

Recent developments in the Jülich model

S. Krewald

**M. Döring, H. Habertzettl, J. Haidenbauer, C. Hanhart, F. Huang,
U.-G. Meißner, K. Nakayama, D. Rönchen,**
Forschungszentrum Jülich, Universität Bonn, University of Georgia, GWU

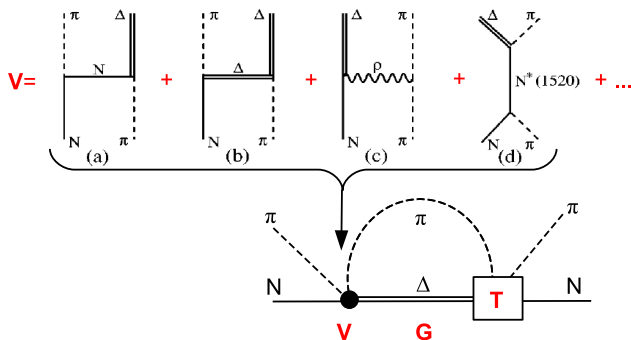
The Jülich model of meson-baryon interaction

Theoretical background

- Coupled channels $\pi N, \eta N, K\Lambda, K\Sigma; \sigma N, \rho N, \pi\Delta$.
(effective $\pi\pi N$ channels)
- Chiral Lagrangian of Wess and Zumino [PR163 (1967), Phys.Rept. 161 (1988)].
- Baryonic resonances up to $J = 7/2$ with derivative couplings.

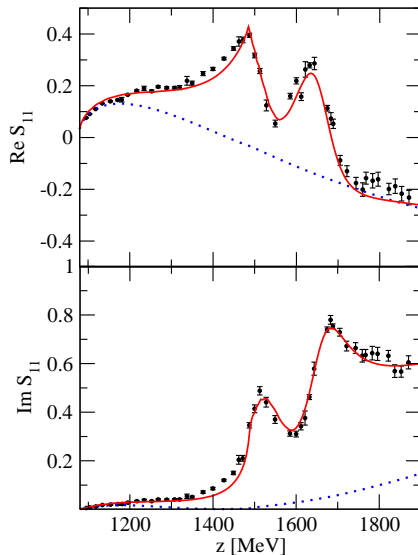
Scattering equation in the JLS basis

$$\langle L' S' k' | T_{\mu\nu}^{IJ} | L S k \rangle = \langle L' S' k' | V_{\mu\nu}^{IJ} | L S k \rangle + \sum_{\gamma, L'' S''} \int_0^\infty k''^2 dk'' \langle L' S' k' | V_{\mu\gamma}^{IJ} | L'' S'' k'' \rangle \frac{1}{Z - E_\gamma(k'') + i\epsilon} \langle L'' S'' k'' | T_{\gamma\nu}^{IJ} | L S k \rangle$$



The S_{11} partial wave in πN scattering

[Data: Arndt et al., FA08, EPJA 35 (2008)]



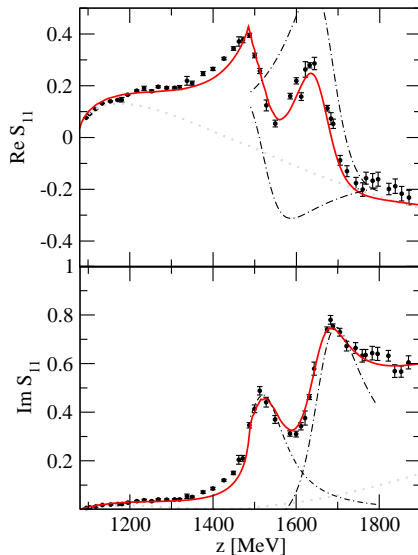
- Laurent series,

$$T^{(2)ij} = \frac{a_{-1}^{ij}}{z - z_0} + a_0^{ij} + \dots$$

- Resonance interference of $N^*(1535)$ and $N^*(1650)$.

The S_{11} partial wave in πN scattering

[Data: Arndt et al., FA08, EPJA 35 (2008)]



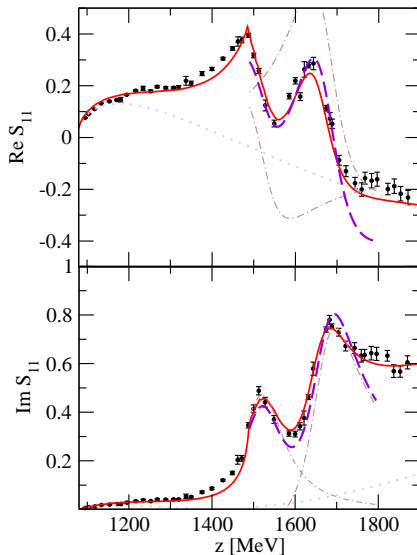
- Laurent series,

$$T^{(2)ij} = \frac{a_{-1}^{ij}}{z - z_0} + a_0^{ij} + \dots$$

- Resonance interference of $N^*(1535)$ and $N^*(1650)$.

The S_{11} partial wave in πN scattering

[Data: Arndt et al., FA08, EPJA 35 (2008)]



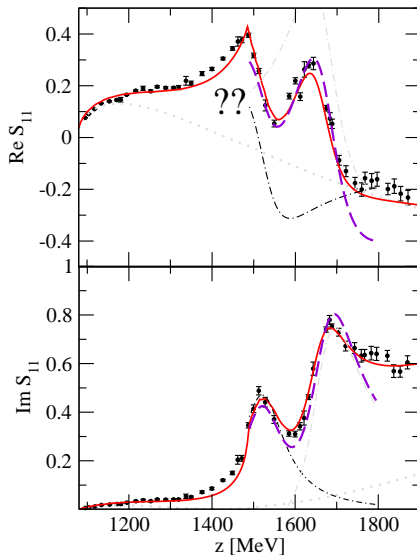
- Laurent series,

$$T^{(2)ij} = \frac{a_{-1}^{ij}}{z - z_0} + a_0^{ij} + \dots$$

- Resonance interference of $N^*(1535)$ and $N^*(1650)$.

The S_{11} partial wave in πN scattering

[Data: Arndt et al., FA08, EPJA 35 (2008)]



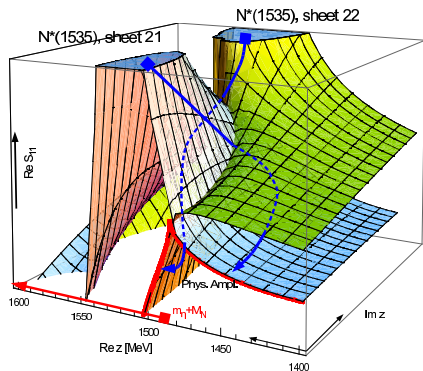
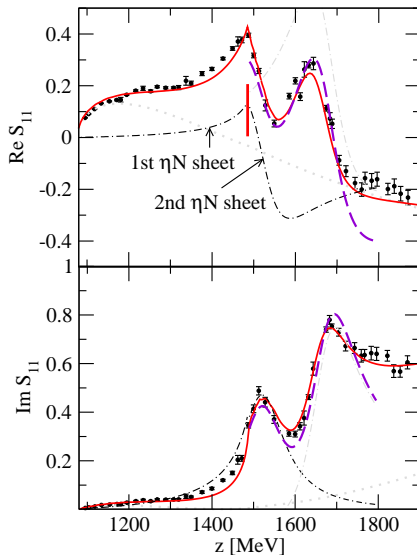
- Laurent series,

$$T^{(2)ij} = \frac{a_{-1}^{ij}}{z - z_0} + a_0^{ij} + \dots$$

- Resonance interference of $N^*(1535)$ and $N^*(1650)$.

The S_{11} partial wave in πN scattering

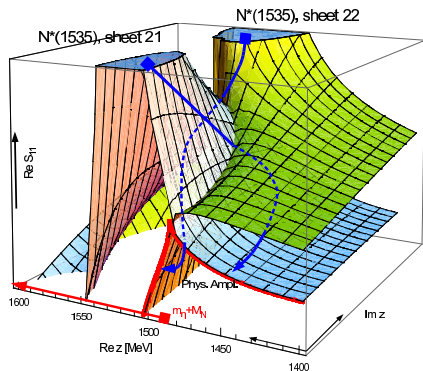
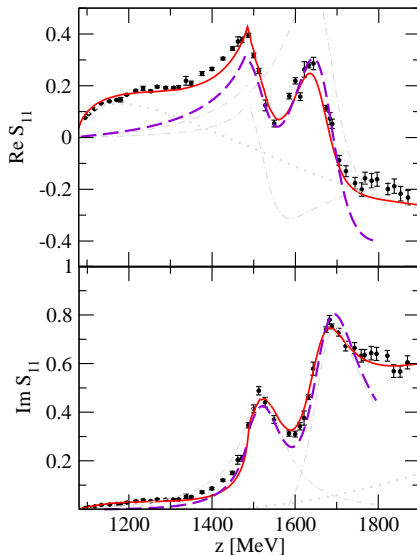
[Data: Arndt et al., FA08, EPJA 35 (2008)]



- Different poles on different sheets produce the cusp.

The S_{11} partial wave in πN scattering

[Data: Arndt et al., FA08, EPJA 35 (2008)]



- Different poles on different sheets produce the cusp.

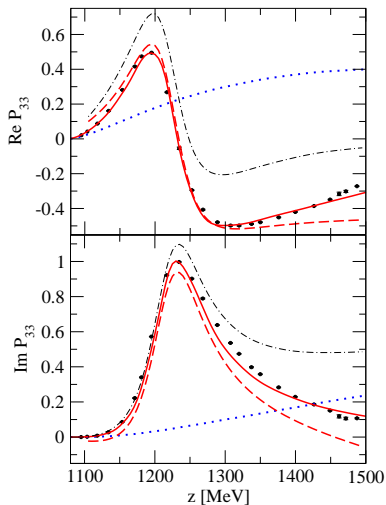
Zeros of T-matrix

Zeros of T in [MeV]. [FA02]: Arndt et al., PRC 69 (2004).

| first sheet | | second sheet | | [FA02] |
|-------------|---------------|--------------|---------------|----------------|
| P_{11} | $1235 - 0 i$ | S_{11} | $1587 - 45 i$ | $1578 - 38 i$ |
| D_{33} | $1396 - 78 i$ | S_{31} | $1585 - 17 i$ | $1580 - 36 i$ |
| | | P_{31} | $1848 - 83 i$ | $1826 - 197 i$ |
| | | P_{13} | $1607 - 38 i$ | $1585 - 51 i$ |
| | | P_{33} | $1702 - 64 i$ | - |
| | | D_{13} | $1702 - 64 i$ | $1759 - 64 i$ |

Zeros of full T-matrix :
information on poles and final state interaction.

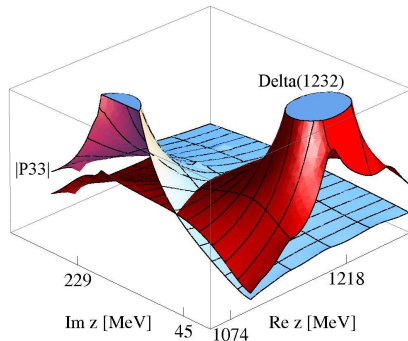
The non-pole T-matrix: P_{33}



- $T = T^{\text{NP}} + T^{\text{P}}$
- $T = \frac{a_{-1}}{z-z_0} + a_0 + \dots$

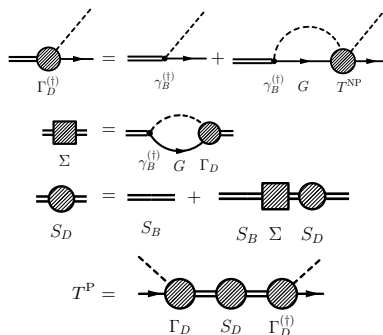
$$a_0 = T^{\text{NP}} + a_0^{\text{P}}$$

$$a_0^{\text{P}} = \frac{a_{-1}}{\Gamma_D \Gamma_D^{(\dagger)}} \left(\frac{\partial}{\partial z} (\Gamma_D \Gamma_D^{(\dagger)}) + \frac{a_{-1}}{2} \frac{\partial^2}{\partial z^2} \Sigma \right).$$



Couplings and dressed vertices

Residue a_{-1} vs. dressed vertex Γ vs. bare vertex γ .



$$a_{-1} = \frac{\Gamma_d \Gamma_d^{(\dagger)}}{1 - \frac{\partial}{\partial Z} \Sigma}$$

$$g = \sqrt{a_{-1}}$$

$$r = |(\Gamma_D - \gamma_B)/\Gamma_D|,$$

$$r' = |1 - \sqrt{1 - \Sigma'}|,$$

- Dressed Γ depends on T^{NP} .

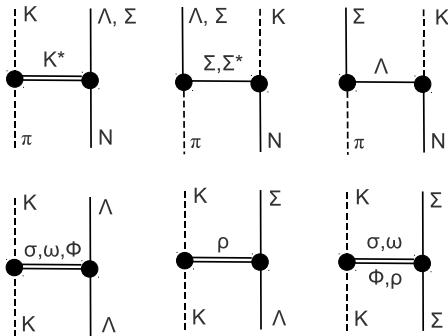
$$\boxed{\sqrt{a_{-1}} \neq \Gamma \neq \gamma}$$

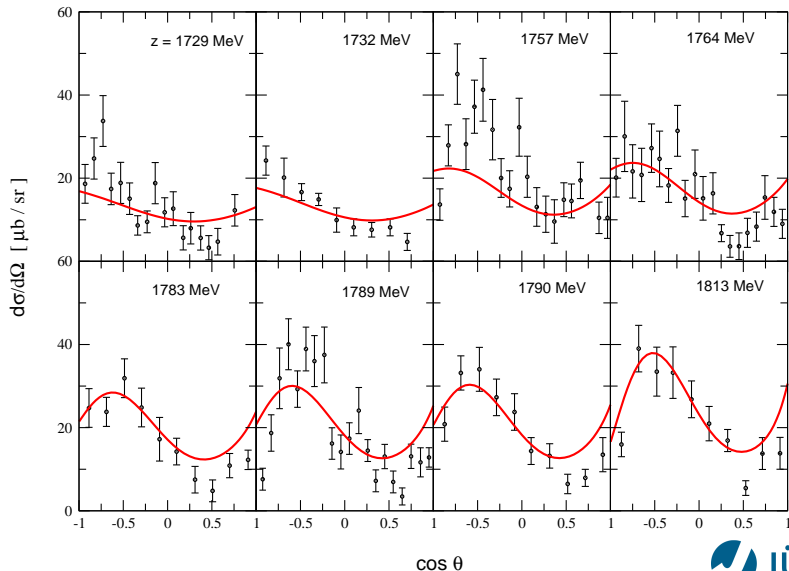
| | γ^C | Γ^C | r [%] | r' [%] |
|-------------------------|---------------|----------------|---------|----------|
| $N^*(1520) D_{13}$ | $6.4 - 0.6i$ | $13.2 + 1.2i$ | 53 | 61 |
| $N^*(1720) P_{13}$ | $-0.1 + 5.4i$ | $0.9 + 4.8i$ | 24 | 45 |
| $\Delta(1232) P_{33}$ | $1.3 + 13.0i$ | $-2.8 + 22.2i$ | 45 | 40 |
| $\Delta^*(1620) S_{31}$ | $0.1 + 14.3i$ | $5.0 + 5.7i$ | 130 | 66 |
| $\Delta^*(1700) D_{33}$ | $5.4 - 0.8i$ | $6.7 + 1.0i$ | 33 | 54 |
| $\Delta^*(1910) P_{31}$ | $9.4 + 0.3i$ | $1.9 - 3.2i$ | 222 | 22 |

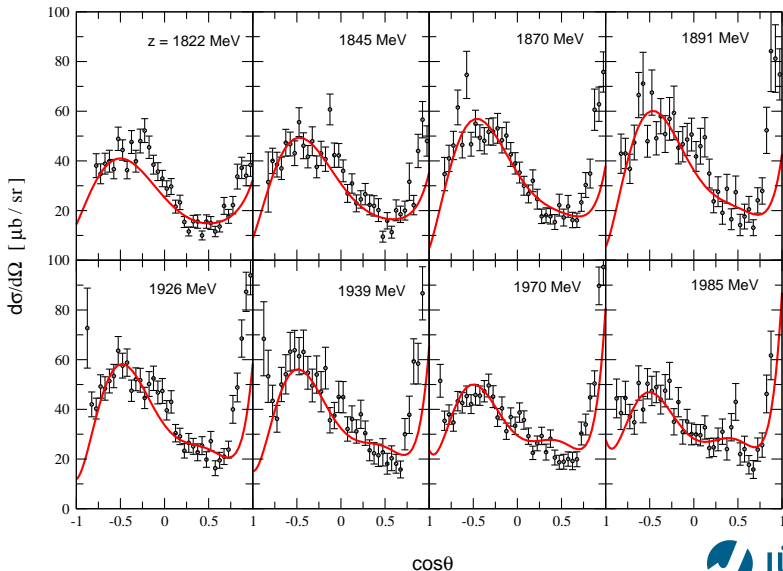
The reaction $\pi^+ p \rightarrow K^+ \Sigma^+$

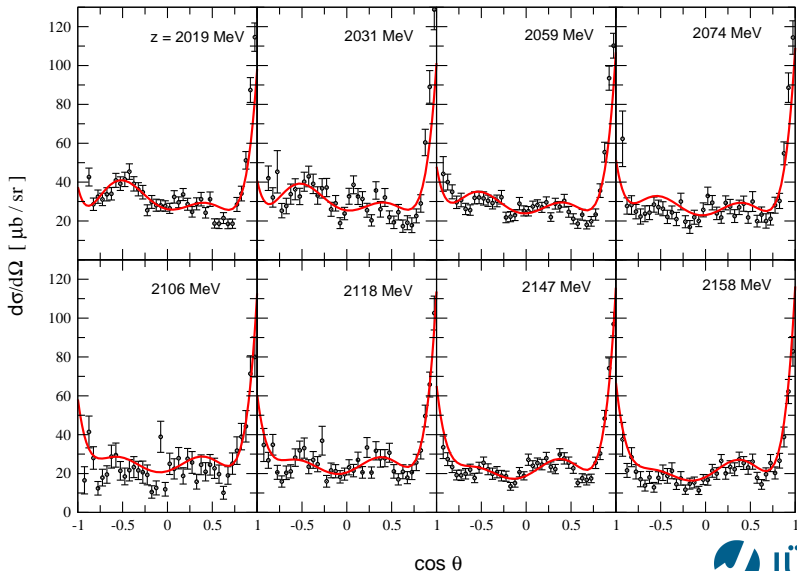
Towards a unified analysis of different final states

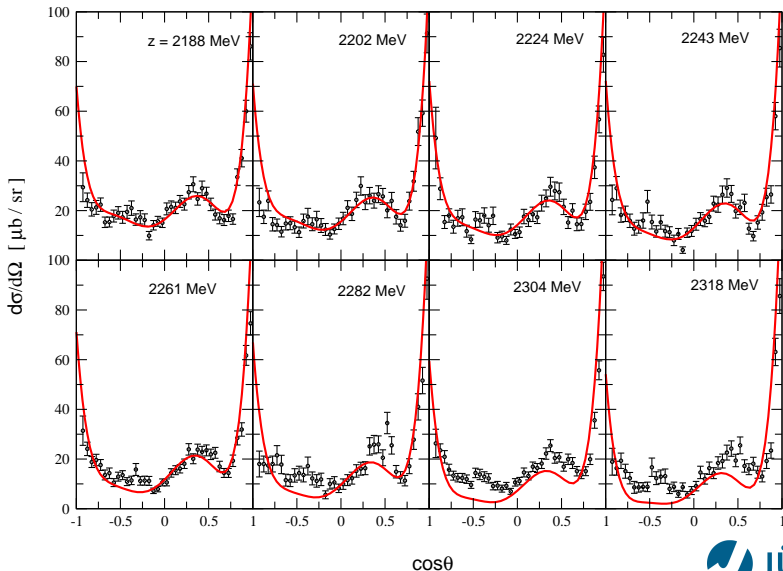
- $\pi^+ p \rightarrow K^+ \Sigma^+$: Pure isospin 3/2; few Δ resonances.
- $SU(3)$ symmetry provides predictive power.
- Guiding principle in the fit: minimal set of Resonances.

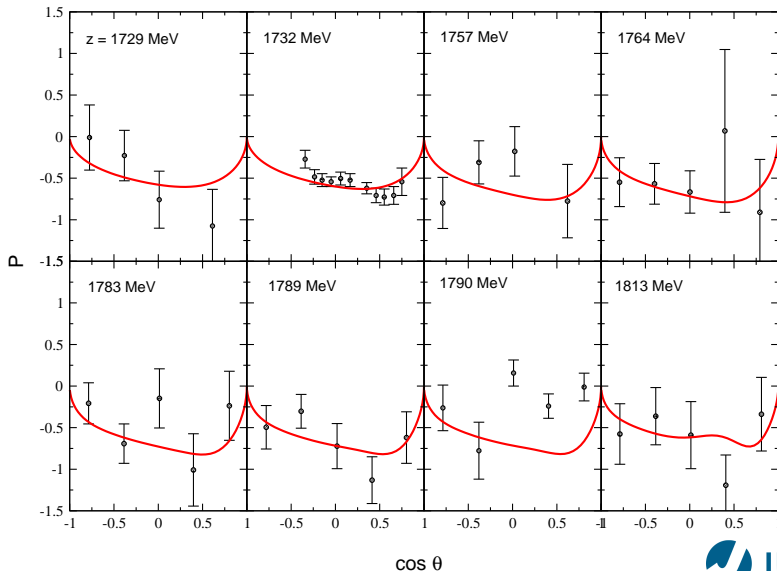


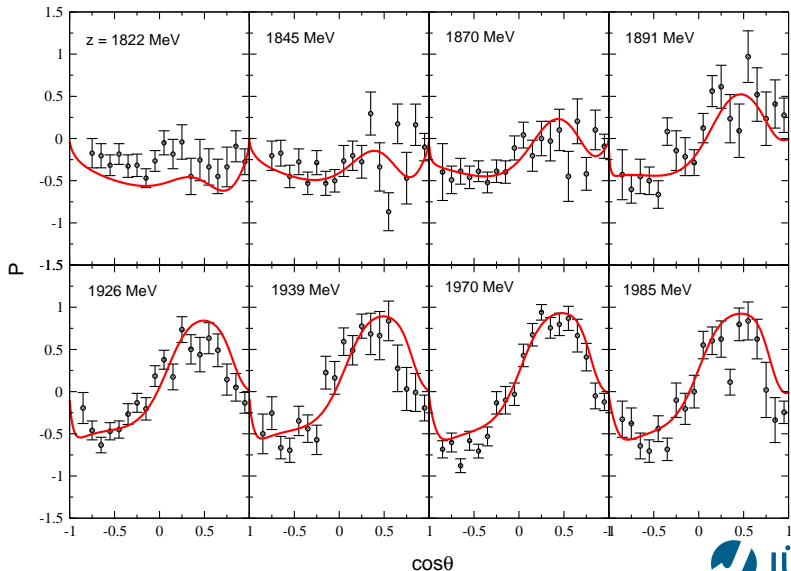
Differential cross section of $\pi^+ p \rightarrow K^+ \Sigma^+$ 

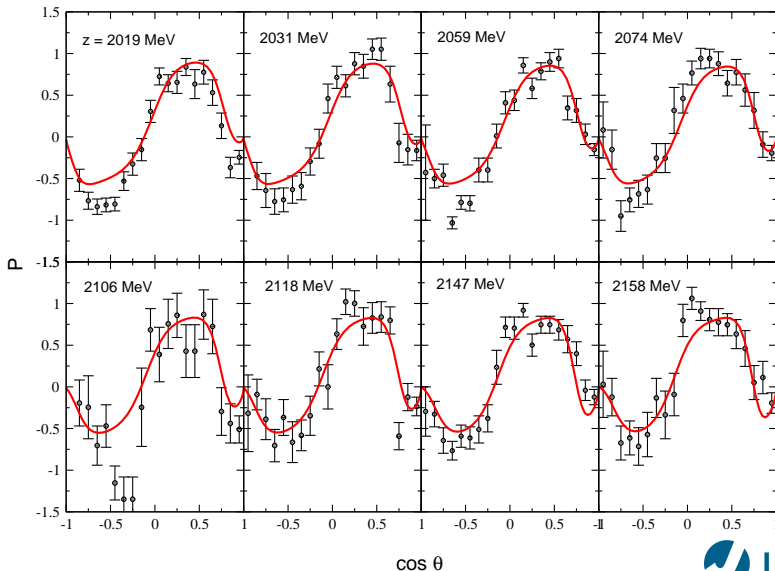
Differential cross section of $\pi^+ p \rightarrow K^+ \Sigma^+$ 

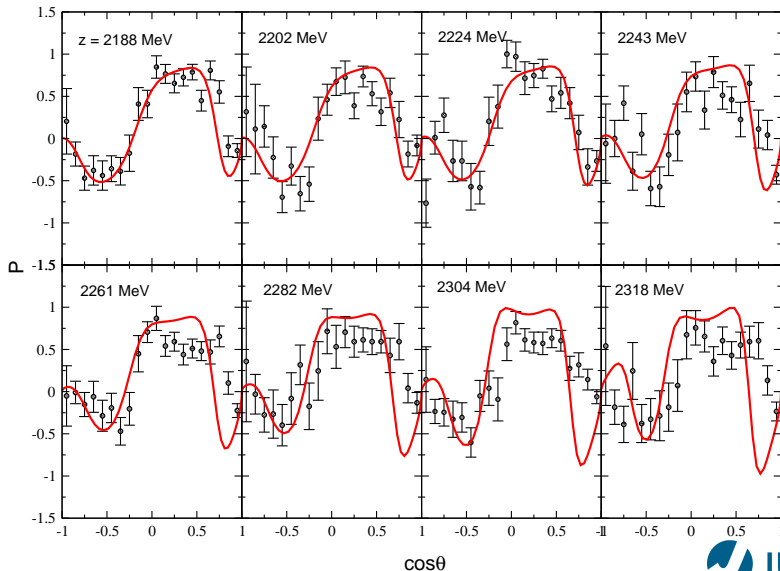
Differential cross section of $\pi^+ p \rightarrow K^+ \Sigma^+$ 

Differential cross section of $\pi^+ p \rightarrow K^+ \Sigma^+$ 

Polarization of $\pi^+ p \rightarrow K^+ \Sigma^+$ 

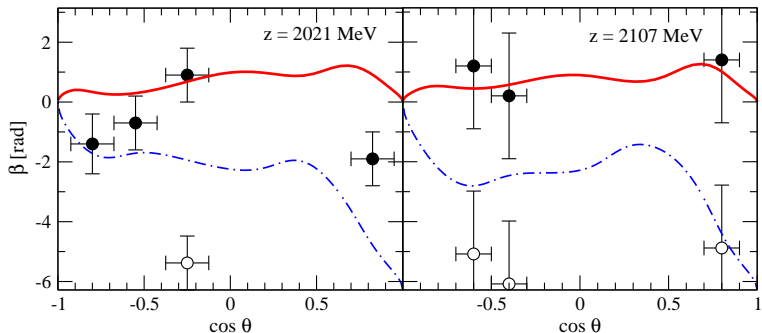
Polarization of $\pi^+ p \rightarrow K^+ \Sigma^+$ 

Polarization of $\pi^+ p \rightarrow K^+ \Sigma^+$ 

Polarization of $\pi^+ p \rightarrow K^+ \Sigma^+$ 

Spin rotation parameter β of $\pi^+ p \rightarrow K^+ \Sigma^+$

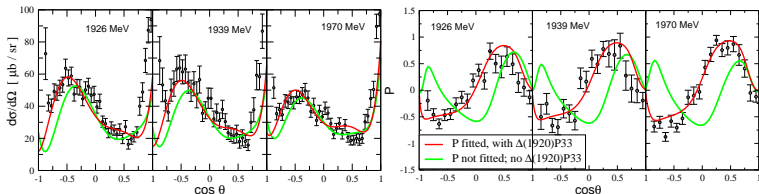
Definitions Observables



important to reduce ambiguities.

The importance of spin observables: $\Delta(1920)P_{33}$

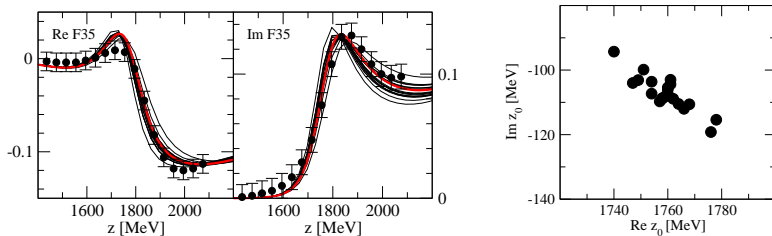
- $\Delta(1920)P_{33}$: invisible in $\pi N \rightarrow \pi N$, but needed in $\pi^+ p \rightarrow K^+ \Sigma^+$.
- GREEN: no $\Delta(1920)P_{33}$, fit elastic πN , $d\sigma/d\Omega(\pi^+ K^+)$, but NOT P .
- RED: $\Delta(1920)P_{33}$, fit elastic πN , $d\sigma/d\Omega(\pi^+ K^+)$ and P .



- Without polarization measurement P , one could have overlooked the $\Delta(1920)P_{33}$.

Error analysis: $\Delta(1905)F_{35}$

- Determination of the non-linear parameter error
 - $\chi^2 + 1$ criterion.
 - Varying 39 of 40 parameters to get parameter error.
- Get error on derived quantities like pole positions and residues.
- So far, simplified consideration (error from πN not available, because energy dependent GWU/SAID solution is fitted [PRC74 (2006)]).



Error estimates for masses: $\Delta(1905)F_{35}$ **Table:** Error estimates of bare mass m_b and bare coupling f for the $\Delta(1905)F_{35}$ resonance.

| m_b [MeV] | πN | ρN | $\pi \Delta$ | ΣK |
|--------------------|------------------------------|-------------------------|---------------------------|-----------------------------|
| 2258^{+44}_{-43} | $0.0500^{+0.0011}_{-0.0012}$ | $-1.62^{+1.29}_{-1.61}$ | $-1.15^{+0.030}_{-0.022}$ | $0.120^{+0.0065}_{-0.0059}$ |

Table: Error estimates of pole position and residues for the $\Delta(1905)F_{35}$ resonance.

| | | | $\pi N \rightarrow \pi N$ | $\pi N \rightarrow K \Sigma$ |
|----------------|--------------------|--------------|---------------------------|------------------------------|
| Re z_0 [MeV] | 1764^{+18}_{-20} | $ r $ [MeV] | $11^{+1.7}_{-1.4}$ | $1.4^{+0.24}_{-0.21}$ |
| Im z_0 [MeV] | -109^{+13}_{-12} | θ [°] | $-45^{+3.8}_{-11}$ | $-313^{+4.2}_{-10}$ |

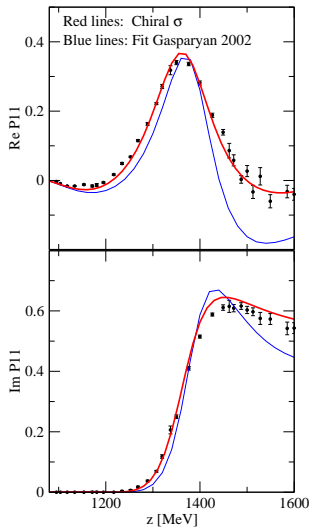
Resonance content from $\pi N \rightarrow \pi N$ plus $\pi^+ p \rightarrow K^+ \Sigma^+$

| | Re z_0 [MeV] -2 Im z_0 [MeV] | $ r $ [MeV] θ [°] | $(\Gamma_{\pi N}^{1/2} \Gamma_{K\Sigma}^{1/2}) / \Gamma_{\text{tot}}$ [%] | | |
|--|-------------------------------------|-----------------------------|---|----------------|---------------|
| | | | This study | Candlin (1983) | Gießen (2004) |
| $\Delta(1905) F_{35}$ 5/2 ⁺ **** | 1764 218 | 1.4 -313 | 1.23 | 1.5(3) | <1 |
| $\Delta(1910) P_{31}$ 1/2 ⁺ **** | 1721 323 | 5.5 -6 | 2.98 | <3 | 1.1 |
| $\Delta(1920) P_{33}$ 3/2 ⁺ *** | 1884 229 | 5.9 -38 | 5.07 | 5.2(2) | 2.1(3) |
| $\Delta(1930) D_{35}$ 5/2 ⁻ *** | 1865 147 | 1.6 -43 | 2.14 | <1.5 | |
| $\Delta(1950) F_{37}$ 7/2 ⁺ **** | 1873 206 | 2.7 -255 | 2.54 | 5.3(5) | — |

Summary

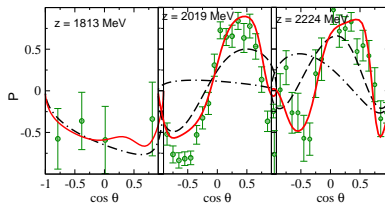
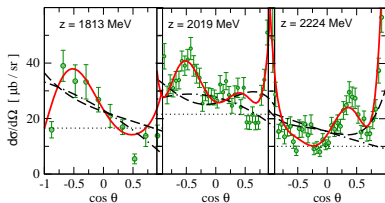
- Analytical Structure:
Poles, residues, branch points, zeros.
- Interpretation: model insensitive quantities
dressed vertices
- Results: coupled channel approach applied for energies below 2.2GeV.
Room for more resonances!
ERROR ANALYSIS
- In progress:
 $K\Lambda$, $K\Sigma$
Photoproduction, Electroproduction.
Determine Lagrangian coupling constants directly from data analysis.

Chiral sigma, P_{11}

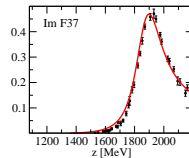
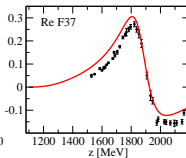
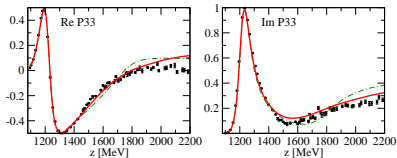


The reaction $\pi^+ p \rightarrow K^+ \Sigma^+$

M.D., C. Hanhart, F. Huang, S. Krewald, U.-G. Meißner, D. Rönchen, [NPA851 (2011)]



Data upper: Candlin 1983, NPB 226 (1983), lower: GWU/SAID, PRC74 (2006)



Couplings “ $g = \sqrt{a_{-1}}$ ” to other channels

◀ back

| | $N\pi$ | $N\rho^{(1)} (S = 1/2)$ | $N\rho^{(2)} (S = 3/2)$ | $N\rho^{(3)} (S = 3/2)$ |
|-------------------------|----------------------------|----------------------------|----------------------------|----------------------------|
| $N^*(1535) S_{11}$ | $S_{11} \quad 8.1 + 0.5i$ | $S_{11} \quad 2.2 - 5.4i$ | – | $D_{11} \quad 0.5 - 0.1i$ |
| $N^*(1650) S_{11}$ | $S_{11} \quad 8.6 - 2.8i$ | $S_{11} \quad 0.9 - 9.1i$ | – | $D_{11} \quad 0.3 - 0.1i$ |
| $N^*(1440) P_{11}$ | $P_{11} \quad 11.2 - 5.0i$ | $P_{11} \quad -1.3 + 3.2i$ | $P_{11} \quad 3.6 - 2.6i$ | – |
| $\Delta^*(1620) S_{31}$ | $S_{31} \quad 2.9 - 3.7i$ | $S_{31} \quad 0.0 - 0.0i$ | – | $D_{31} \quad 0.0 - 0.1i$ |
| $\Delta^*(1910) P_{31}$ | $P_{31} \quad 1.2 - 3.5i$ | $P_{31} \quad 0.2 - 0.4i$ | $P_{31} \quad -0.2 - 0.4i$ | – |
| $N^*(1720) P_{13}$ | $P_{13} \quad 3.7 - 2.6i$ | $P_{13} \quad 0.1 + 0.8i$ | $P_{13} \quad -1.1 + 0.1i$ | $F_{13} \quad 0.1 - 0.1i$ |
| $N^*(1520) D_{13}$ | $D_{13} \quad 8.4 - 0.8i$ | $D_{13} \quad -0.6 + 0.7i$ | $D_{13} \quad 0.9 - 2.0i$ | $S_{13} \quad -2.5 - 0.1i$ |
| $\Delta(1232) P_{33}$ | $P_{33} \quad 17.9 - 3.2i$ | $P_{33} \quad -1.3 - 0.8i$ | $P_{33} \quad -0.9 - 3.0i$ | $F_{33} \quad 0.0 - 0.1i$ |
| $\Delta^*(1700) D_{33}$ | $D_{33} \quad 4.9 - 1.0i$ | $D_{33} \quad -0.2 + 0.9i$ | $D_{33} \quad -0.4 - 0.4i$ | $S_{33} \quad -0.1 - 0.1i$ |

| | $N\eta$ | $\Delta\pi^{(1)}$ | $\Delta\pi^{(2)}$ | $N\sigma$ |
|-------------------------|-----------------------------|--|-----------------------------|----------------------------|
| $N^*(1535) S_{11}$ | $S_{11} \quad 11.9 - 2.3i$ | – | $D_{11} \quad -5.9 + 4.8i$ | $P_{11} \quad -1.4 - 0.1i$ |
| $N^*(1650) S_{11}$ | $S_{11} \quad -3.0 + 0.5i$ | – | $D_{11} \quad 4.3 + 0.4i$ | $P_{11} \quad -2.1 - 0.1i$ |
| $N^*(1440) P_{11}$ | $P_{11} \quad -0.1 + 0.0i$ | $P_{11} \quad -4.6 - 1.7i$ | – | $S_{11} \quad -8.3 - 0.1i$ |
| $\Delta^*(1620) S_{31}$ | – | – | $D_{31} \quad 11.1 - 4.0i$ | – |
| $\Delta^*(1910) P_{31}$ | – | $P_{31} \quad 15.0 - 0.3i$ | – | – |
| $N^*(1720) P_{13}$ | $P_{13} \quad -7.7 + 5.5i$ | $P_{13} \quad -14.1 + 3.0i$ | $F_{13} \quad 0.0 - 0.3i$ | $D_{13} \quad -0.8 - 0.1i$ |
| $N^*(1520) D_{13}$ | $D_{13} \quad 0.16 - 0.60i$ | $D_{13} \quad 0.0 + 0.4i$ | $S_{13} \quad -12.9 - 0.7i$ | $P_{13} \quad -0.6 - 0.1i$ |
| $\Delta(1232) P_{33}$ | – | $P_{33} \quad -(4 \text{ to } 5) + i(0 \text{ to } 0.5)$ | $F_{33} \quad \sim 0$ | – |
| $\Delta^*(1700) D_{33}$ | – | $D_{33} \quad -0.7 - 0.3i$ | $S_{33} \quad -19.7 + 4.5i$ | – |

Resonance couplings g_i [$10^{-3} \text{ MeV}^{-1/2}$] to the coupled channels i . Also, the LJS type of each coupling is indicated. For the ρN channels, the total spin S is also indicated.

| first sheet | | second sheet | | [FA02] |
|-------------|---------------|--------------|---------------|----------------|
| P_{11} | $1235 - 0 i$ | S_{11} | $1587 - 45 i$ | $1578 - 38 i$ |
| D_{33} | $1396 - 78 i$ | S_{31} | $1585 - 17 i$ | $1580 - 36 i$ |
| | | P_{31} | $1848 - 83 i$ | $1826 - 197 i$ |
| | | P_{13} | $1607 - 38 i$ | $1585 - 51 i$ |
| | | P_{33} | $1702 - 64 i$ | - |
| | | D_{13} | $1702 - 64 i$ | $1759 - 64 i$ |

Position of **zeros** of the full amplitude T in [MeV]. [FA02]: Arndt et al., PRC 69 (2004).

| | $\Gamma_{\pi N}/\Gamma_{\text{Tot}}$ [%] | $\Gamma_{\eta N}/\Gamma_{\text{Tot}}$ [%] |
|-------------------------|--|---|
| $N^*(1535) S_{11}$ | 48 [33 to 55] | 38 [45 to 60] |
| $N^*(1650) S_{11}$ | 79 [60 to 95] | 6 [3 to 10] |
| $N^*(1440) P_{11}$ | 64 [55 to 75] | 0 [0 ± 1] |
| $\Delta^*(1620) S_{31}$ | 34 [20 to 30] | - |
| $\Delta^*(1910) P_{31}$ | 11 [15 to 30] | - |
| $N^*(1720) P_{13}$ | 13 [10 to 20] | 38 [4 ± 1] |
| $N^*(1520) D_{13}$ | 67 [55 to 65] | 0.10 [0.23 ± 0.04] |
| $\Delta(1232) P_{33}$ | 100 [100] | - |
| $\Delta^*(1700) D_{33}$ | 13 [10 to 20] | - |

Branching ratios into πN and ηN . The values in brackets are from the PDG, [Amsler et al., PLB 667 (2008)].

g_{fi} und h_{fi} in JLS-Basis:

$$g_{fi} = \frac{1}{2\sqrt{k_f k_i}} \sum_j (2j+1) d_{\frac{1}{2}\frac{1}{2}}^j(\theta) \left[\tau^{j(j-\frac{1}{2})\frac{1}{2}} + \tau^{j(j+\frac{1}{2})\frac{1}{2}} \right] \cos \frac{\theta}{2} \\ + \frac{1}{2\sqrt{k_f k_i}} \sum_j (2j+1) d_{-\frac{1}{2}\frac{1}{2}}^j(\theta) \left[\tau^{j(j-\frac{1}{2})\frac{1}{2}} - \tau^{j(j+\frac{1}{2})\frac{1}{2}} \right] \sin \frac{\theta}{2}$$

$$h_{fi} = \frac{-i}{2\sqrt{k_f k_i}} \sum_j (2j+1) d_{\frac{1}{2}\frac{1}{2}}^j(\theta) \left[\tau^{j(j-\frac{1}{2})\frac{1}{2}} + \tau^{j(j+\frac{1}{2})\frac{1}{2}} \right] \sin \frac{\theta}{2} \\ + \frac{i}{2\sqrt{k_f k_i}} \sum_j (2j+1) d_{-\frac{1}{2}\frac{1}{2}}^j(\theta) \left[\tau^{j(j-\frac{1}{2})\frac{1}{2}} - \tau^{j(j+\frac{1}{2})\frac{1}{2}} \right] \cos \frac{\theta}{2}$$

$$\begin{aligned}
 \frac{d\sigma}{d\Omega} &= \frac{k_f}{k_i} (|g_{fi}|^2 + |h_{fi}|^2) \\
 &= \frac{1}{2k_i^2} \frac{1}{2} \cdot \left(\left| \sum_j (2j+1) (\tau^{j(j-\frac{1}{2})\frac{1}{2}} + \tau^{j(j+\frac{1}{2})\frac{1}{2}}) \cdot d_{\frac{1}{2}\frac{1}{2}}^j(\Theta) \right|^2 \right. \\
 &\quad \left. + \left| \sum_j (2j+1) (\tau^{j(j-\frac{1}{2})\frac{1}{2}} - \tau^{j(j+\frac{1}{2})\frac{1}{2}}) \cdot d_{-\frac{1}{2}\frac{1}{2}}^j(\Theta) \right|^2 \right)
 \end{aligned}$$

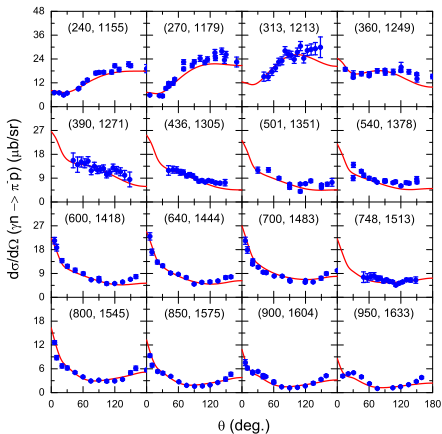
$$\vec{P}_f = \frac{2\text{Re}(g_{fi}h_{fi}^*)}{|g_{fi}|^2 + |h_{fi}|^2} \cdot \hat{n}$$

$$\beta = \arctan \left(\frac{2\text{Im}(h_{fi}^*g_{fi})}{|g_{fi}|^2 - |h_{fi}|^2} \right)$$

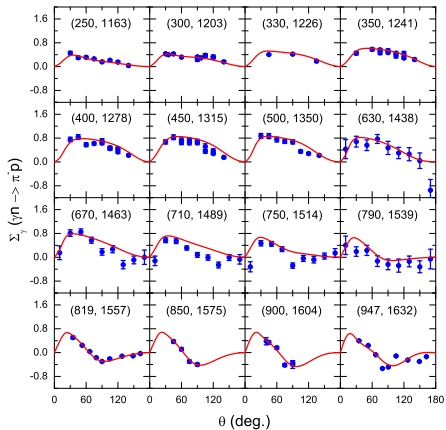
$d\sigma/d\Omega$ and Σ_γ for $\gamma n \rightarrow \pi^- p$

preliminary

◀ back



Differential cross section for $\gamma n \rightarrow \pi^- p$

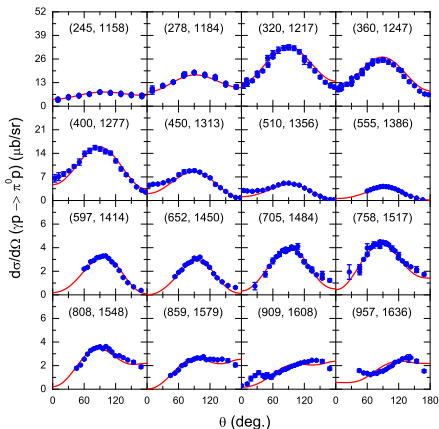


Photon spin asymmetry for $\gamma n \rightarrow \pi^- p$

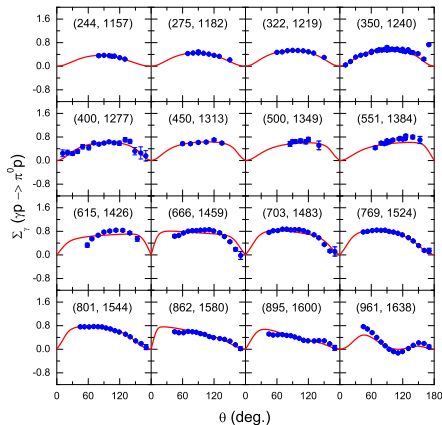
$d\sigma/d\Omega$ and Σ_γ for $\gamma p \rightarrow \pi^0 p$

preliminary

◀ back



Differential cross section for $\gamma p \rightarrow \pi^0 p$



Photon spin asymmetry for $\gamma p \rightarrow \pi^0 p$

χ Pert. Theory

- Systematic expansion in p , m .
- Perturbative unitarity.
- Low energy.

χ unitary approaches

- Systematics lost.
- Unitarity gained.
- Usually restriction to S -wave.
→ limited tool for data analysis.
- Dynamical generation of resonances.
 $\sigma(600)$, $f_0(980)$, $\Lambda(1405)$, ...
(Lutz, Meißner, Oller, Oset, Pelaez, Weise,...)
- Including NLO contributions in a “true” solution of the BSE (Mai, Meißner,...).

Dynamical CC models

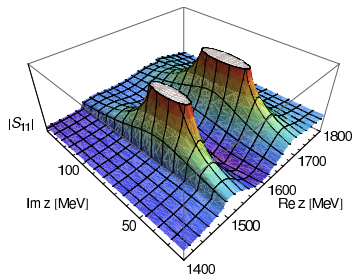
- Unitarity, dispersive parts.
- Lagrangian based, field theoretical approach.
- Chiral constraints.
- Dynamical generation of resonances plus genuine s -channel states.
- practical restrictions as data analysis tool.
- EBAC, DMT, Jülich, Nijmegen,...

K matrix approaches

- Unitarity.
- No dispersive parts.
- Lagrangian based (Gießen, Groningen,...).
- ...or phenomenological (Bonn-Gatchina).
- Partial wave analyses (GWU/Said).
- Most flexible tool for data analysis.

Poles and residues: Parameterization of the resonance content

[M.D., C. Hanhart, F. Huang, S. Krewald and U.-G. Meißner, NPA 829 (2009), PLB 681 (2009)]



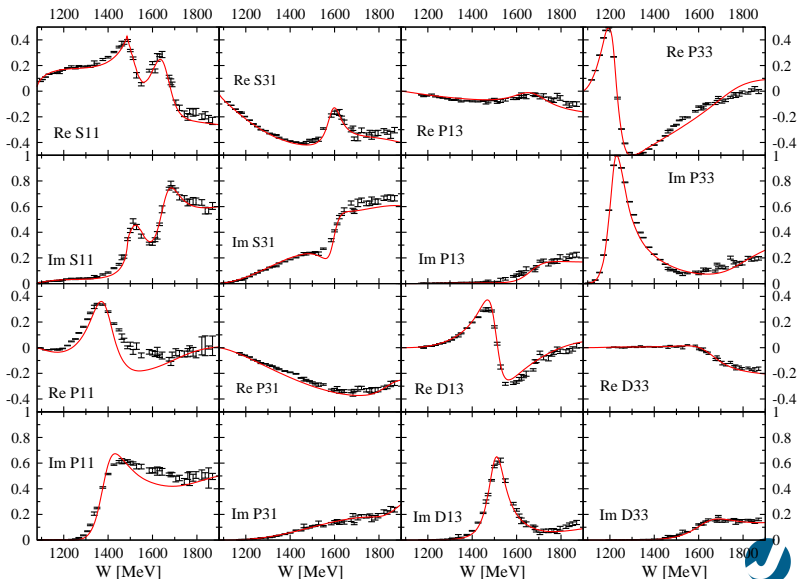
| | Re z_0 [MeV] | -2 Im z_0 [MeV] | $ R $ [MeV] | θ [deg] [$^\circ$] | | | | | |
|-------------------------|-------------------|----------------------|----------------|--------------------------------|-------------------------|---------------|--------------|------------|--------------|
| $N^*(1535) S_{11}$ | 1519 | 129 | 31 | -3 | | | | | |
| ARN | 1502 | 95 | 16 | -16 | | | | | |
| HOE | 1487 | | | | | | | | |
| CUT | 1510 ± 50 | 260 ± 80 | 120 ± 40 | $+15 \pm 45$ | | | | | |
| $N^*(1650) S_{11}$ | 1669 | 136 | 54 | -44 | | | | | |
| ARN | 1648 | 80 | 14 | -69 | | | | | |
| HOE | 1670 | 163 | 39 | -37 | | | | | |
| CUT | 1640 ± 20 | 150 ± 30 | 60 ± 10 | -75 ± 25 | | | | | |
| $N^*(1720) P_{13}$ | 1663 | 212 | 14 | -82 | | | | | |
| ARN | 1666 | 355 | 25 | -94 | | | | | |
| HOE | 1686 | 187 | 15 | | | | | | |
| CUT | 1680 ± 30 | 120 ± 40 | 8 ± 12 | -160 ± 30 | | | | | |
| $\Delta(1232) P_{33}$ | 1218 | 90 | 47 | -37 | | | | | |
| ARN | 1211 | 99 | 52 | -47 | | | | | |
| HOE | 1209 | 100 | 50 | -48 | | | | | |
| CUT | 1210 ± 1 | 100 ± 2 | 53 ± 2 | -47 ± 1 | | | | | |
| $\Delta^*(1620) S_{31}$ | 1593 | 72 | 12 | -108 | | | | | |
| ARN | 1595 | 135 | 15 | -92 | | | | | |
| HOE | 1608 | 116 | 19 | -95 | | | | | |
| CUT | 1600 ± 15 | 120 ± 20 | 15 ± 2 | -110 ± 20 | | | | | |
| $N^*(1440) P_{11}$ | 1387 | 147 | 48 | -64 | $\Delta^*(1700) D_{33}$ | 1637 | 236 | 16 | -38 |
| ARN | 1359 | 162 | 38 | -98 | ARN | 1632 | 253 | 18 | -40 |
| HOE | 1385 | 164 | 40 | | HOE | 1651 | 159 | 10 | |
| CUT | 1375 ± 30 | 180 ± 40 | 52 ± 5 | -100 ± 35 | CUT | 1675 ± 25 | 220 ± 40 | 13 ± 3 | -20 ± 25 |
| $N^*(1520) D_{13}$ | 1505 | 95 | 32 | -18 | $\Delta^*(1910) P_{31}$ | 1840 | 221 | 12 | -153 |
| ARN | 1515 | 113 | 38 | -5 | ARN | 1771 | 479 | 45 | +172 |
| HOE | 1510 | 120 | 32 | -8 | HOE | 1874 | 283 | 38 | |
| CUT | 1510 ± 5 | 114 ± 10 | 35 ± 2 | -12 ± 5 | CUT | 1880 ± 30 | 200 ± 40 | 20 ± 4 | -90 ± 30 |

[ARN]: Arndt et al., PRC 74 (2006), [HOE]: Höhler, πN Newsl. 9 (1993), [CUT]: Cutkowski et al., PRD 20 (1979).

Residues to ηN , σN , ρN , $\pi \Delta$. Zeros. Branching ratios to πN , ηN .

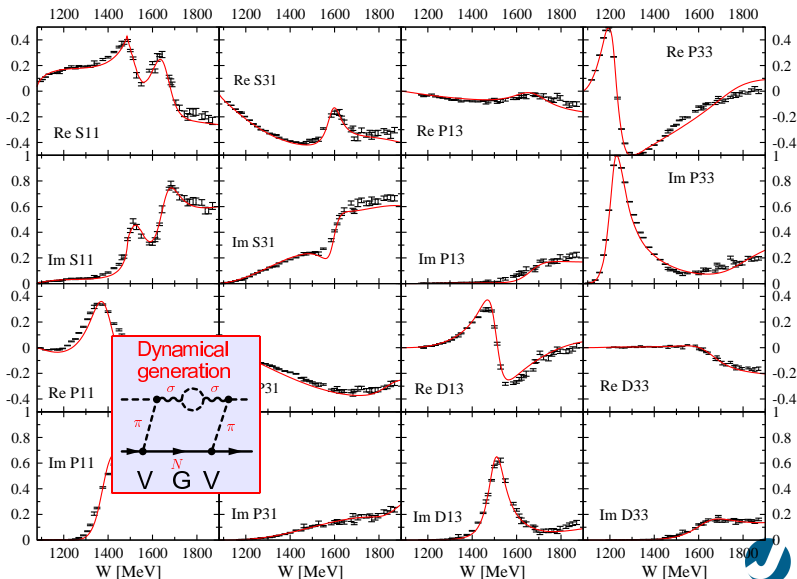
Some partial waves in $\pi N \rightarrow \pi N$

Jülich approach, solution 2002; "Data": GWU/SAID, PRC74 (2006)



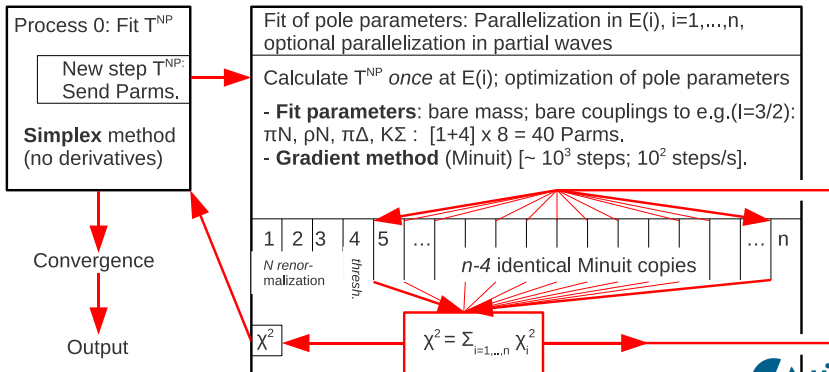
Some partial waves in $\pi N \rightarrow \pi N$

Jülich approach, solution 2002; "Data": GWU/SAID, PRC74 (2006)



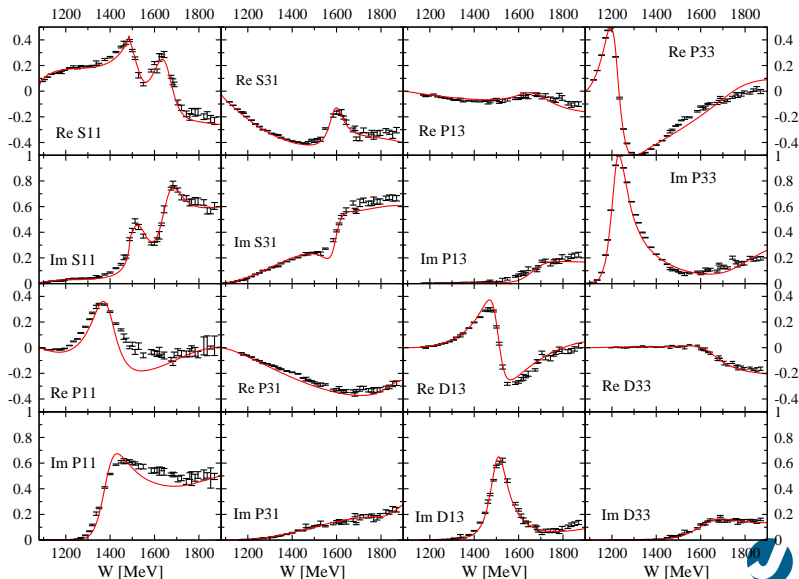
Requirements:

- 1) Maintain speed advantage of ($\times 100$) of calculation of T^P from T^{NP} ($T=T^{NP}+T^P$)
 - > 2 nested Minuit runs: full fit of T^P [-40 parms.] for every step in T^{NP}
 - > requires separated memory spaces/ **mpi** parallelization on **Juropa/FZ Julich**
- 2) Scaling with # processes
- 3) Adding large amounts of data to χ^2 without increase of execution time



Partial waves in $\pi N \rightarrow \pi N$ (Solution 2002)

"Data": GWU/SAID, PRC74 (2006)

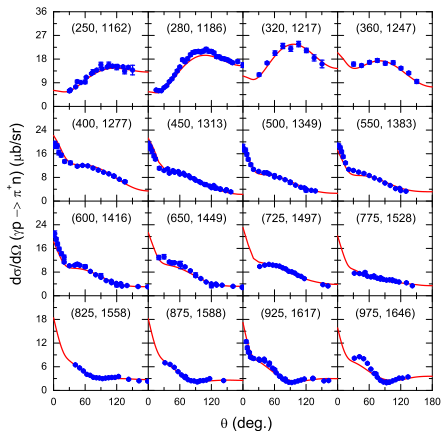


Linking reactions: $d\sigma/d\Omega$ and Σ_γ for $\gamma p \rightarrow \pi^+ n$

F. Huang, M.D., K. Nakayama, *et al.*, in prep. (see also arXiv:1103.2065)

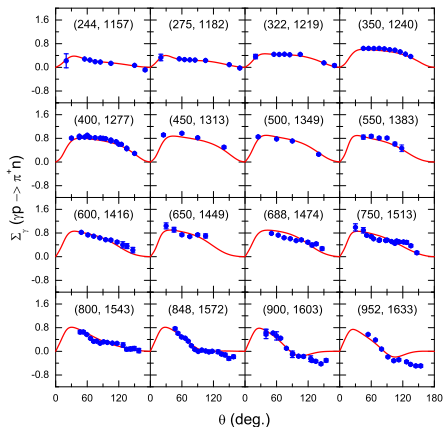
Gauge invariance respected

Further results $\gamma N \rightarrow \pi N$



Differential cross section for $\gamma p \rightarrow \pi^+ n$

Data: CNS Data analysis center [CBELSA/TAPS, JLAB, MAMI,...]



Photon spin asymmetry for $\gamma p \rightarrow \pi^+ n$

The reaction $\pi^+ p \rightarrow K^+ \Sigma^+$

Towards a unified analysis of different final states

- Different reaction channels provide more information on resonance content than higher precision in one channel.
- SU(3) symmetry provides predictive power.
- Linking partial waves and **different reactions** puts more constraints on the resonance content.
- $\pi^+ p \rightarrow K^+ \Sigma^+$: Pure isospin 3/2; relatively few Δ resonances.
- Simultaneous fit to $\pi N \rightarrow \pi N$ partial waves [energy dependent GWU SAID solution, PRC74 (2006)] plus $\pi^+ p \rightarrow K^+ \Sigma^+$ observables.
- Guiding principle in the fit: Resonances are the last resort (while technically being the easiest way to improve the χ^2).
- Error analysis on pole positions is crucial.