left(g^{\mathcal{H}}_{\varphi\pi\pi\ 000} + \left(\mu^2 - \frac{M_R^2}{2}\right) g^{\mathcal{H}}_{\varphi\pi\pi\ 110} - \mu^2 g^{\mathcal{H}}_{\varphi\pi\pi\ 020} + \dots\right)}_{g^{0m}_{\pi\pi\varphi}} \frac{1}{t - M_R^2} V^{N\bar{N}\varphi} +$$

{ Smooth part } + { Part that is zero on-shell }

Minimal Hamiltonian

$$H^{0m}_{\pi\pi\varphi} = \frac{1}{2} g^{0m}_{\pi\pi\varphi} \pi\pi\varphi .$$

Minimal Triple πNR Vertices

Baryon resonances $I = \frac{1}{2}, \frac{3}{2}$, $J = L + \frac{1}{2}$, $\mathcal{N} = P(-1)^L = \pm 1$:

$$H(\pi N N^*) = g_{N^*} \overline{N} \vec{\sigma} \Gamma N^*_{\mu_1 \ldots \mu_L} \partial^{\mu_1} \ldots \partial^{\mu_L} \vec{\pi} + H.c.;$$

$$H(\pi N \Delta) = g_{\Delta} \overline{N} \Gamma P_{3/2} \Delta_{\mu_1 \ldots \mu_L} \partial^{\mu_1} \ldots \partial^{\mu_L} \vec{\pi} + H.c.;$$

where

$$\Gamma = \begin{cases} 1_{4\times 4}, & \text{for } \mathcal{N} = -1; \\ i\gamma_5, & \text{for } \mathcal{N} = +1. \end{cases}$$

Meson resonances $J = 0, 2, \ldots$ $I = 0$, $P = +1$ and $J = 1, 3, \ldots$ $I = 1$, $P = -1$:

$$H(S\pi\pi) = \frac{1}{2} g_{S\pi\pi} S_{\mu_1 \ldots \mu_J} (\vec{\pi} \cdot \partial^{\mu_1} \ldots \partial^{\mu_J} \vec{\pi}) \; ;$$

$$H(SNN) = \left[g^{(1)}_{NNS} \overline{N} \partial_{\mu_1} \ldots \partial_{\mu_J} N + i g^{(2)}_{NNS} J \partial_{\mu_1} \ldots \partial_{\mu_{J-1}} \overline{N} \gamma_{\mu_J} N \right] S^{\mu_1 \cdots \mu_J}$$

$$H(V\pi\pi) = \frac{1}{2} g_{V\pi\pi} \vec{V}_{\mu_1 \ldots \mu_J} (\vec{\pi} \times \partial^{\mu_1} \ldots \partial^{\mu_J} \vec{\pi}) \; ;$$

$$H(VNN) =$$

$$\left[i g^{(1)}_{NNV} \overline{N} \vec{\sigma} \partial_{\mu_1} \ldots \partial_{\mu_J} N + g^{(2)}_{NNV} J \overline{N} \gamma_{\mu_J} \vec{\sigma} \partial_{\mu_1} \ldots \partial_{\mu_{J-1}} N \right] \vec{V}^{\mu_1 \cdots \mu_J} \; .$$

Cauchy Forms

- *meromorphic* function of two variables (no cuts, the only singularities are poles) polynomially bounded in layers $B_x\{x \in (a,b) \in \mathbb{R};\ \nu \in \mathbb{C}\ \}$
- number of poles may be infinite; example: Veneziano string amplitude
- The Cauchy integral formula solves the problem of construction of polynomially bounded in B_x meromorphic function $f(x,\nu)$ with the given set of singularities (poles in ν at $\nu = p_n(x)$ with residues $r_n(x)$)

The Cauchy form:

- The Cauchy form allows one to present the N-bounded in the layer B_x function $f(x,\nu)$ as the uniformly converging series of poles contributions with the following structure:

$$f(x,\nu) = \left[\begin{matrix}\text{Background polynomial} \\ \text{of power } N \text{ in } \nu\end{matrix}\right] + \sum_n \left[\frac{r_n(x)}{\nu - p_n(x)} + \begin{matrix}\text{Correcting} \\ \text{polynomial}\end{matrix}\right].$$

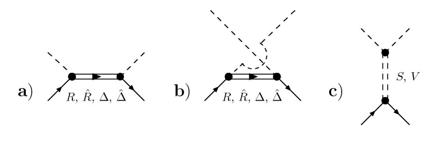
Necessary Ingredients

- We construct well-defined expressions for the invariant amplitudes $X = A^{\pm},\ B^{\pm}$ in the system of layers ($x = s,\ t,\ u$):

$$B_x\,\{x \sim 0, \quad \nu_x \in \mathbb{C}\} \qquad (\nu_s = u - t,\ \nu_t = s - u,\ \nu_u = t - s)$$

To construct the Cauchy forms for the $2 \longrightarrow 2$ tree-level πN amplitudes we need:

1. The asymptotic behavior. In accordance with our uniformity principle it is dictated by the corresponding Regge intercepts: $(B_t : A^{+} \sim \nu_t^{1},\ A^{-} \sim \nu_t^{0.5},\ B^{+} \sim \nu_t^{0})$.
2. The residues at poles relevant for the corresponding B_x layer:



These are just the relevant on-shell spin sums dotted by the minimal triple coupling constants:

$$g_R^2 \sum_j \mathcal{U}_{\rho,\nu_1,\dots\nu_l}(j,q)\,\overline{\mathcal{U}}^{\tau,\mu_1,\dots\mu_L}(j,q)\big|_{q^2=M_R^2}.$$

The Cauchy Forms for $X^{\pm}$ in the Layer $B_t\,\{t \sim 0, \quad \nu_t \equiv s - u \in \mathbb{C}\}$

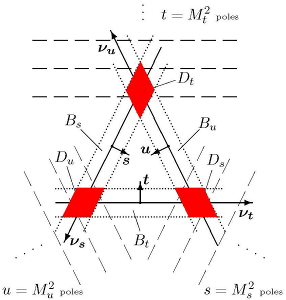
- In B_t the only singularities are poles in s and u:

$$X^{\pm}(\nu_t, t)\Big|_{B_t} =$$

$$\underbrace{\sum_{k=0}^{N} \frac{1}{k!}\alpha_k(t)\nu_t^k}_{\text{Background term}} + \sum_{n}^{\infty} \left\{ \frac{p_n(t)}{s - M_{s_n}^2} + \frac{q_n(t)}{u - M_{u_n}^2} - \underbrace{\sum_{m=0}^{N} \beta_{n,m}(t)\nu_t^k}_{\text{Correcting polynomial}} \right\}.$$

- $\beta_{n,m}(t)$ is fully specified by the parameters of s- and u-channel resonances. $\alpha_k(t)$ are at this stage unknown.
- This fixes the tree-level amplitude X in the layer B_t up to few unknown functions.

The Source of Bootstrap Constrains



- Bootstrap constrains: the Cauchy forms (different in different layers) should coincide in the domains of intersection of layers (duality).
- Analogue of finite energy sum rules (R.Dolen, D.Horn, C.Schmid'68 but at a given order of loop expansion)

E.g. in D_s domain ($t \sim 0$ $u \sim 0$):

$$\sum_n \left(\begin{matrix} \text{poles in} \\ s \text{ and } u \end{matrix} + \text{C.p.}(t,\, \nu_t) \right) + \text{Background}(t,\, \nu_t) =$$

$$\sum_{n'} \left(\begin{matrix} \text{poles in} \\ s \text{ and } t \end{matrix} + \text{C.p.}(u,\, \nu_u) \right) + \text{Background'}(u,\, \nu_u)$$

Bootstrap Constrains

- Bootstrap constrains allow to fix the unknown smooth contribution of 4-point contact vertices & cross-channel poles in terms of minimal triple couplings and masses.

- Bootstrap constrains are renorm-invariant in the sense that they are the equations for physical renormalization prescriptions (RPs).

- These constrains lower down the number of independent RPs and restrict the admissible values of spectrum parameters.

Numerical Tests of Bootstrap Constrains

- Bootstrap constrains connect physical quantities, so they can be tested against the experimental data.

On the parametrization of resonances:

- No kind of partial resummation is allowed. Breit-Wigner looses its meaning in terms of minimal parameters.
- When comparing to the data we are forced to use say PDG data on πN resonance spectrum. We formally express triple couplings $g_{R\pi N}$ through $\Gamma_{R\to\pi N}$.
- For well separated narrow resonances different methods result in approximately the same values of mass and coupling

Bootstrap for A^- Invariant Amplitude in D_s domain

In D_s : [Cauchy form in B_u] − [Cauchy form in B_t] $\equiv \Psi_s(t,u)$

$$\partial_t^m \, \partial_u^n \, \Psi_s(t,u)\Big|_{t=u=0} = 0\,, (m,n=0,1,\ldots)\,.$$

The set of sum rules:

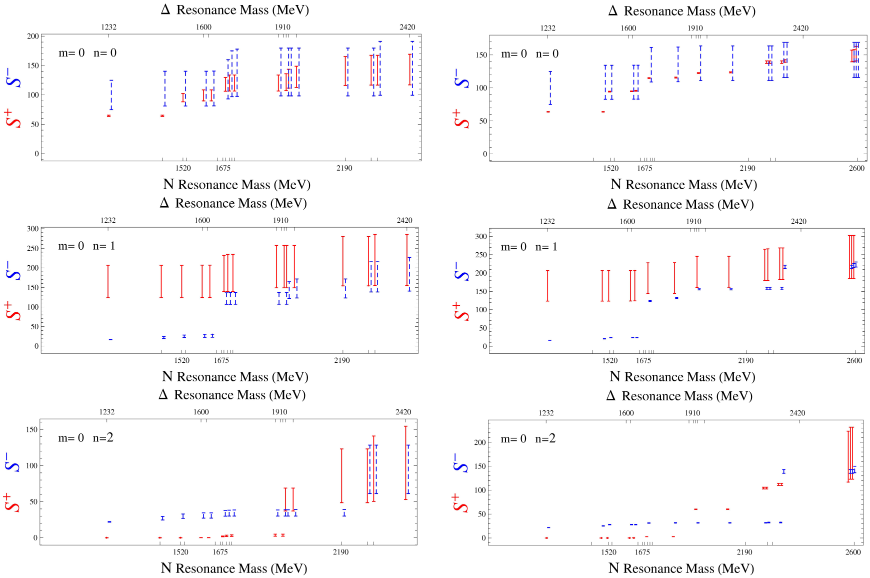
$$\sum_{\text{Baryons}} g^2_{R_B \pi N} V_{m,n}(M_{R_B}, L, \mathcal{N}, I) -$$

$$\sum_{\substack{\text{Mesons with} \\ \text{I=1, odd J, P=}-1}} g_{R_M \pi\pi} \cdot g_{R_M N \bar{N}} W_{m,n}(M_{R_M}, J) = 0$$

$$S^+(M) = \sum_{\substack{R_B,\ R_M, \\ M_R < M}} \left\{ \begin{array}{c} \text{Positive contributions} \\ \text{into sum rule} \end{array} \right\}$$

$$S^-(M) = -\sum_{\substack{R_B,\ R_M \\ M_R < M}} \left\{ \begin{array}{c} \text{Negative contributions} \\ \text{into sum rule} \end{array} \right\}$$

Saturation of SR: PDG v.s. SP06 πN spectrum



Our approach suggests the possible scheme of data fitting

- The vicinity of poles should be removed from the fitting domain
- Fit with amplitudes of fixed loop order (real poles)
- Bootstrap constrains should be respected by fitting procedure since it provides the consistency of the perturbative scheme.

Conclusions

- We develop the logically complete scheme of Effective scattering theory (EST) suitable for the description of hadronic scattering processes. Corner stones:
  1. Physical: QFT, phenomenology (Regge-like)
  2. Mathematical principles: summability and uniformity
- Numerical test (πN, $\pi\pi$ and KN) show that our approach does not contradict the known phenomenology.
- Possible scheme of data fitting
  1. The vicinity of poles should be removed from the fitting domain
  2. Bootstrap constrains should be respected by fitting procedure since this provides the consistency of the perturbation theory scheme.

Weinberg's Scheme of Constructing QFT

Asymptotic states (including vacuum)

$\Downarrow$

Creation (annihilation) operators $a^{+}(p)$, $(a^{-}(p))$ depending on momenta; their commutation relations

$\Downarrow$

Free causal fields (local operators) $\phi(x)$

$\Downarrow$

Interaction Hamiltonian H_{int}: Lorentz invariant sum of local monomials constructed from free fields and their derivatives of arbitrary high orders and powers

$\Downarrow$

Formal Dyson series for S-matrix elements (here T_W stands for Wick's T-product)

$$S_{fi} = \langle f | T_W exp \left\{ -i \int H_{int} dx \right\} | i \rangle \, .$$