Evgeny Epelbaum, RUB

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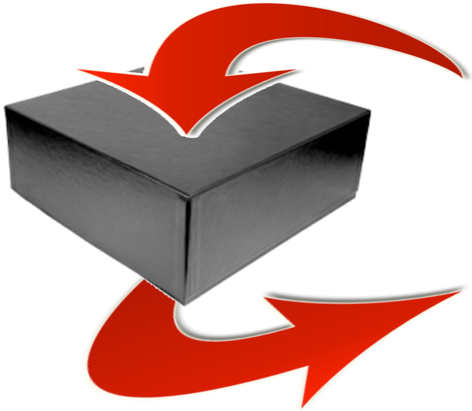
Chiral perturbation theory with explicit spin-3/2 DOF

Outline

- Introduction
- Heavy Baryon ChEFT with explicit Δ(1232)
  - πN scattering, nuclear forces
- Beyond the Heavy Baryon approach
  - V^2CS & spin-dependent polarizabilities of the nucleon
- Summary and outlook

Chiral perturbation theory

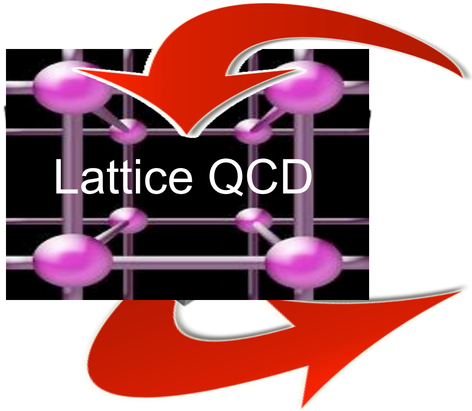


1. QCD $\mathcal{L}_{\rm QCD} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \bar{q}(i\not{D} - \mathcal{M})q$

2. Effective Lagrangian for hadronic DOF (π, N, ...)

Most general form (infinitely many terms), restricted only by symmetries, approximate spontaneously-broken chiral symmetry

Chiral perturbation theory



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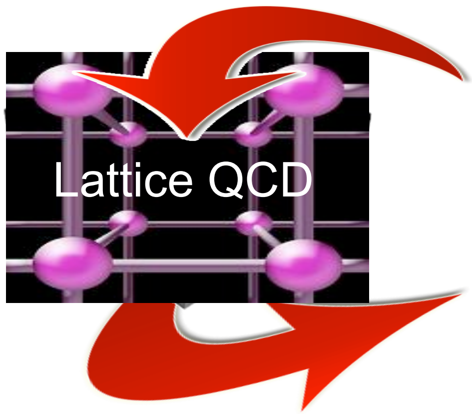

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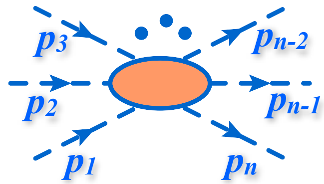

Perturb. expansion in powers of soft scales over the χ symmetry breaking scale $Q \in (p_i/\Lambda_\chi,\ M_\pi/\Lambda_\chi)$

At each order only a finite number of LECs to be determined from the data

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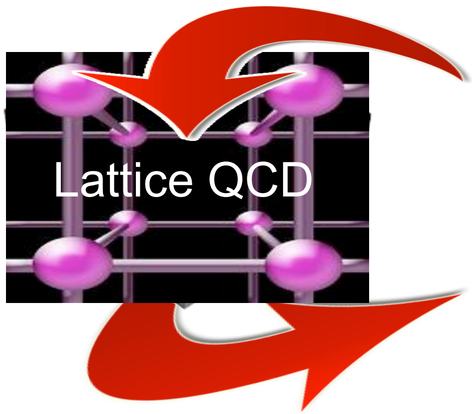

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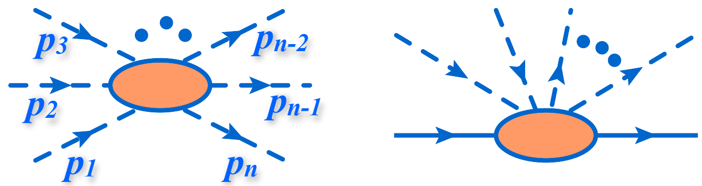

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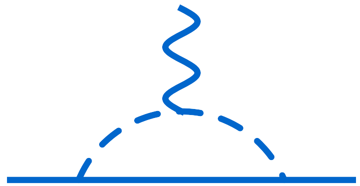
4. Heavy baryon expansion

Special care needed to ensure that the nucleon mass does not spoil the power counting

$$\delta m_N = -\frac{3 g_A^2 m_N^3}{(4\pi F_\pi)^2} \left(16\pi^2 L(\mu) + \frac{1}{2} \ln \frac{m_N^2}{\mu^2} \right) + \mathcal{O}(d-4)$$

HB approach: (covariant) nonrelativistic expansion of the Lagrangian

ChPT: beyond heavy baryon



The imaginary part of the triangle diagram is proportional to

$\arctan x$ with $x = \frac{\sqrt{(t-4M_\pi^2)(4m_N^2-t)}}{t-2M_\pi^2}$

Near threshold, formally: $x = \mathcal{O}\left(m_N/M_\pi\right)$ ⟹ the HB approach corresponds to the expansion:

$$\arctan x = \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^2} + \ldots$$

which converges for $|x| > 1$. This condition is violated for $|t-4M_\pi^2| \leq M_\pi^4/m_N^2$ ...

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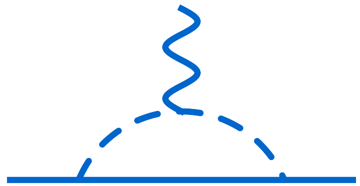

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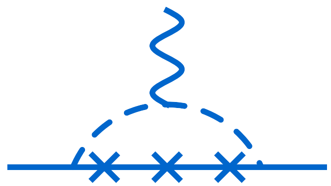
Solutions (extraction of the soft part of the amplitude):



- Infrared regularization: expand the integrand, evaluate the integrals using DR and resum...
  Ellis & Tang; Becher & Leutwyler

- Extended on-mass-shell renormalization: covariant approach + DR + properly chosen subtraction to get rid of hard pieces.
  Fuchs, Gegelia, Japaridze, Scherer

Inclusion of the spin-3/2 DOF

Why to include Δ(1232) as an explicit DOF?

- Low excitation energy: $\Delta \equiv m_\Delta - m_N = 293\ \mathrm{MeV} \sim 2M_\pi$
- Strong coupling to the pion-nucleon system
- In standard ChPT, effects of the Δ are included implicitly (through LECs)

⇒ large values of the (Δ-saturated) LECs may spoil convergence

⇒ explicit treatment in SSE: $\Delta \sim \mathcal{O}(M_\pi)$ Hemmert, Holstein, Kambor

Expansion parameter: $\epsilon \in \left(\frac{M_\pi}{\Lambda_\chi}, \frac{p_i}{\Lambda_\chi}, \frac{\Delta}{\Lambda_\chi} \right)$

Price to pay: more LECs, calculations considerably more involved...

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Inclusion of spin-3/2 fields (Rarita-Schwinger formalism) in chiral EFT is non-trivial...

- Maintaining the proper number of DOF in the interacting theory, chiral & gauge invariance

  Pascalutsa; EE, Krebs, Meißner; Wies, Gegelia, Scherer; Shklyar, Lenske

- Off-shell parameters in the effective Lagrangian

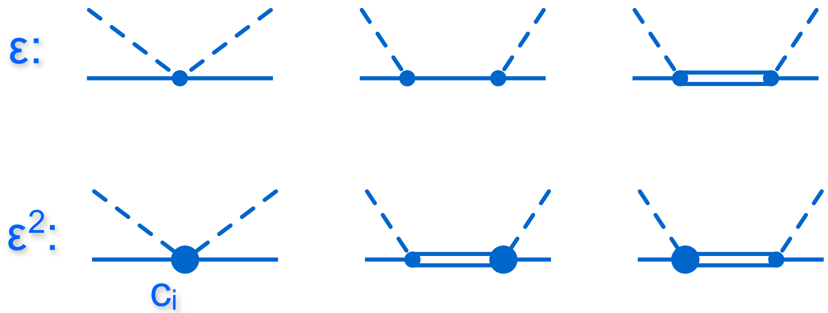
  Ellis, Tang; Pascalutsa; EE, Krebs, Meißner

Heavy baryon Chiral EFT

with explicit Δ (1232)

πN scattering: Δ-less vs Δ-full

Tree level: πN scattering at NLO in SSE:



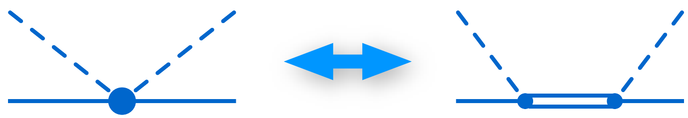
	Q^2, no Δ	Q^2 with Δ	Q^3 no Δ	EM98
a^+_{0+}	0.41	0.41	0.49	0.41 ± 0.09
b^+_{0+}	−4.46	−4.46	−5.23	−4.46
a^-_{0+}	7.74	7.74	7.72	7.73 ± 0.06
b^-_{0+}	3.34	3.34	1.62	1.56
a^-_{1-}	−0.05	−1.32	−1.19	-1.19 ± 0.08
a^+_{1-}	−2.81	−5.30	−5.38	-5.46 ± 0.10
a^-_{1+}	−6.22	−8.45	−8.16	-8.22 ± 0.07
a^+_{1+}	9.68	12.92	13.66	13.13 ± 0.13

(in units of $10^{-2}\ M_\pi^{\,n}$); from: Krebs, EE, Meißner, EPJA 32 (07) 127

The LECs c_1, c_2, c_3, c_4 are determined from a fit to S- and P-wave threshold parameters.

One finds:

- Q^2 with Δ is performing better/worth than Q^2/Q^3 in the Δ-less theory
- Numerical values of c_2, c_3 and c_4 strongly reduced once Δ is included explicitly



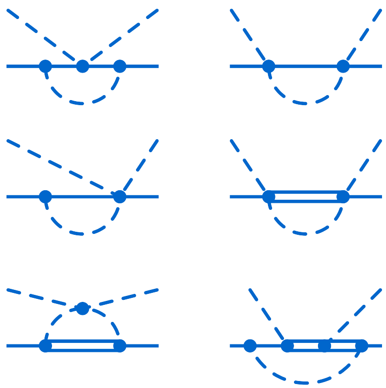
$$c_2 = -2.84 \to -0.25\ , \quad c_3 = -3.87 \to -0.79\ , \quad c_4 = 2.89 \to 1.33$$

(all values in units of GeV^{-1})

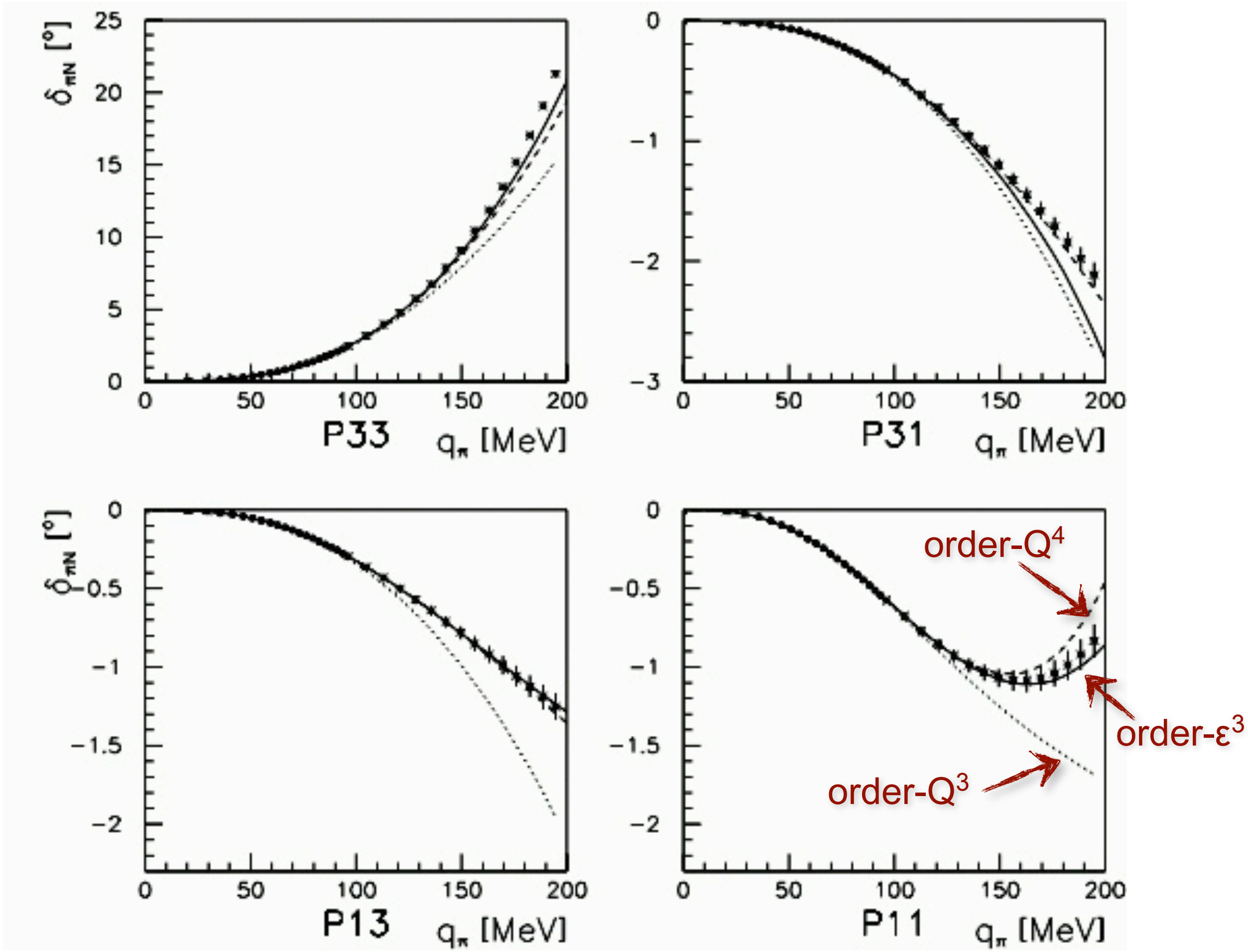
πN scattering: Δ-less vs Δ-full

Similar conclusions from the leading loop analysis: ε^3 more accurate than Q^3

Fettes, Meißner '98



+ many more diagrams...

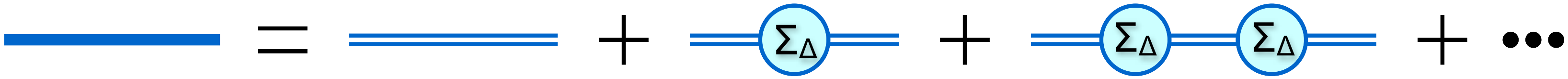


from: Fettes, Meißner, Nucl. Phys. A679 (01) 629

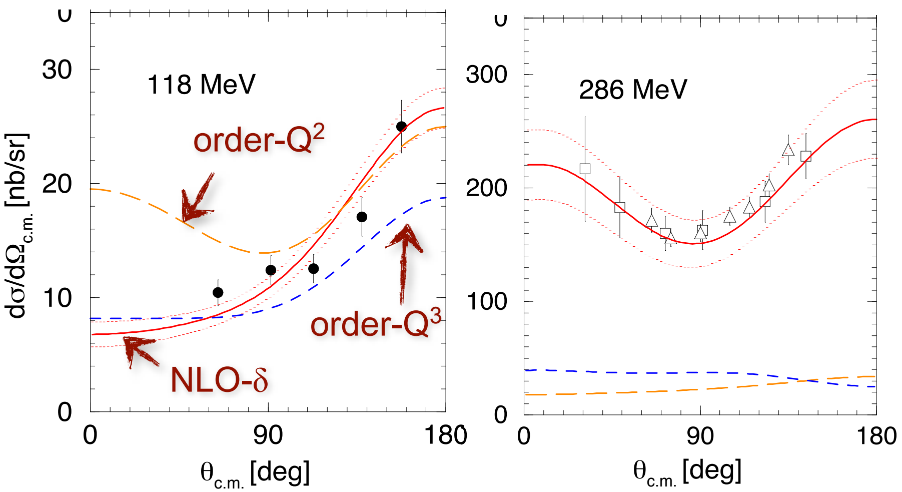
Extensions to the Δ-region

The generic assignment $\omega \sim M_\pi \sim \Delta$ in the SSE does not account for enhancement of the one-delta-reducible graphs in the delta region (and thus converges only for ω well below Δ). Extension to the Delta-region requires resummation of 1ΔR graphs.

Ellis,Tang '98; Pascalutsa, Phillips '03; Pascalutsa, Vanderhaeghen '05-'08; Long, van Kolck '10

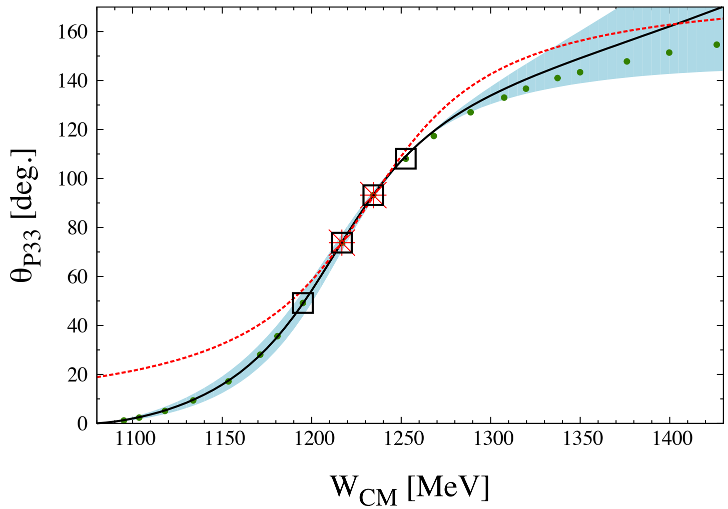


γp differential cross section in the δ-expansion



from: Pascalutsa, Phillips, PRC67 (03) 055202

P$_{33}$ πN phase shift

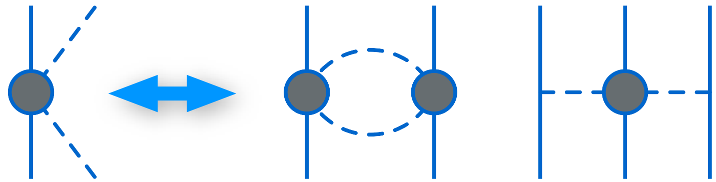


from: Long, van Kolck, NPA840 (10) 39

δ-expansion: $\delta \in \left(\frac{\Delta}{\Lambda_\chi}, \frac{M_\pi}{\Delta}\right)$

Implications for nuclear forces

Given the importance of the Δ in the πN system, one expects implications for nuclear forces as well...



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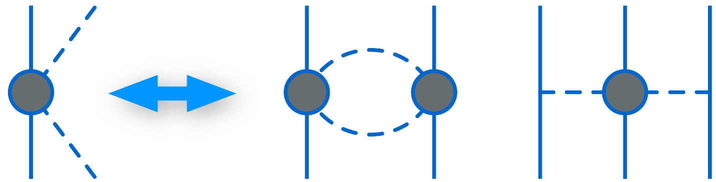
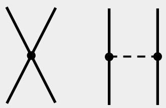
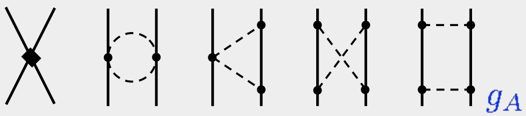
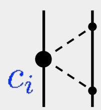


Two-nucleon force in EFT with and without Δ

	Δ-less theory	Δ-full theory: additional graphs
LO		
NLO		
N^2LO		

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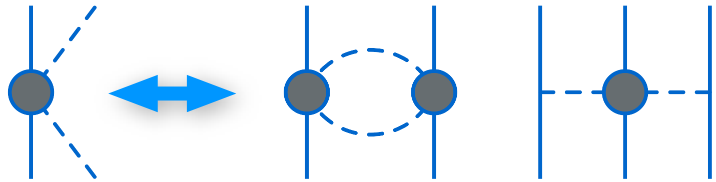

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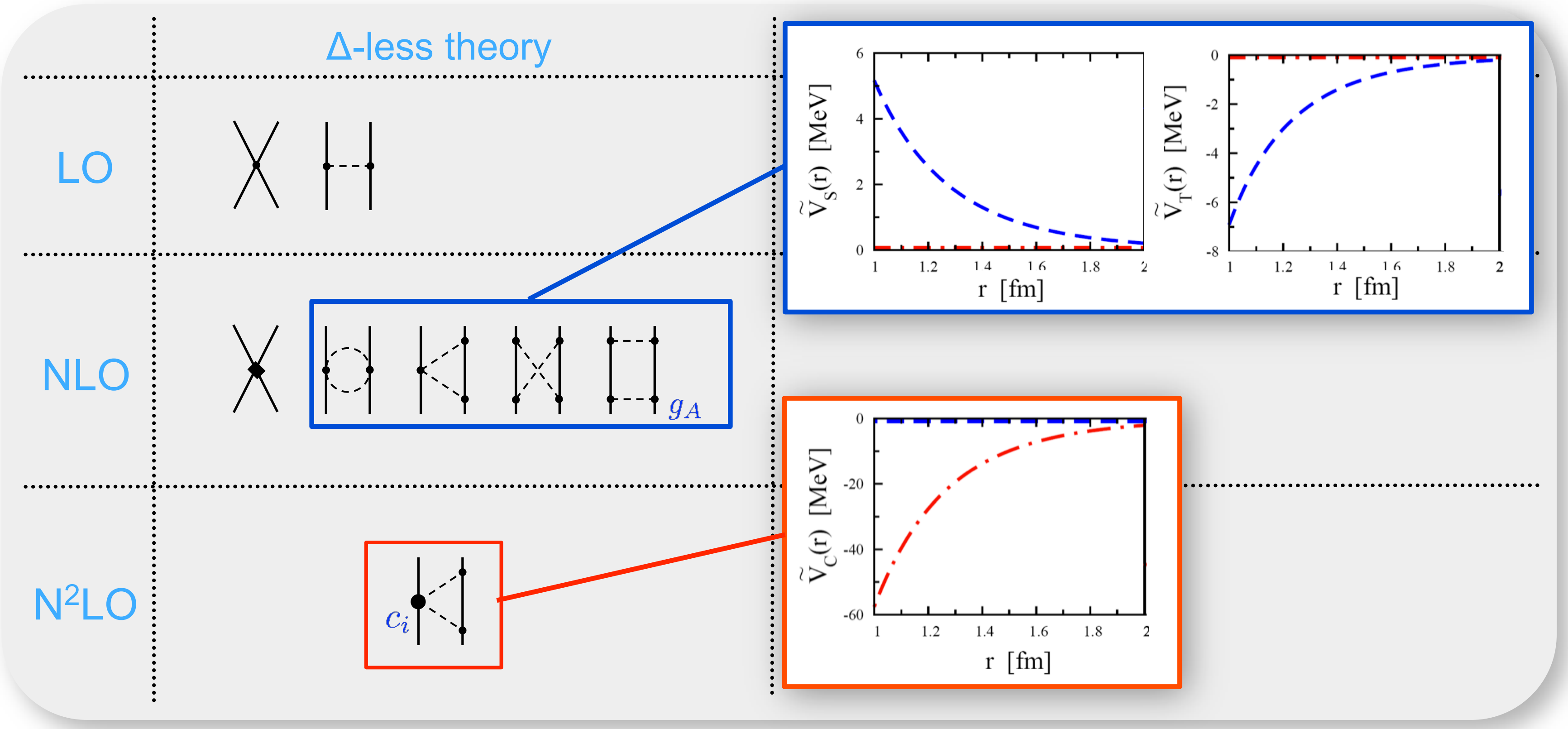

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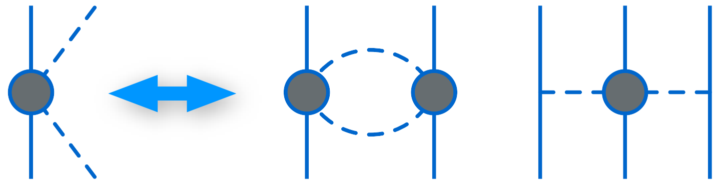

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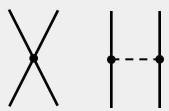
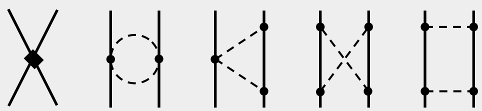
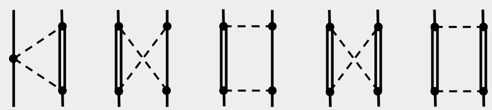
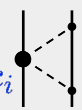
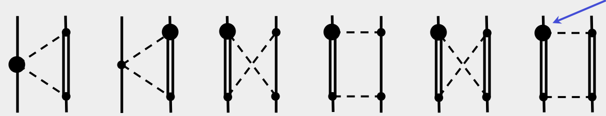
LO		—
NLO		 Ordonez, Ray & van Kolck.'96, Kaiser, Gerstendorfer & Weise '98
N^2LO		 Krebs, E.E., Meißner EPJA 32 (2007) 127

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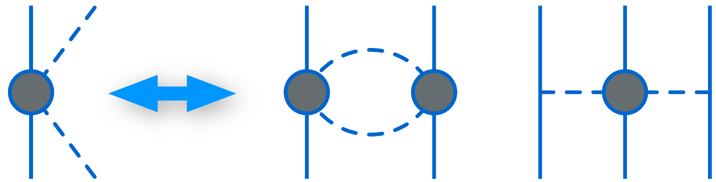
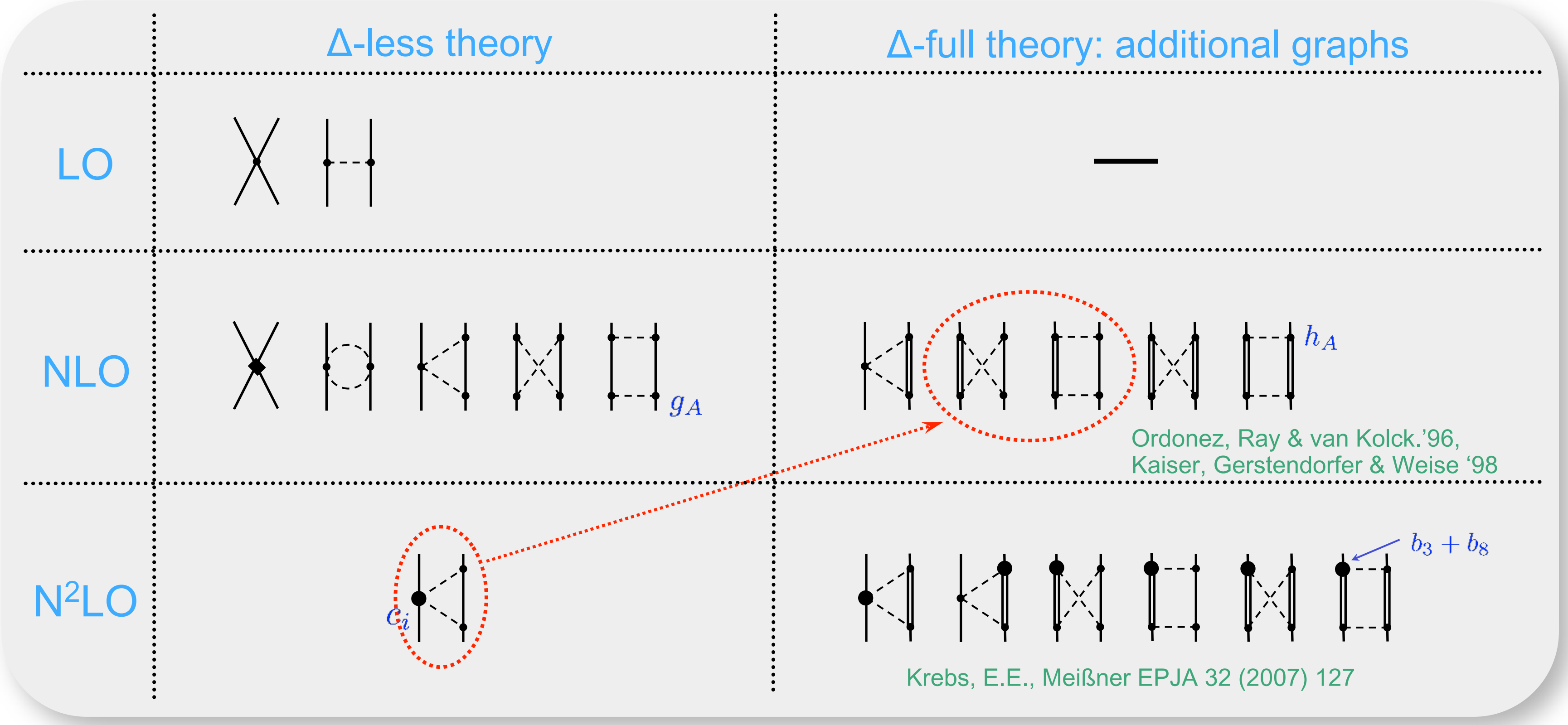


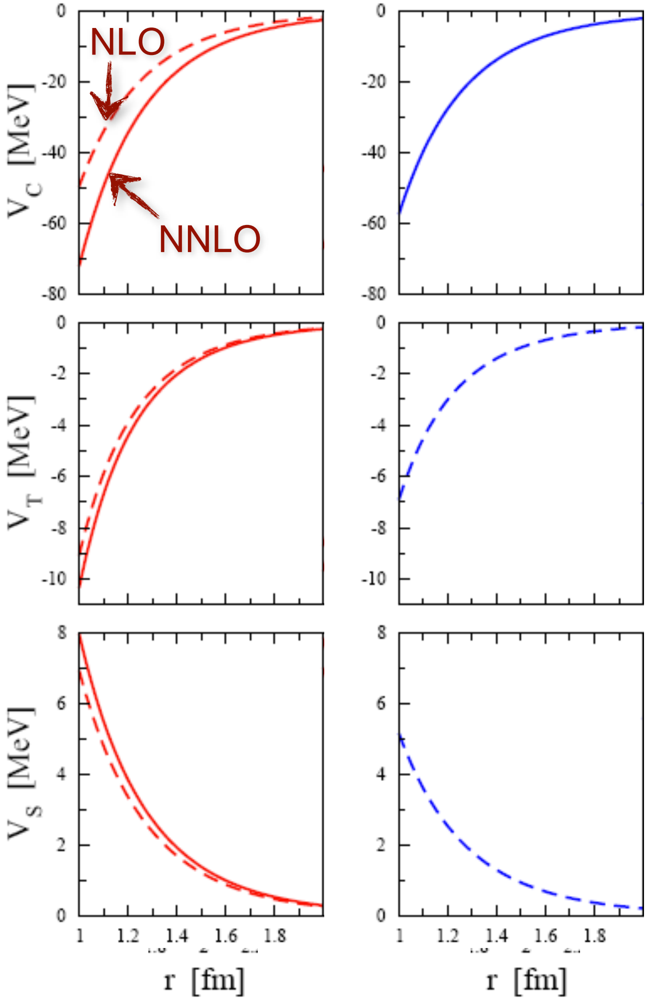
Two-nucleon force in EFT with and without Δ



Implications for nuclear forces

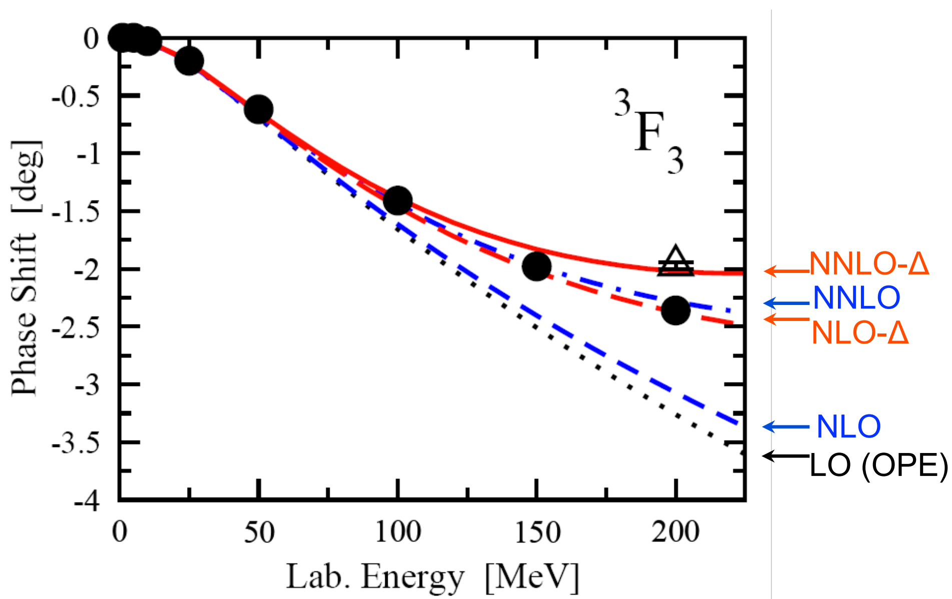
2π-exchange up to N^{2}LO

with Δ   without Δ



- a much better convergence for the potential when Δ is included explicitly
- clearly visible in NN peripheral waves

$^3F_3$ partial wave up to N^{2}LO



V^2CS in EFT with explicit Δ: Lorentz-invariant approach

in collaboration with Veronique Bernard, Hermann Krebs & Ulf-G. Meißner

V²CS and polarizabilities

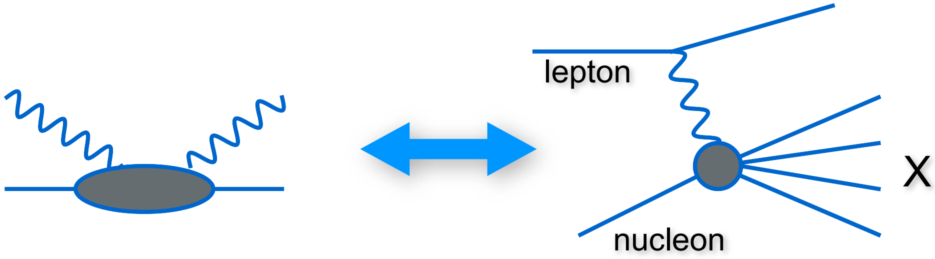
Spin-dependent forward tensor for V²CS:

photon energy $\nu = P \cdot q / m_N$

$$i \int d^4x \, e^{iq\cdot x} \langle PS | T J^\mu(x) J^\nu(0) | PS \rangle = -\frac{i}{2} \epsilon^{\mu\nu\alpha\beta} q_\alpha \left[S_\beta \, S_1(\nu, Q^2) + \frac{1}{m_N^2} (P \cdot q S_\beta - S \cdot q P_\beta) S_2(\nu, Q^2) \right]$$

photon virtuality

The structure functions $S_{1,2}$ are related via dispersion integrals to the ones $G_{1,2}$ measured in polarized spin-dependent inclusive lepton-nucleon scattering.



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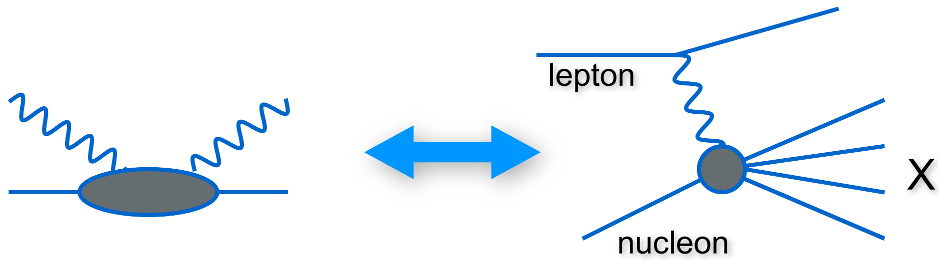

For small photon energy, $\bar{S}_{1,2}$ (elastic contrib. subtracted) can be expanded powers of ν^2:

$$\bar{S}_1(\nu,\, Q^2) = \sum_{i=0}^{\infty}\bar{S}_1^{(2i)}(0,Q^2)\nu^{2i}\,, \qquad \bar{S}_2(\nu,\, Q^2) = \sum_{i=0}^{\infty}\bar{S}_2^{(2i+1)}(0,Q^2)\nu^{2i+1}$$

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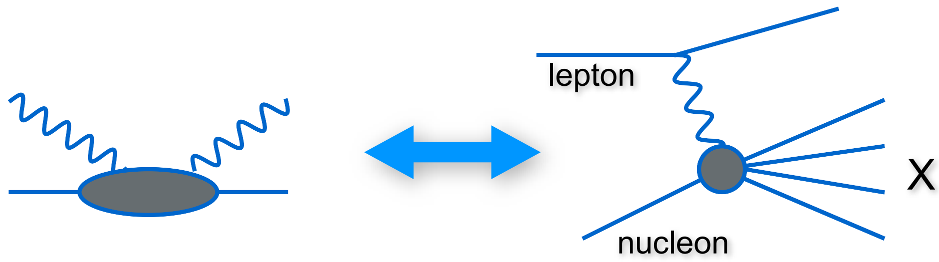

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moments of structure functions

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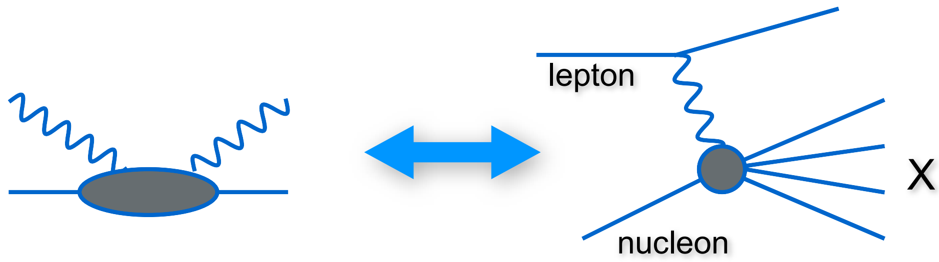

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moments of structure functions

Generalized polarizabilities:

$$\gamma_0(Q^2) = \frac{1}{8\pi}\left(\bar S_1^{(2)}(0,\,Q^2) - \frac{Q^2}{m_N}\bar S_2^{(3)}(0,\,Q^2)\right) \qquad \delta_0(Q^2) = \frac{1}{8\pi}\left(\bar S_1^{(2)}(0,\,Q^2) + \frac{1}{m_N}\bar S_2^{(1)}(0,\,Q^2)\right)$$

can be measured (using dispersion integrals) and computed in ChPT

V^2CS and polarizabilities

Previous calculations within chiral EFT

- HB ChPT up to order Q^4

  Ji, Kao, Osborne, Spitzenberg, Vanderhaeghen, Birse, McGovern, Kumar

  - Only well-known LECs from $\mathcal{L}_{\pi N}^{(2)}$ contribute
  - Large discrepancy for γ_0 even at the photon point

- HB ChPT with explicit Δ up to order ε^3 (leading loop)

  Kao, Spitzenberg, Vanderhaeghen '02

  - Sizable, negative contribution to γ_0 ; δ_0 less sensitive...

- IR ChPT up to order Q^4

  Bernard, Hemmert, Meißner '03

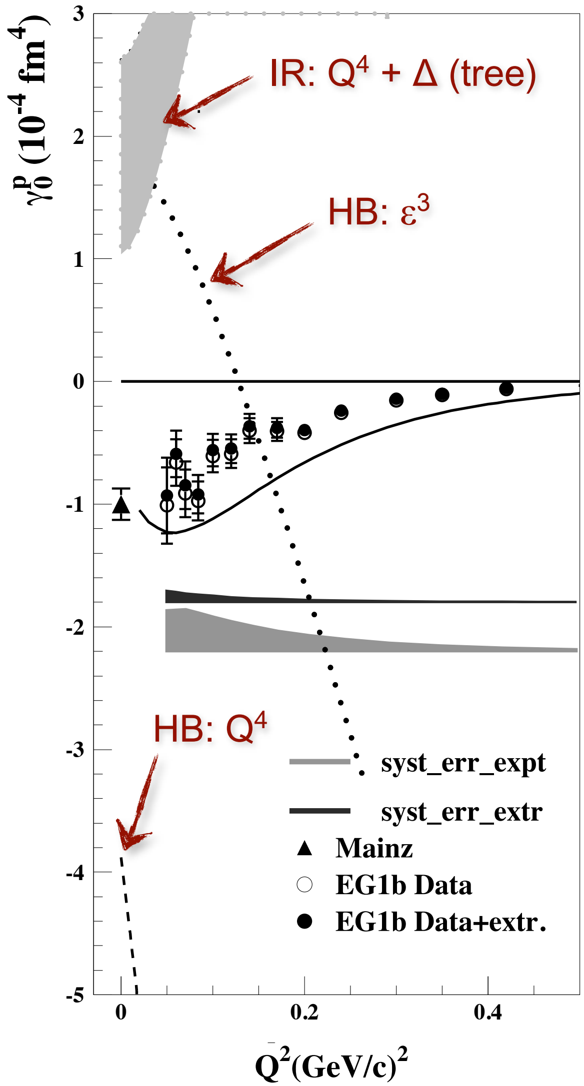
  - Test of the HB expansion: poor/good convergence for γ_0/δ_0:

$$\gamma_0^p = \underbrace{4.45}_{O(\mu^{-2})} - \underbrace{8.31}_{O(\mu^{-1})} + \underbrace{6.03}_{O(\mu^{0})} + \underbrace{3.22}_{O(\mu^{1})} + O(\mu^2) = 4.64$$

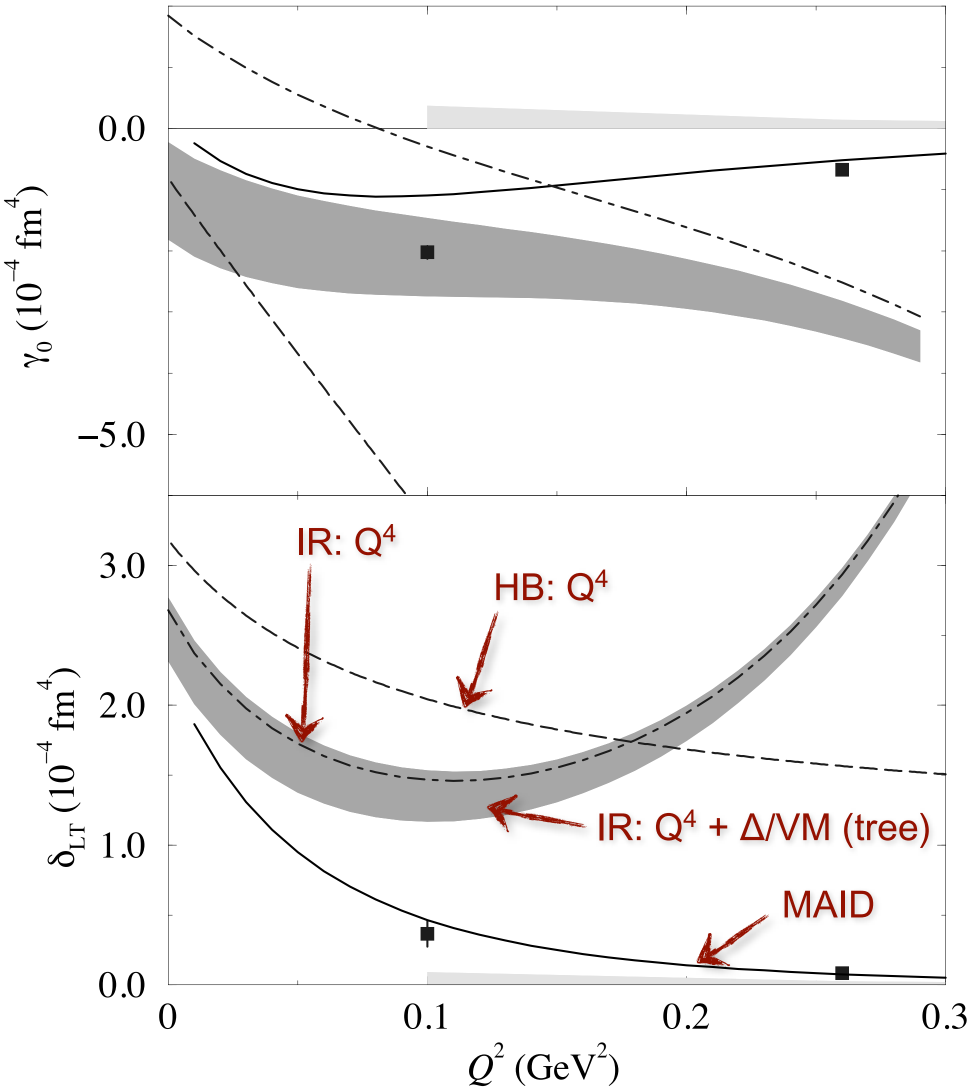
$$\delta_0^p = \underbrace{2.23}_{O(\mu^{-2})} - \underbrace{0.75}_{O(\mu^{-1})} + \underbrace{0.53}_{O(\mu^{0})} + \underbrace{0.12}_{O(\mu^{1})} + O(\mu^2) = 2.04$$

(values at the photon point, all numbers in units of $10^{-4}\mathrm{fm}^4$)

V²CS and polarizabilities



from: Prok et al., PLB672 (09) 12

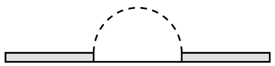


from: Amarian et al. PRL 93 (04) 152301

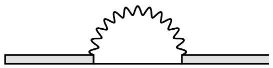
V^2CS and polarizabilities

We worked out the contributions of the Δ upto ε^3 within the covariant framework (both IR and DR)

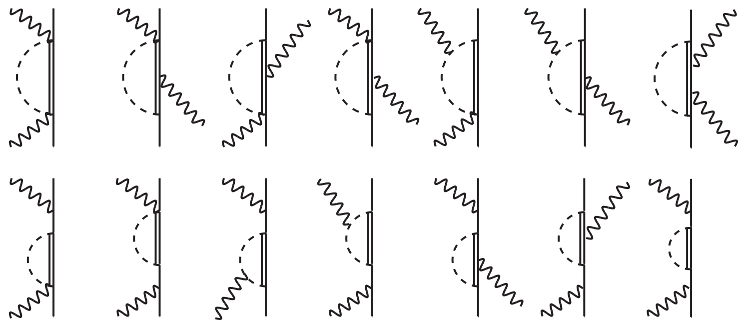
The LECs h_A/b_1 determined from the strong/em width of the Δ:

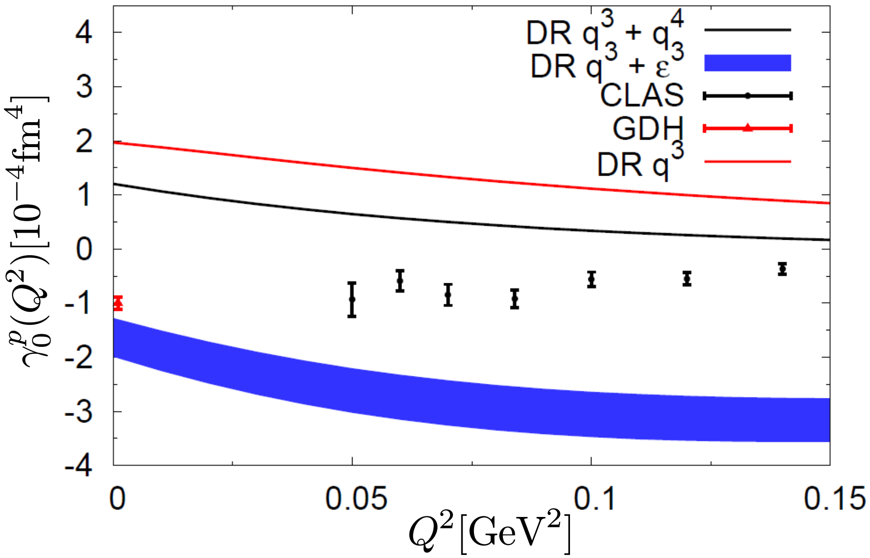


$$h_A = 1.45 \pm 0.02$$



$$b_1 = (4.84 \pm 0.21)\, m_N^{-1}$$





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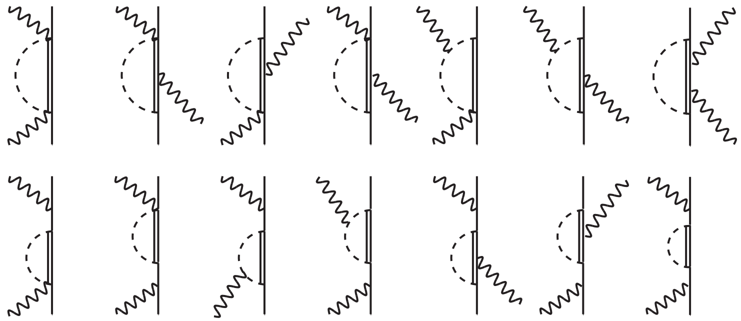

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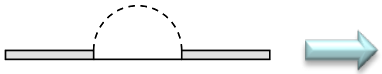

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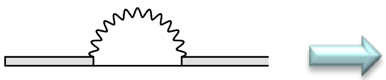

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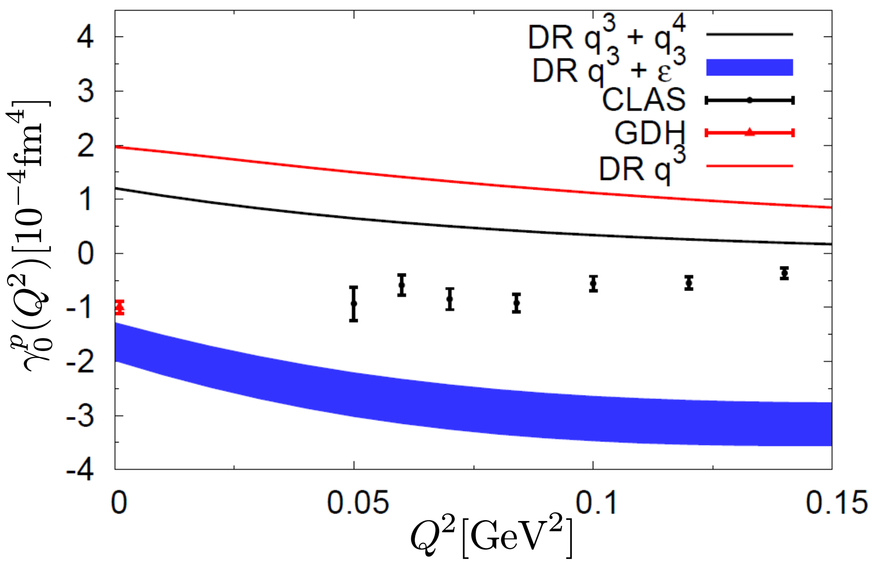
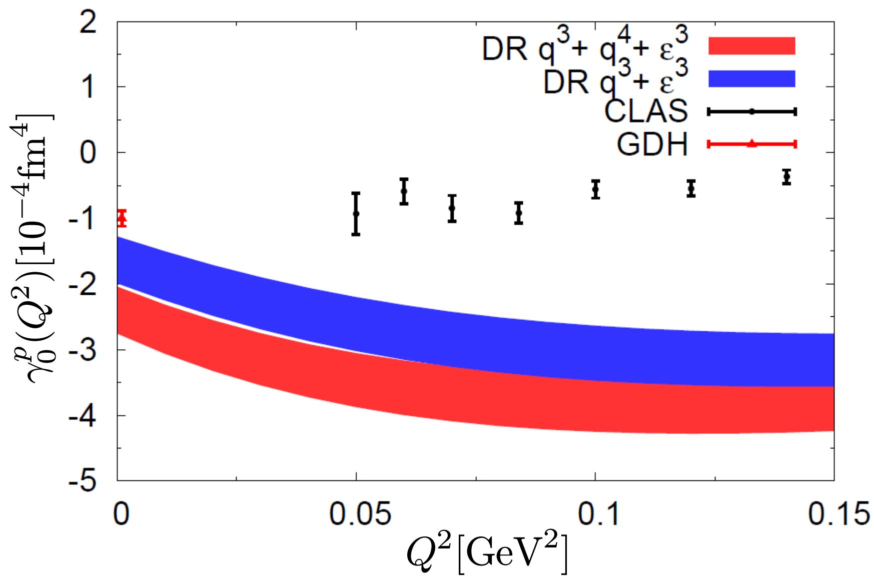
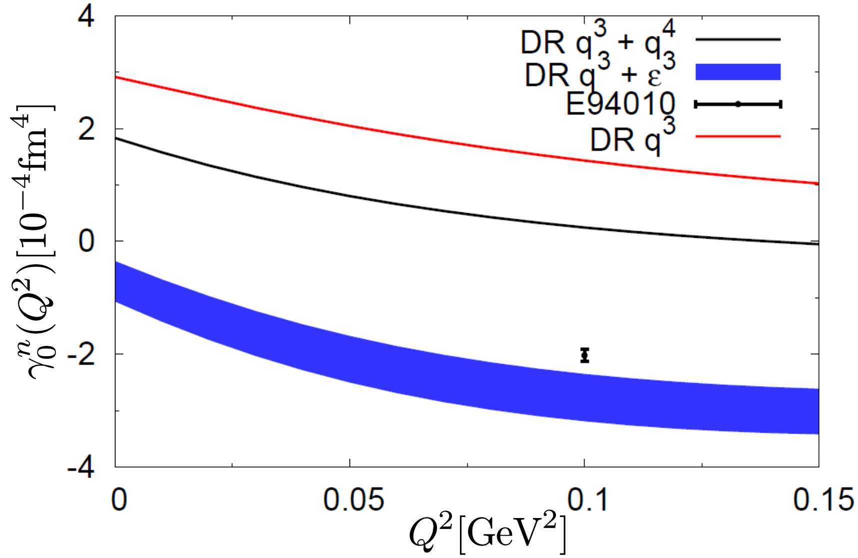
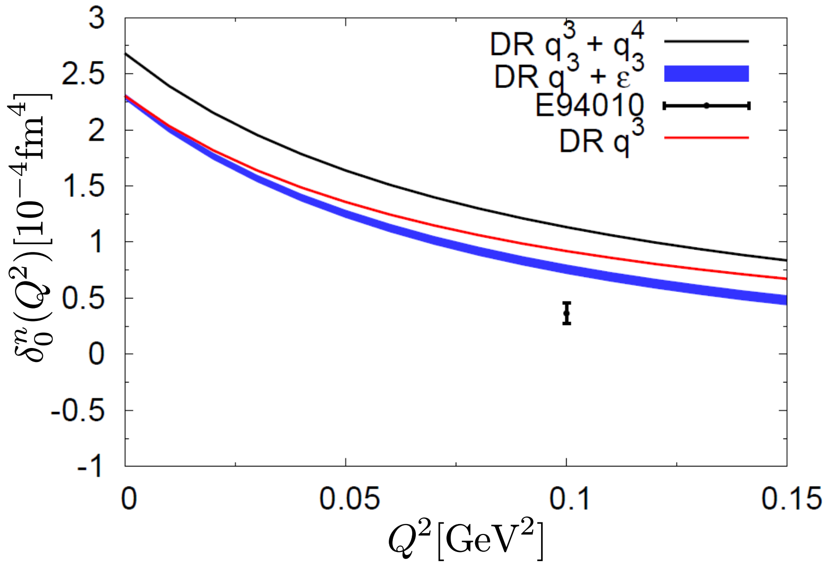




V^2CS and polarizabilities





- Strong improvement for γ_0; insensitivity of $\boldsymbol{\delta}_0$ to the Δ-contributions confirmed
- HB expansion in the presence of Δ seems not to converge even at the photon point
- Consequently, some scheme dependence observed (IR vs DR), further study needed...
- Order-ε^4 calculation needed to draw final conclusions

Summary and outlook

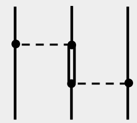
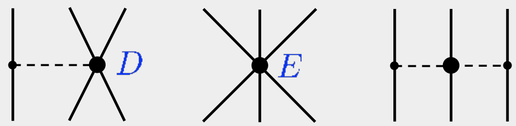
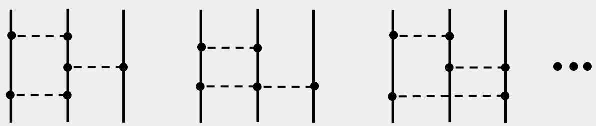
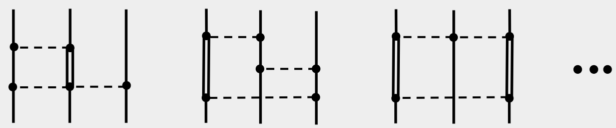
- Explicit treatment of the $\Delta(1232)$ in chiral EFT within the SSE improves the description of the pion-nucleon system and nuclear forces.

- Generalized forward spin polarizabilities are calculated up to order ε^3 in the Lorentz-invariant formulation. Δ loop contributions to γ_0 are large and strongly improve the description of the data; δ_0 appears to be less sensitive.

Still to be done (work in progress):

- Nuclear forces to order ε^4; V^2CS at order ε^4 in Lorentz-invariant ChEFT; global analysis of πN data, form factors, π photo-/electroproduction in Baryon ChEFT with explicit Δ

Implications for nuclear forces

	Δ-less theory	Δ-full theory: additional graphs
NLO	—	
N^2LO		—
N^3LO		

EE, Krebs, Meißner, NPA 806 (08) 65

Implications for nuclear forces

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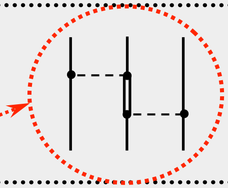
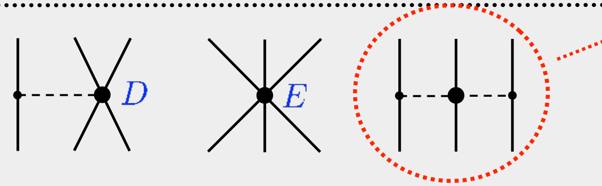
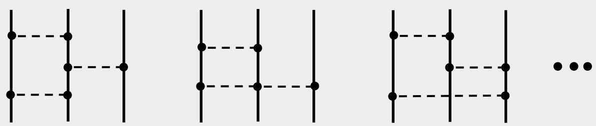
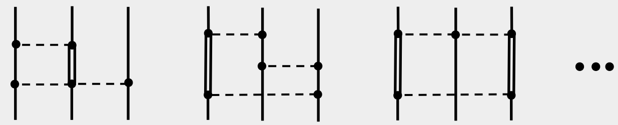
NLO	—	
N^2LO		—
N^3LO		

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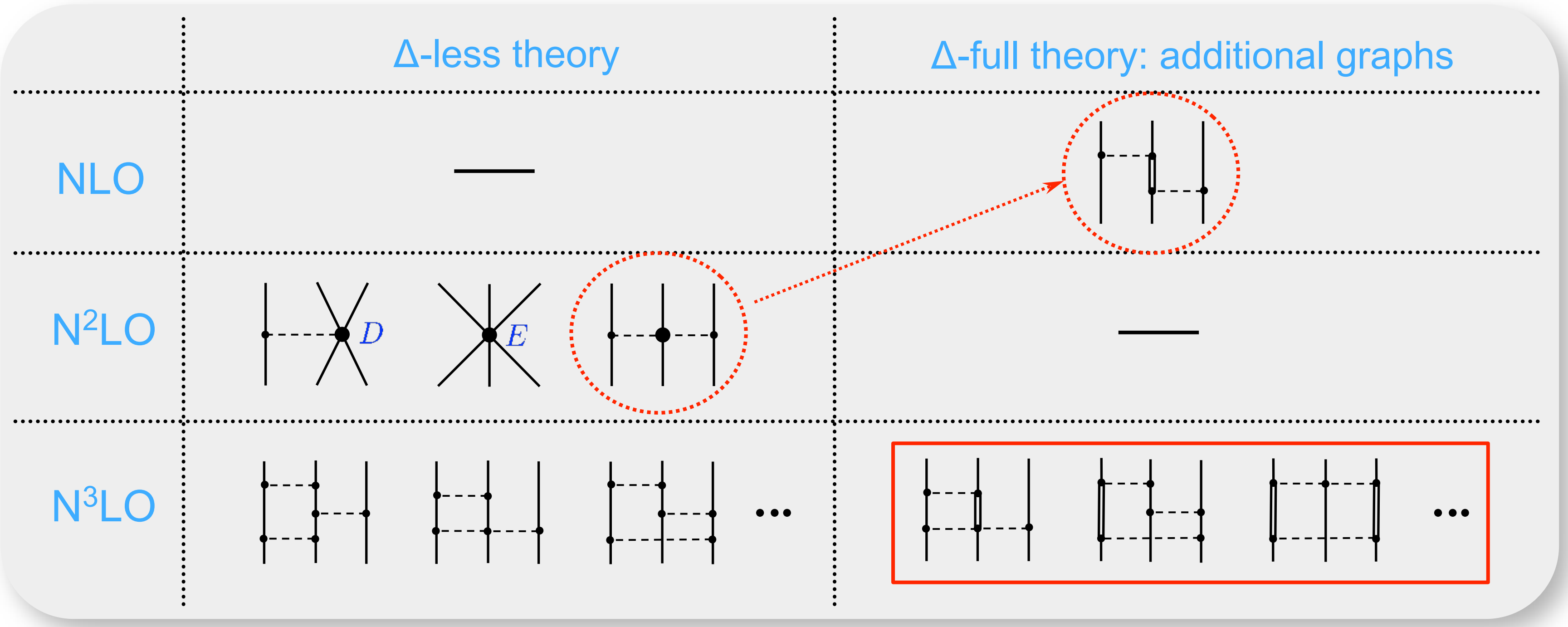
Implications for nuclear forces



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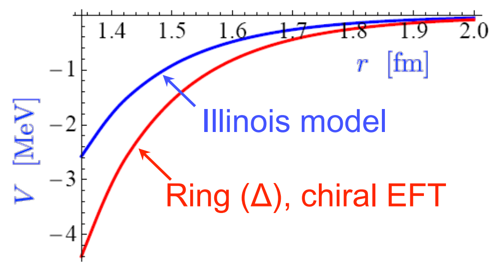
- Δ contributions at N³LO are large!
- Long-range part is parameter free
- Much richer spin/isospin structure compared to the Illinois model
- Complete analysis still to be done
  Krebs, E.E., in progress

isoscalar central potential



$\vec{\sigma}_2 \cdot \hat{r}_{12} \times \hat{r}_{23} \, \vec{\sigma}_3 \cdot \hat{r}_{13} \times \hat{r}_{23}$