Combined analysis of pion-induced reactions in a dynamical coupled-channels approach

M. Döring

H. Haberzettl, J. Haidenbauer, C. Hanhart, F. Huang, S. Krewald, U.-G. Meißner, K. Nakayama, D. Rönchen,
Forschungszentrum Jülich, Universität Bonn, University of Georgia, GWU

The Jülich model of pion-nucleon interaction

Motivation

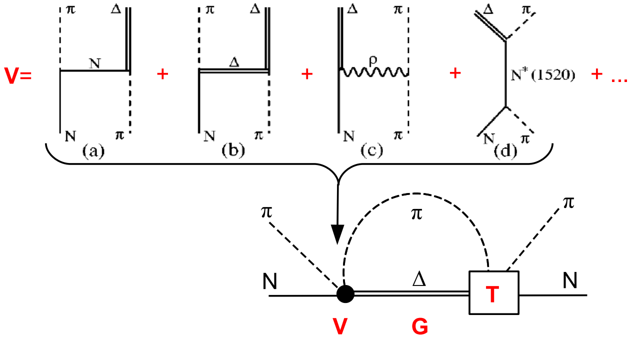
- Coupled channels πN, ηN, KY; effective $\pi\pi N$ channels σN, ρN, $\pi\Delta$.
- Chiral Lagrangian of Wess and Zumino [PR163 (1967), Phys.Rept. 161 (1988)].
- Baryonic resonances up to $J = 7/2$ with derivative couplings.
- General requirements of the S-matrix.
  - Crossed (u-channel) contributions $\rightarrow$ sub-threshold cuts.
  - Dispersive treatment of t-channel exchanges (σ, ρ exchange from $N\bar{N} \rightarrow \pi\pi$).
  - Full analyticity, also of $\pi\pi N$ intermediate states $\rightarrow$ additional branch points in complex plane.
  - 2-body unitarity, some requirements of 3-body unitarity (but not full).

Talks by S. Krewald, F. Huang

- Analytic structure and the "background", The reaction $\pi^+ p \rightarrow K^+\Sigma^+$.
- Photoproduction.

Scattering equation in the JLS basis

$$\langle L'S'k'|T^{IJ}_{\mu\nu}|LSk\rangle = \langle L'S'k'|V^{IJ}_{\mu\nu}|LSk\rangle$$
$$+ \sum_{\gamma,L''S''} \int_0^\infty k''^2\, dk'' \langle L'S'k'|V^{IJ}_{\mu\gamma}|L''S''k''\rangle \frac{1}{Z - E_\gamma(k'') + i\epsilon} \langle L''S''k''|T^{IJ}_{\gamma\nu}|LSk\rangle$$



Scattering equation in the JLS basis

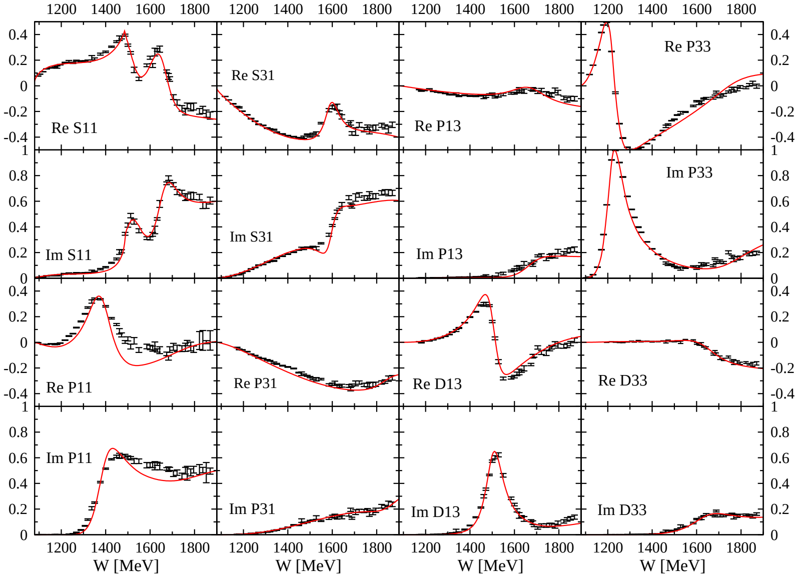
$$\langle L'S'k'|T^{IJ}_{\mu\nu}|LSk\rangle = \langle L'S'k'|V^{IJ}_{\mu\nu}|LSk\rangle$$
$$+ \sum_{\gamma,L''S''} \int_0^\infty k''^2\, dk'' \langle L'S'k'|V^{IJ}_{\mu\gamma}|L''S''k''\rangle \frac{1}{Z - E_\gamma(k'') + i\epsilon} \langle L''S''k''|T^{IJ}_{\gamma\nu}|LSk\rangle$$

Features

- Hadron exchange: provides the relevant dynamics.
- Full analyticity (dispersive parts).
- All partial waves are linked (t-, u-channel processes)
- Channels linked (SU(3) symmetry).
- Minimal resonance content required.
- Dynamical generation of resonances is possible, but not easy (strong constraints).

Partial waves in $\pi N \to \pi N$ (Solution 2002)

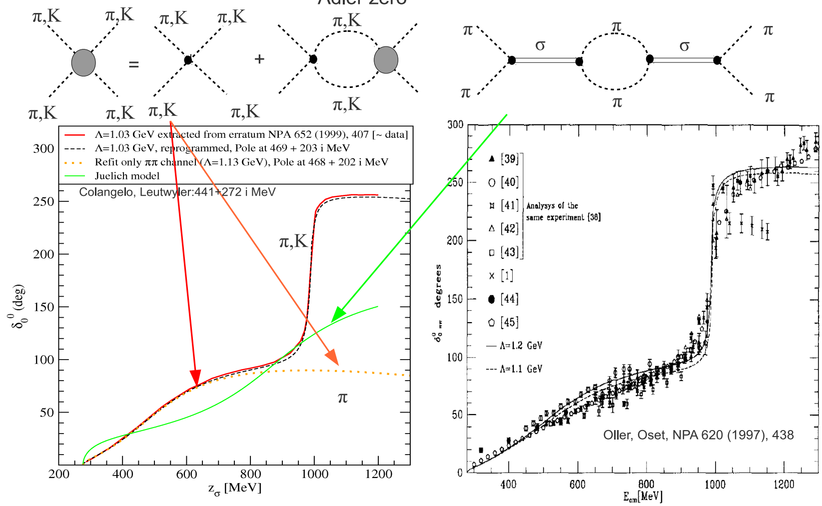
"Data": GWU/SAID, PRC74 (2006)



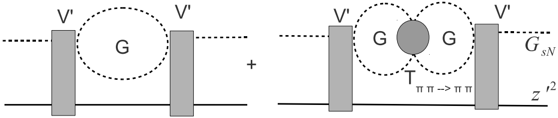
Example of chiral constraints: $\pi\pi$ scattering

$T_{\pi\pi} = V\chi + V\chi\, G\, T_{\pi\pi}$; $V\chi = \underbrace{(1/2m_\pi^2 - s)}_{\text{Adler zero}} / f^2$

$T_{\pi\pi} = (V_{\sigma\pi\pi})^2 / (z - M - \Sigma_{\pi\pi})$

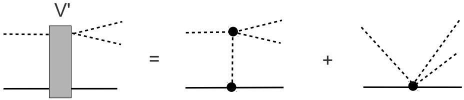


Implementation of the chiral σ

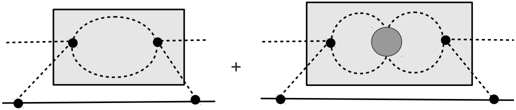


$$G_{sN} = c(z,p)\frac{G(z')}{1 - V(z')G(z')}$$

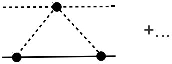
$$z'^2 = z^2 + m_N^2 - 2z\sqrt{p^2 + m_N^2}$$



Independence on parameterization
of the pion fields (off-shell cancellatio

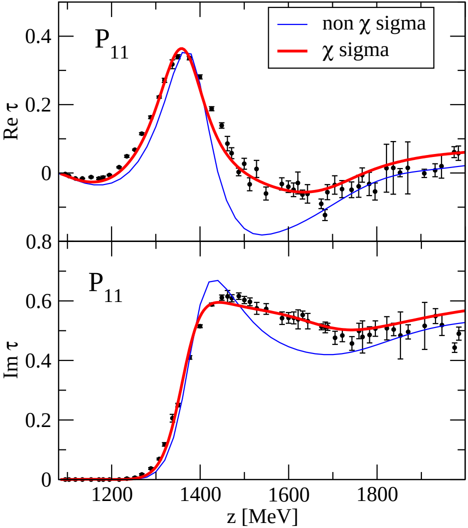


ππ scattering as subprocess
w/o tree level term



Tree level: Not included (double counting
with ρ, σ t-channel exchanges)

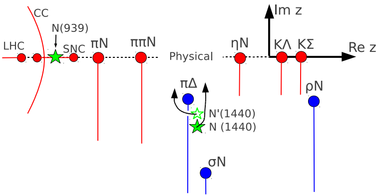
Effects of the χ unitary σ meson in the P11 πN amplitude



- Dynamical generation of Roper does not depend on details of the model
- Chiral σ provides better description.

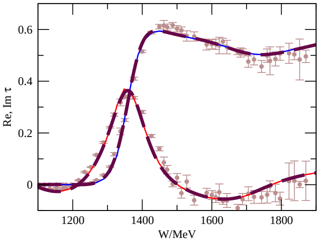
Structure of the P11 partial wave (Roper)

analytic continuation



- σN interaction strongly attractive $\rightarrow$ dynamical generation of the Roper.
- Roper pole + $\pi\Delta$ branch point $\rightarrow$ non-standard resonance shape.

- Where is the $3*$ N(1710)?
  [S. Ceci, M.D. et al, arXiv 1104.3490]



Fit of a model without ρN branch point (CMB type) [solid lines] to the Jülich amplitude [dashed lines]

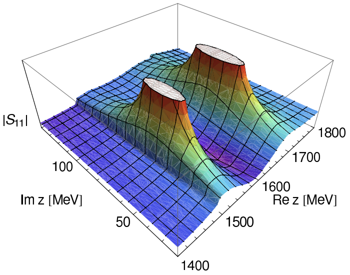
- CMB fit to JM has pole at $1698 - 130\,i$ MeV, simulates missing branch point.

Branch points in $\gamma n \rightarrow \eta n$

Inclusion of full analytic structure important to avoid false pole signals.

Poles and residues

[M.D., C. Hanhart, F. Huang, S. Krewald and U.-G. Meißner, NPA 829 (2009), PLB 681 (2009)]



	Re z_0 [MeV]	-2 Im z_0 [MeV]	$\|R\|$ [MeV]	θ [deg] [^{0}]
$N^*(1535)\ S_{11}$	1519	129	31	-3
ARN	1502	95	16	-16
HOE	1487			
CUT	1510±50	260±80	120±40	+15±45
$N^*(1650)\ S_{11}$	1669	136	54	-44
ARN	1648	80	14	-69
HOE	1670	163	39	-37
CUT	1640±20	150±30	60±10	-75±25
$N^*(1720)\ P_{13}$	1663	212	14	-82
ARN	1666	355	25	-94
HOE	1686	187	15	
CUT	1680±30	120±40	8±12	-160±30
$\Delta(1232)\ P_{33}$	1218	90	47	-37
ARN	1211	99	52	-47
HOE	1209	100	50	-48
CUT	1210±1	100±2	53±2	-47±1
$\Delta^*(1620)\ S_{31}$	1593	72	12	-108
ARN	1595	135	15	-92
HOE	1608	116	19	-95
CUT	1600±15	120±20	15±2	-110±20
$\Delta^*(1700)\ D_{33}$	1637	236	16	-38
ARN	1632	253	18	-40
HOE	1651	159	10	
CUT	1675±25	220±40	13±3	-20±25
$\Delta^*(1910)\ P_{31}$	1840	221	12	-153
ARN	1771	479	45	+172
HOE	1874	283	38	
CUT	1880±30	200±40	20±4	-90±30

	Re z_0 [MeV]	-2 Im z_0 [MeV]	$\|R\|$ [MeV]	θ [deg] [^{0}]
$N^*(1440)\ P_{11}$	1387	147	48	-64
ARN	1359	162	38	-98
HOE	1385	164	40	
CUT	1375±30	180±40	52±5	-100±35
$N^*(1520)\ D_{13}$	1505	95	32	-18
ARN	1515	113	38	-5
HOE	1510	120	32	-8
CUT	1510±5	114±10	35±2	-12±5

[ARN]: Arndt et al., PRC 74 (2006), [HOE]: Höhler, πN Newsl. 9 (1993), [CUT]: Cutkowski et al., PRD 20 (1979).

Residues to ηN, σN, ρN, $\pi\Delta$. Zeros. Branching ratios to πN, ηN.

Parallelization

[Project *JIKP07* on JUROPA/FZ Jülich, 384,000 CPU hours granted]

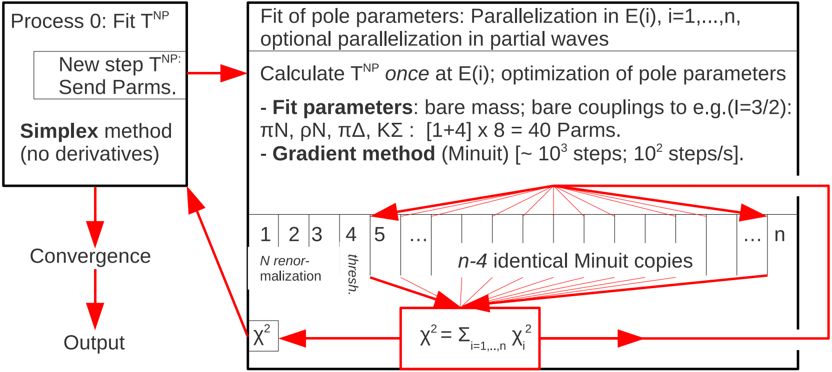
Fixing free parameters from s-channel "pole" processes [fast!] and $t-$, $u-$ processes [$\sim$ 100 x slower]

Requirements:

1) Maintain speed advantage of (x 100) of calculation of T^P from T^{NP} ($T=T^{NP}+T^P$)
   - > 2 nested Minuit runs: full fit of T^P [~40 parms.] for every step in T^{NP}
   - > requires separated memory spaces/ **mpi** parallelization on **Juropa/FZ Julich**

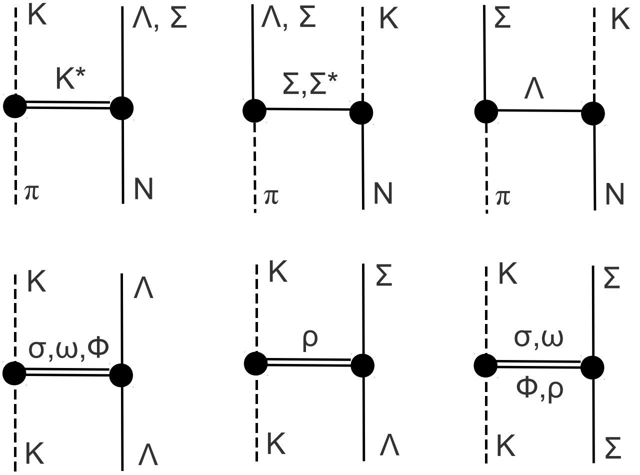
2) Scaling with # processes

3) Adding large amounts of data to χ^2 without increase of execution time



Inclusion of the KY channels [see talk S. Krewald]

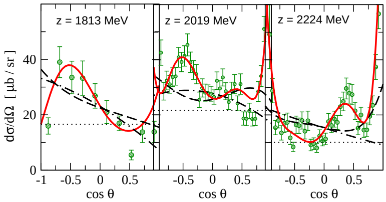
Inclusion via $SU(3)$ symmetry plus s-channel resonance vertices to KY (not shown; 40 parms.).

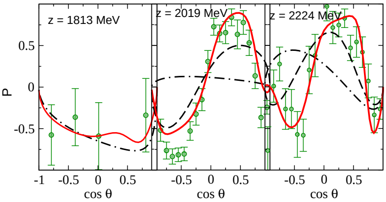


The reaction $\pi^+ p \to K^+\Sigma^+$

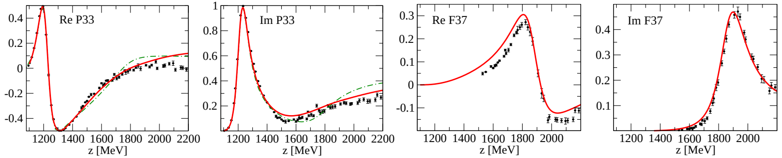
Link to full Results & Error analysis

M.D., C. Hanhart, F. Huang, S. Krewald, U.-G. Meißner, D. Rönchen, [NPA851 (2011)]





Data upper: Candlin 1983, NPB 226 (1983), lower: GWU/SAID, PRC74 (2006)



Linking partial waves and different reactions puts more constraints on resonance content.

Pole Structure of the Amplitudes

extracted from analytic continuation

[Nucl.Phys.A851:58-98,2011]

Established 4-star resonances:

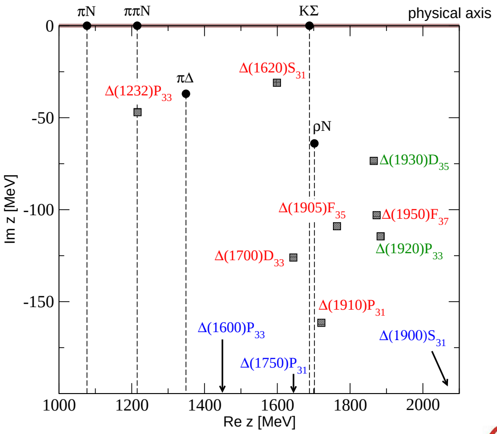
$\Delta(1232)P_{33}$, $\Delta(1700)D_{33}$, $\Delta(1905)F_{35}$, $\Delta(1950)F_{37}$, $\Delta(1620)S_{31}$, $\Delta(1910)P_{31}$

3-star resonances:

$\Delta(1920)P_{33}$, $\Delta(1930)D_{35}$
constrains from $K^+\Sigma^+$ data lead to positions in vicinity of 3-star PDG resonances

Dynamically generated poles:

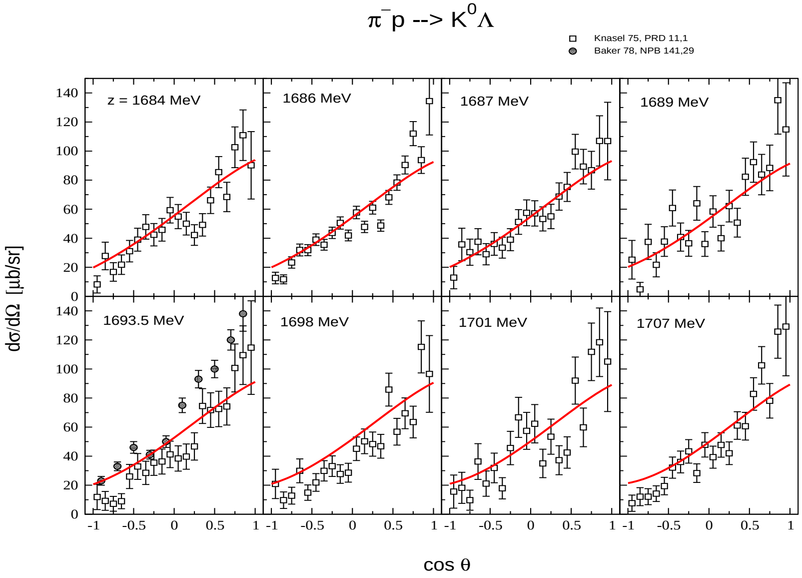
$\Delta(1600)P_{33}$

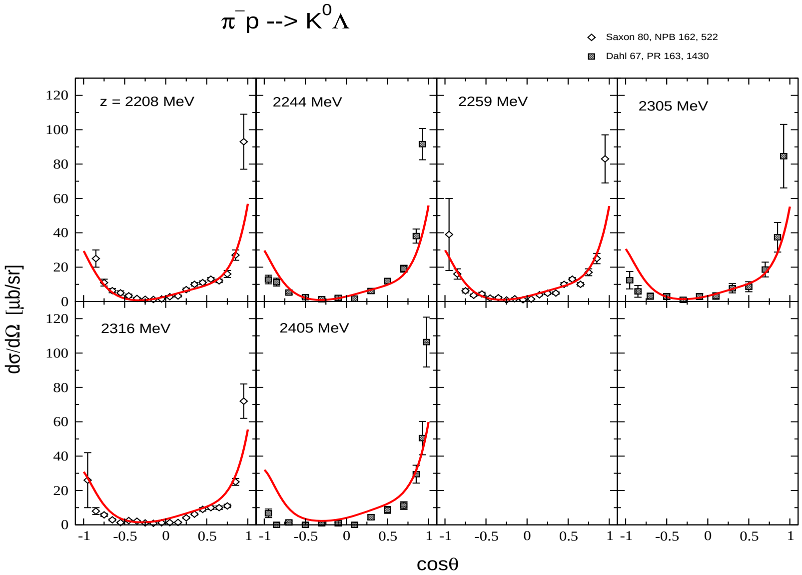


Comparison of poles (extracted from $\pi N \to \pi N$ & $\pi^+ \to K^+\Sigma^+$)

Data:	$\pi N + K^+\Sigma^+$ $(+\cdots)$		πN					$K^+\Sigma^+$	$\pi\pi N$	Quark Models	
Analysis:	Jülich	Gießen	GWU	KH	CMB	EBAC	DMT	CdI	Mnly	LMP, $\mathcal{A}$	CI
Type:	DCM	KM	KM/DA	DA	DA	DCM	DCM	IA	KM	–	–
Pole/BW:	P	BW	P	SP	P	P	P	BW	BW	–	–
$\Delta(1232)P_{33}$	1216	1228(1)	1211	1209	1210	1211	1212	–	1232	1261	1230
$3/2^+$ ****	96	106(1)	99	100	100	100	98		118	–	–
$\Delta(1600)P_{33}$	1455[a]	1667(1)	1457	1550	1550	–	1544	–	1706	1810	1795
$3/2^+$ ***	694	397(10)	400	–	200		190		430	–	–
$\Delta(1620)S_{31}$	1599	1612(2)	1595	1608	1600	1563	1589	–	1672	1654	1555
$1/2^-$ ****	62	202(7)	135	116	120	190	148		154	–	–
$\Delta(1700)D_{33}$	1644	1678(1)	1632	1651	1675	1604	1604	–	1762	1628	1620
$3/2^-$ ****	252	606(15)	253	159	220	212	142		599	–	–
$K^+\Sigma^+(1688)$											
$\Delta(1750)P_{31}$	1668[a]	1712(1)	1771	–	–	–	–	–	1744	1866	–
$1/2^+$ *	892	643(17)	479						299	–	
$\Delta(1900)S_{31}$	–	1984	–	1780	1870	–	1774	–	1920	2100	2035
$1/2^-$ **		237		170	180		72		263	–	–
$\Delta(1905)F_{35}$	1764	1845(15)	1819	1829	1830	1738	1760	1960	1881	1897	1910
$5/2^+$ ****	218	426(26)	247	303	280	220	200	270	327	–	–
$\Delta(1910)P_{31}$	1721	1975	1771	1874	1880	–	1900	–	1882	1906	1875
$1/2^+$ ****	323	676	479	283	200		174		239	–	–
$\Delta(1920)P_{33}$	1884	2057(1)	–	1900	1900	–	–	1840	2014	1871	1915
$3/2^+$ ***	229	525(32)		–	300		300	200	152	–	–
$\Delta(1930)D_{35}$	1865	–	2001	1850	1890	–	1989	–	1956	2179	2155
$5/2^-$ ***	147		387	180	260		280		526	–	–
$\Delta(1940)D_{33}$	–	–	–	–	–	–	–	–	2057	2089	2080
$3/2^-$ *									460	–	–
$\Delta(1950)F_{37}$	1873	–	1876	1878	1890	1858	1858	1925	1945	1956	1940
$7/2^+$ ****	206		227	230	260	200	208	330	300	–	–

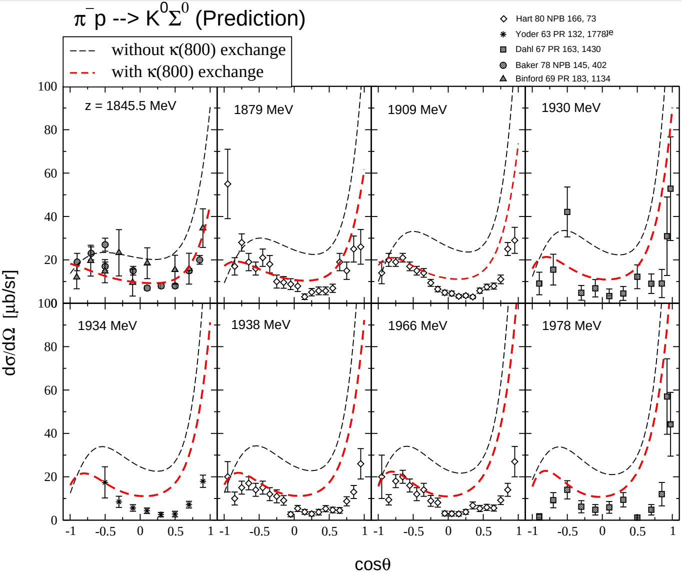
Example of other final states [preliminary, no N(1710)P11 needed so far]




π⁻p --> K⁰Λ
Saxon 80, NPB 162, 522
Dahl 67, PR 163, 1430
z = 2208 MeV
2244 MeV
2259 MeV
2305 MeV
2316 MeV
2405 MeV
dσ/dΩ [μb/sr]
cosθ

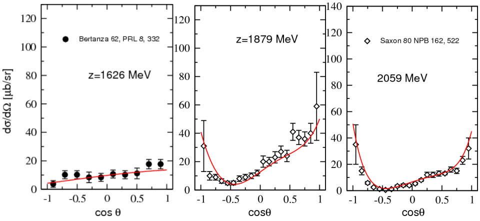
The power of SU(3)

Fixing u-, t-channel exchanges from $\pi N \to \pi N$, $K^+\Sigma^+$, $K^0\Lambda$, ηN

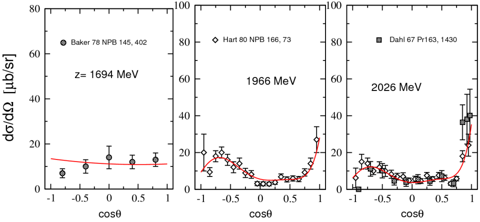


First results other KY channels: differential cross section

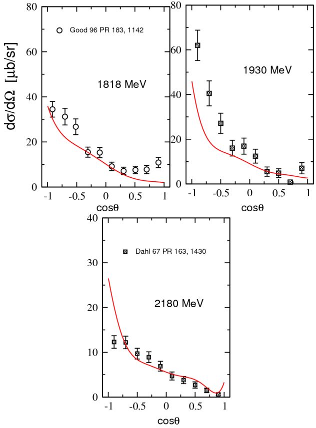
$\pi^- p \to K^0\Lambda$:



$\pi^- p \to K^0\Sigma^0$:

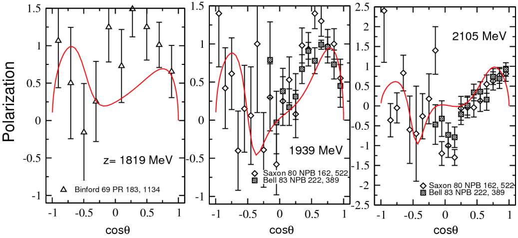


$\pi^- p \to K^+\Sigma^-$:

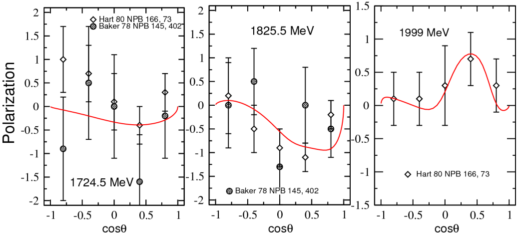


First results other KY channels: Polarization

$\pi^- p \to K^0\Lambda$:



$\pi^- p \to K^0\Sigma^0$:



$\pi^- p \to K^+\Sigma^-$:
no data

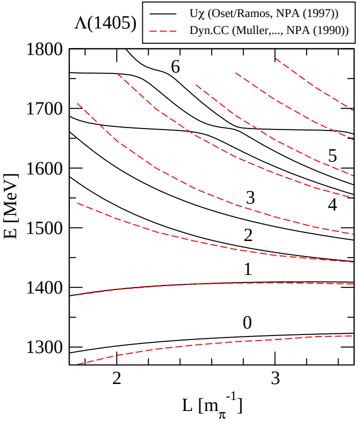
- HADES proposal: Measurement of π^- induced reactions
  HADES Symp., May 13, 2011, Seillac, France.
- c.m. energies from 1.7 to 2 GeV.
- Additional motivation most welcome!

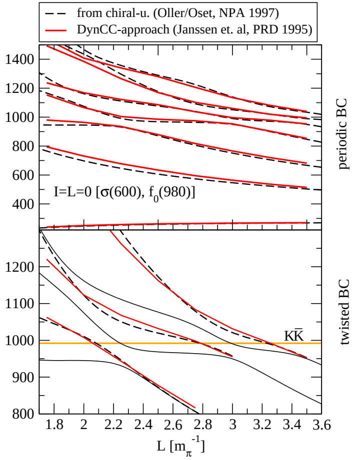
Dynamical coupled channels models in a box

Discretization & twisted BC

[M.D., J. Haidenbauer, A. Rusetsky, U.-G. Meißner, E. Oset, in preparation]

- Variation of box size $L \to$ reconstruction of phase shifts (Lüscher)
- Prediction of lattice levels & including coming lattice-data in analysis.
- Examples: $\Lambda(1405)$, $\sigma(600)$, $f_0(980)$ on the lattice.

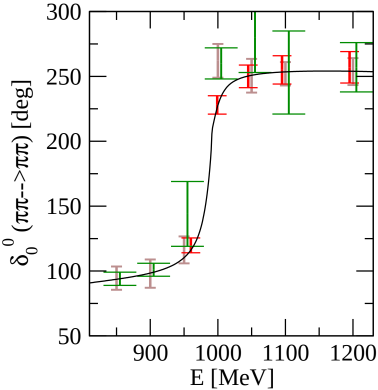




Multi-channel dynamics

Error propagation from pseudo lattice-data [M.D., U.-G. Meißner, E. Oset, A. Rusetsky, in prep.]

- Coupled channels $\pi\pi$, $\bar{K}K$: three unknwons
  - $V(\pi\pi \to \pi\pi)$
  - $V(\pi\pi \to \bar{K}K)$
  - $V(\bar{K}K \to \bar{K}K)$
- How good is the reconstructed phase shift using different lattice data?
- Use pseudo-data generated from hadronic model.
- Hadronic input can reduce the error.



- red: twisted boundary conditions
- green: Asymmetric boxes
- brown: different levels

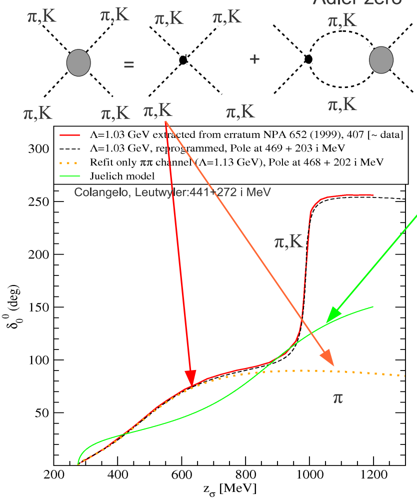
Conclusions

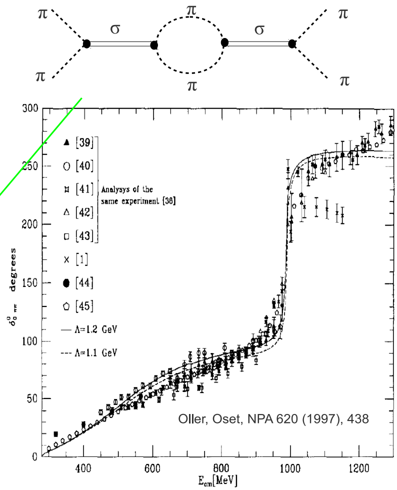
- Meson and baryon exchange: relevant degrees of freedom in the 2^{nd} and 3^{rd} resonance region.
- Exchange provides constraints, because all partial waves & reactions are linked $\to$ minimal resonance content.
- Lagrangian based, field theoretical description of meson-baryon interaction. Unitarity and analyticity are ensured.
- Constructed to fulfill general requirements of the S-matrix (dispersive t-, u-channel [crossing]), branch points in complex plane, $\cdots$
- $\Rightarrow$ precise determination of model independent resonance parameters (poles).
- Parallelization & program structure: Inclusion of large amounts of data possible.
- Dynamical coupled channel models on a momentum lattice: predict levels, error propagation, analyse coming lattice data.

Chiral unitary approach to $\pi\pi$ scattering

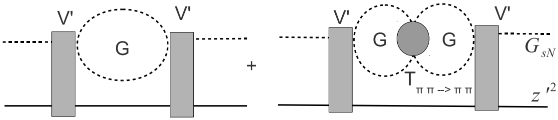
$T_{\pi\pi} = V\chi + V\chi\, G\, T_{\pi\pi}$; $V\chi = (1/2m_\pi^2 - s) / f^2$ (Adler zero)

$T_{\pi\pi} = (V_{\sigma\pi\pi})^2 / (z - M - \Sigma_{\pi\pi})$



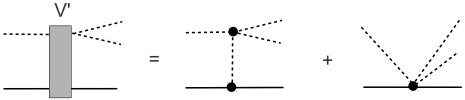


Implementation of the chiral σ

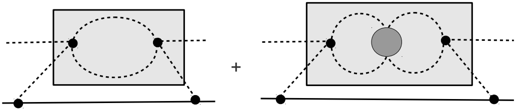


$$G_{sN} = c(z,p)\frac{G(z')}{1-V(z')G(z')}$$

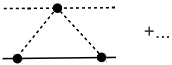
$$z'^2 = z^2 + m_N^2 - 2z\sqrt{p^2 + m_N^2}$$



Independence on parameterization of the pion fields (off-shell cancellatio



ππ scattering as subprocess w/o tree level term



Tree level: Not included (double counting with ρ, σ t-channel exchanges)

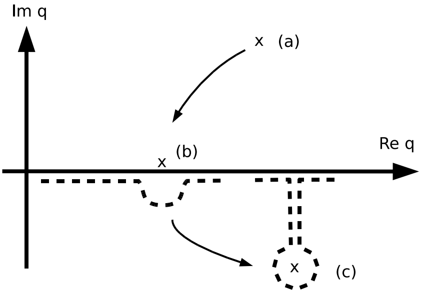
Analytic continuation via Contour deformation

...enables access to all Riemann sheets

$$\Pi_\sigma(z) = \int_0^\infty q^2\, dq\, \frac{(v^{\sigma\pi\pi}(q))^2}{z - E_1 - E_2 + i\epsilon}$$

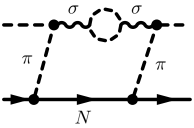
$$z - E_1 - E_2 = 0 \Leftrightarrow q = q_{c.m.}$$

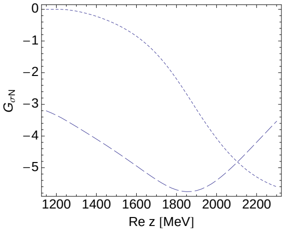
$$q_{c.m.} = \frac{1}{2z}\sqrt{[z^2 - (m_1 - m_2)^2][z^2 - (m_1 + m_2)^2]}$$

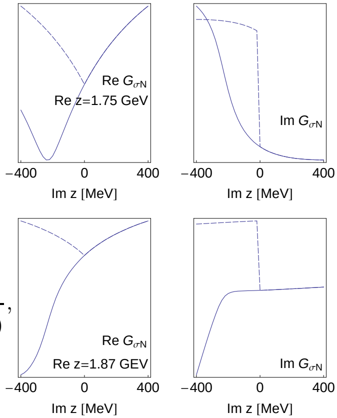


- Plot $q_{c.m.}(z)$ in the q plane of integration (X: Pole positions).
- case (a), Im $z > 0$: straight integration from $q = 0$ to $q = \infty$.
- case (b), Im $z = 0$: Pole is on real q axis.
- case (c), Im $z < 0$: Deformation gives analytic continuation.
- Special case: Pole at $q = 0$ $\Leftrightarrow$ branch point at $z = m_1 + m_2$ (= threshold).

Propagator of effective $\pi\pi N$ channels σN, ρN, $\pi\Delta$





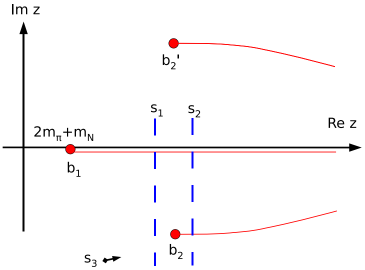


$$g_{\sigma N}(z,k) = \frac{1}{z - \sqrt{m_N^2 + k^2} - \sqrt{(m_\sigma^0)^2 + k^2} - \Pi_\sigma(z_\sigma(z,k),k)},$$

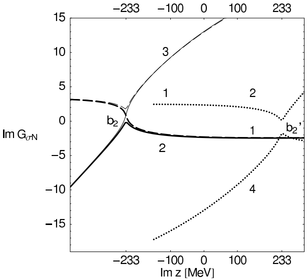
$$G_{\sigma N}(z) = \int_0^\infty dk\, k^2\, F(k)\, g_{\sigma N}(z,k),$$

$$z_\sigma(z,k) = z + m_\sigma^0 - \sqrt{k^2 + (m_\sigma^0)^2} - \sqrt{k^2 + m_N^2}$$

→ Branch point in the complex z plane.

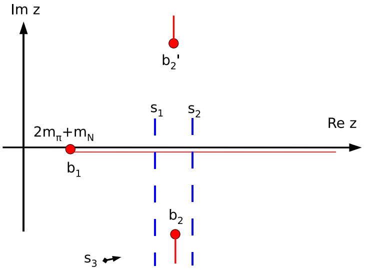


Three branch points and four sheets for each of the σN, ρN, and $\pi\Delta$ propagators.

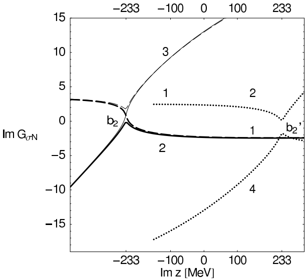


- The cut along Im $z = 0$ is induced by the cut of the self energy of the unstable particle.
- The poles of the unstable particle (σ) induce branch points in the σN propagator at

$$z_{b_2} = m_N + z_0, \; z_{b_2'} = m_N + z_0^*$$



Three branch points and four sheets for each of the σN, ρN, and $\pi\Delta$ propagators.



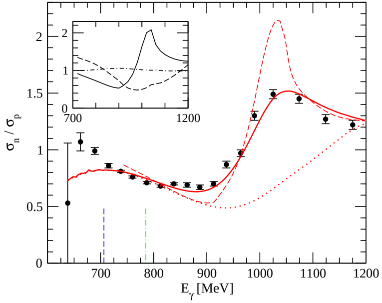
- The cut along Im $z = 0$ is induced by the cut of the self energy of the unstable particle.
- The poles of the unstable particle (σ) induce branch points in the σN propagator at

$$z_{b_2} = m_N + z_0, \; z_{b_2'} = m_N + z_0^*$$

Branch points in coupled channels ($\gamma N \to \eta N$)

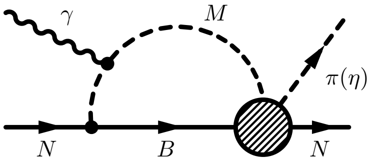
[M.D., K. Nakayama, EPJA43 (2010), PLB683 (2010)]

back



[Data: I. Jaegle et al., CBELSA/TAPS, PRL 100 (2008)]

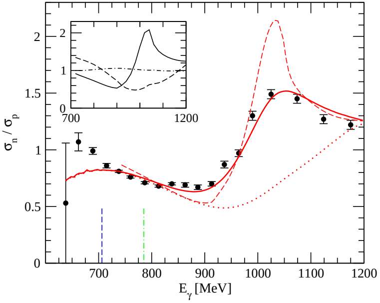
- Intermediate states in photon loops, $Q = 0,\ 1$:
- $\pi^- p,\ \pi^0 n,\ \eta n,\ K^0\Lambda,\ K^+\Sigma^-,\ K^0\Sigma^0$
- $\pi^0 p,\ \pi^+ n,\ \eta p,\ K^+\Lambda,\ K^+\Sigma^0,\ K^0\Sigma^+$



- Pronounced cusp from dispersive ("real") part of the loop.
- Peak in σ_n/σ_p: Direkt consequence of Weinberg-Tomozawa driving term from LO χ Lagrangian.

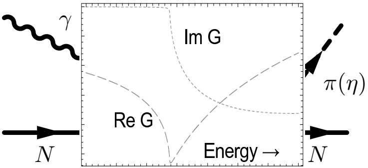
Branch points in coupled channels ($\gamma N \to \eta N$)

[M.D., K. Nakayama, EPJA43 (2010), PLB683 (2010)]



[Data: I. Jaegle et al., CBELSA/TAPS, PRL 100 (2008)]

- Intermediate states in photon loops, $Q = 0,\ 1$:
- $\pi^- p,\ \pi^0 n,\ \eta n,\ K^0\Lambda,\ K^+\Sigma^-,\ K^0\Sigma^0$
- $\pi^0 p,\ \pi^+ n,\ \eta p,\ K^+\Lambda,\ K^+\Sigma^0,\ K^0\Sigma^+$



- Pronounced cusp from dispersive ("real") part of the loop.
- Peak in σ_n/σ_p: Direkt consequence of Weinberg-Tomozawa driving term from LO χ Lagrangian.

Couplings “ $g = \sqrt{a_{-1}}$ ” to other channels

back

	$N\pi$	$N\rho^{(1)}$ $(S = 1/2)$	$N\rho^{(2)}$ $(S = 3/2)$	$N\rho^{(3)}$ $(S =$
$N^*(1535)\ S_{11}$	S_{11} $8.1 + 0.5i$	S_{11} $2.2 - 5.4i$	–	D_{11} 0.5 -
$N^*(1650)\ S_{11}$	S_{11} $8.6 - 2.8i$	S_{11} $0.9 - 9.1i$	–	D_{11} 0.3 -
$N^*(1440)\ P_{11}$	P_{11} $11.2 - 5.0i$	P_{11} $-1.3 + 3.2i$	P_{11} $3.6 - 2.6i$	–
$\Delta^*(1620)\ S_{31}$	S_{31} $2.9 - 3.7i$	S_{31} $0.0 - 0.0i$	–	D_{31} 0.0 -
$\Delta^*(1910)\ P_{31}$	P_{31} $1.2 - 3.5i$	P_{31} $0.2 - 0.4i$	P_{31} $-0.2 - 0.4i$	–
$N^*(1720)\ P_{13}$	P_{13} $3.7 - 2.6i$	P_{13} $0.1 + 0.8i$	P_{13} $-1.1 + 0.1i$	F_{13} 0.1 -
$N^*(1520)\ D_{13}$	D_{13} $8.4 - 0.8i$	D_{13} $-0.6 + 0.7i$	D_{13} $0.9 - 2.0i$	S_{13} -2.5 -
$\Delta(1232)\ P_{33}$	P_{33} $17.9 - 3.2i$	P_{33} $-1.3 - 0.8i$	P_{33} $-0.9 - 3.0i$	F_{33} 0.0 -
$\Delta^*(1700)\ D_{33}$	D_{33} $4.9 - 1.0i$	D_{33} $-0.2 + 0.9i$	D_{33} $-0.4 - 0.4i$	S_{33} -0.1 -

	$N\eta$	$\Delta\pi^{(1)}$	$\Delta\pi^{(2)}$	$N\sigma$
$N^*(1535)\ S_{11}$	S_{11} $11.9 - 2.3i$	–	D_{11} $-5.9 + 4.8i$	P_{11} -1.4 -
$N^*(1650)\ S_{11}$	S_{11} $-3.0 + 0.5i$	–	D_{11} $4.3 + 0.4i$	P_{11} -2.1 -
$N^*(1440)\ P_{11}$	P_{11} $-0.1 + 0.0i$	P_{11} $-4.6 - 1.7i$	–	S_{11} -8.3 -
$\Delta^*(1620)\ S_{31}$	–	–	D_{31} $11.1 - 4.0i$	–
$\Delta^*(1910)\ P_{31}$	–	P_{31} $15.0 - 0.3i$	–	–
$N^*(1720)\ P_{13}$	P_{13} $-7.7 + 5.5i$	P_{13} $-14.1 + 3.0i$	F_{13} $0.0 - 0.3i$	D_{13} -0.8 -
$N^*(1520)\ D_{13}$	D_{13} $0.16 - 0.60i$	D_{13} $0.0 + 0.4i$	S_{13} $-12.9 - 0.7i$	P_{13} -0.6 -
$\Delta(1232)\ P_{33}$	–	P_{33} $-(4$ to $5) + i(0$ to $0.5)$	F_{33} ~ 0	–
$\Delta^*(1700)\ D_{33}$	–	D_{33} $-0.7 - 0.3i$	S_{33} $-19.7 + 4.5i$	–

Resonance couplings g_i $[10^{-3}\ \mathrm{MeV}^{-1/2}]$ to the coupled channels i. Also, the LJS type of each coupling is indicated. For the ρN channels, the total spin S is also indicated.

first sheet		second sheet		[FA02]
P_{11}	$1235 - 0\,i$	S_{11}	$1587 - 45\,i$	$1578 - 38\,i$
D_{33}	$1396 - 78\,i$	S_{31}	$1585 - 17\,i$	$1580 - 36\,i$
		P_{31}	$1848 - 83\,i$	$1826 - 197\,i$
		P_{13}	$1607 - 38\,i$	$1585 - 51\,i$
		P_{33}	$1702 - 64\,i$	–
		D_{13}	$1702 - 64\,i$	$1759 - 64\,i$

Position of zeros of the full amplitude T in [MeV]. [FA02]: Arndt et al., PRC 69 (2004).

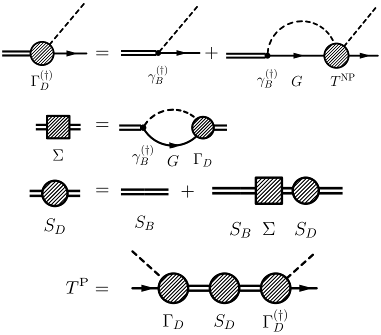
	$\Gamma_{\pi N}/\Gamma_{\text{Tot}}$ [%]	$\Gamma_{\eta N}/\Gamma_{\text{Tot}}$ [%]
$N^*(1535)\ S_{11}$	48 [33 to 55]	38 [45 to 60]
$N^*(1650)\ S_{11}$	79 [60 to 95]	6 [3 to 10]
$N^*(1440)\ P_{11}$	64 [55 to 75]	$0\ [0 \pm 1]$
$\Delta^*(1620)\ S_{31}$	34 [20 to 30]	–
$\Delta^*(1910)\ P_{31}$	11 [15 to 30]	–
$N^*(1720)\ P_{13}$	13 [10 to 20]	$38\ [4 \pm 1]$
$N^*(1520)\ D_{13}$	67 [55 to 65]	$0.10\ [0.23 \pm 0.04]$
$\Delta(1232)\ P_{33}$	100 [100]	–
$\Delta^*(1700)\ D_{33}$	13 [10 to 20]	–

Branching ratios into πN and ηN. The values in brackets are from the PDG, [Amsler et al., PLB 667 (2008)].

Couplings and dressed vertices

Residue a_{-1} vs. dressed vertex Γ vs. bare vertex γ.

back



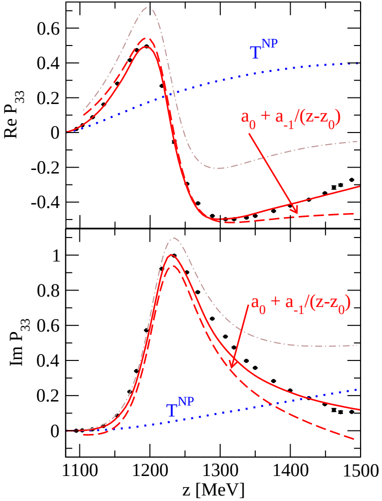
$$
\begin{aligned}
a_{-1} &= \frac{\Gamma_d \Gamma_d^{(\dagger)}}{1-\frac{\partial}{\partial Z}\Sigma} \\
g &= \sqrt{a_{-1}} \\
r &= |(\Gamma_D - \gamma_B)/\Gamma_D|, \\
r' &= |1-\sqrt{1-\Sigma'}|,
\end{aligned}
$$

- Dressed Γ depends on $T^{\rm NP}$.
- $\sqrt{a_{-1}} \neq \Gamma \neq \gamma$

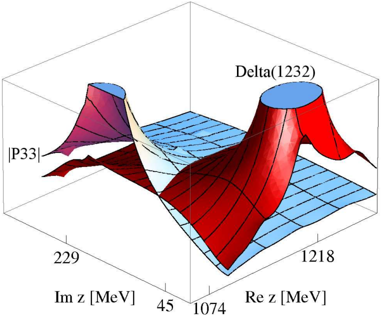
	γ^C	Γ^C	r [%]	r' [%]
$N^*(1520)\ D_{13}$	$6.4-0.6i$	$13.2+1.2i$	53	61
$N^*(1720)\ P_{13}$	$-0.1+5.4i$	$0.9+4.8i$	24	45
$\Delta(1232)\ P_{33}$	$1.3+13.0i$	$-2.8+22.2i$	45	40
$\Delta^*(1620)\ S_{31}$	$0.1+14.3i$	$5.0+5.7i$	130	66
$\Delta^*(1700)\ D_{33}$	$5.4-0.8i$	$6.7+1.0i$	33	54
$\Delta^*(1910)\ P_{31}$	$9.4+0.3i$	$1.9-3.2i$	222	22

Pole repulsion in P_{33}

back



- Poles in T^{NP} may occur $\Rightarrow$ pole repulsion in $T = T^{\mathrm{NP}} + T^{\mathrm{P}}$!



g_{fi} und h_{fi} in JLS-Basis:

$$g_{fi} = \frac{1}{2\sqrt{k_f k_i}} \sum_j (2j+1)\, d^j_{\frac{1}{2}\frac{1}{2}}(\theta) \left[\tau^{j(j-\frac{1}{2})\frac{1}{2}} + \tau^{j(j+\frac{1}{2})\frac{1}{2}}\right] \cos\frac{\theta}{2} + \frac{1}{2\sqrt{k_f k_i}} \sum_j (2j+1)\, d^j_{-\frac{1}{2}\frac{1}{2}}(\theta) \left[\tau^{j(j-\frac{1}{2})\frac{1}{2}} - \tau^{j(j+\frac{1}{2})\frac{1}{2}}\right] \sin\frac{\theta}{2}$$

$$h_{fi} = \frac{-i}{2\sqrt{k_f k_i}} \sum_j (2j+1)\, d^j_{\frac{1}{2}\frac{1}{2}}(\theta) \left[\tau^{j(j-\frac{1}{2})\frac{1}{2}} + \tau^{j(j+\frac{1}{2})\frac{1}{2}}\right] \sin\frac{\theta}{2} + \frac{i}{2\sqrt{k_f k_i}} \sum_j (2j+1)\, d^j_{-\frac{1}{2}\frac{1}{2}}(\theta) \left[\tau^{j(j-\frac{1}{2})\frac{1}{2}} - \tau^{j(j+\frac{1}{2})\frac{1}{2}}\right] \cos\frac{\theta}{2}$$

$$\frac{d\sigma}{d\Omega} = \frac{k_f}{k_i}\left(|g_{fi}|^2 + |h_{fi}|^2\right)$$

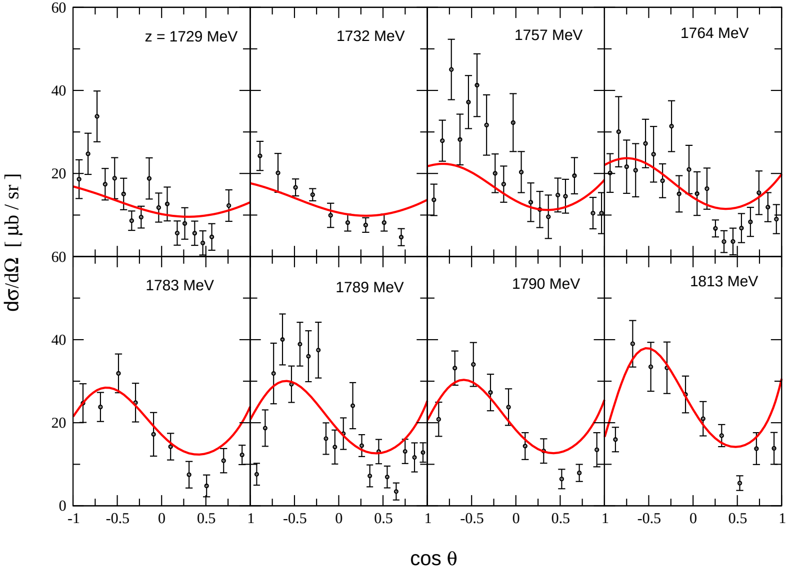
$$= \frac{1}{2k_i^2}\frac{1}{2}\cdot\left(\left|\sum_j (2j+1)(\tau^{j(j-\frac{1}{2})\frac{1}{2}} + \tau^{j(j+\frac{1}{2})\frac{1}{2}})\cdot d^j_{\frac{1}{2}\frac{1}{2}}(\Theta)\right|^2 + \left|\sum_j (2j+1)(\tau^{j(j-\frac{1}{2})\frac{1}{2}} - \tau^{j(j+\frac{1}{2})\frac{1}{2}})\cdot d^j_{-\frac{1}{2}\frac{1}{2}}(\Theta)\right|^2\right)$$

$$\vec{P}_f = \frac{2Re(g_{fi}h^*_{fi})}{|g_{fi}|^2 + |h_{fi}|^2}\cdot\hat{n}$$

$$\beta = \arctan\left(\frac{2Im(h^*_{fi}g_{fi})}{|g_{fi}|^2 - |h_{fi}|^2}\right)$$

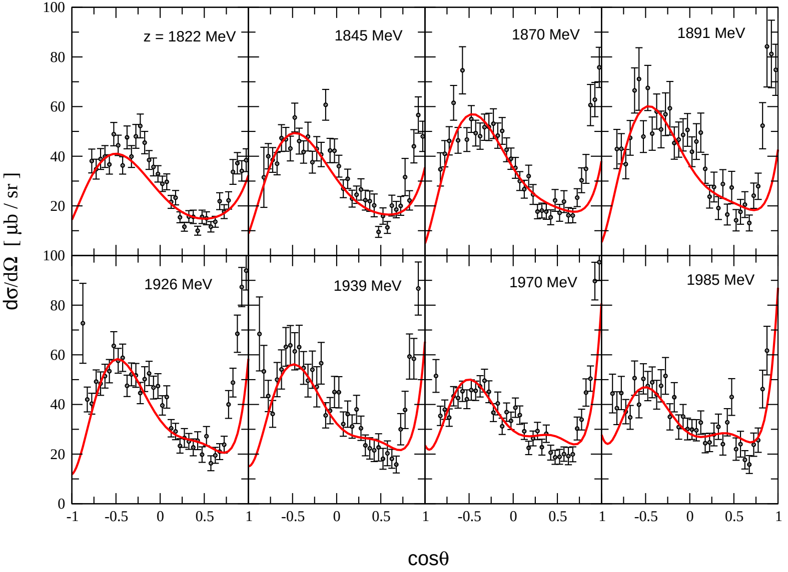
Differential cross section of $\pi^+ p \to K^+ \Sigma^+$

back



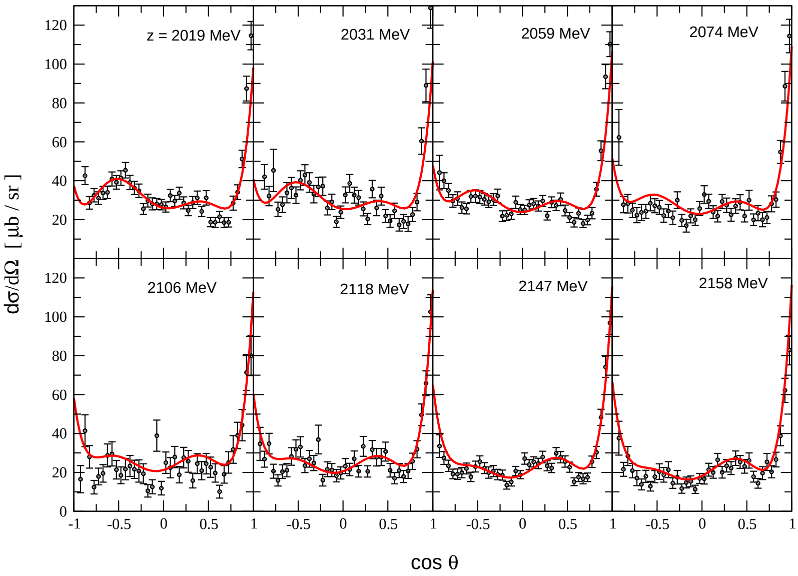
Differential cross section of $\pi^+ p \to K^+ \Sigma^+$

back



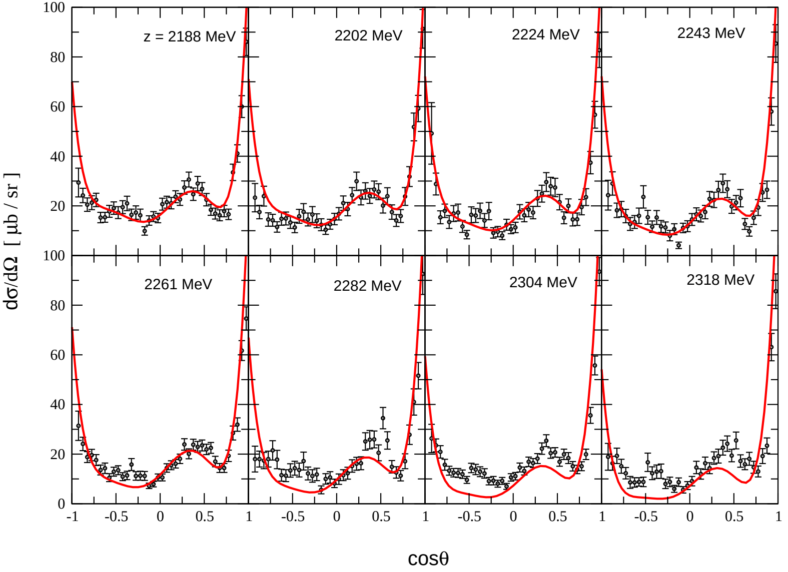
Differential cross section of $\pi^+ p \to K^+ \Sigma^+$

back

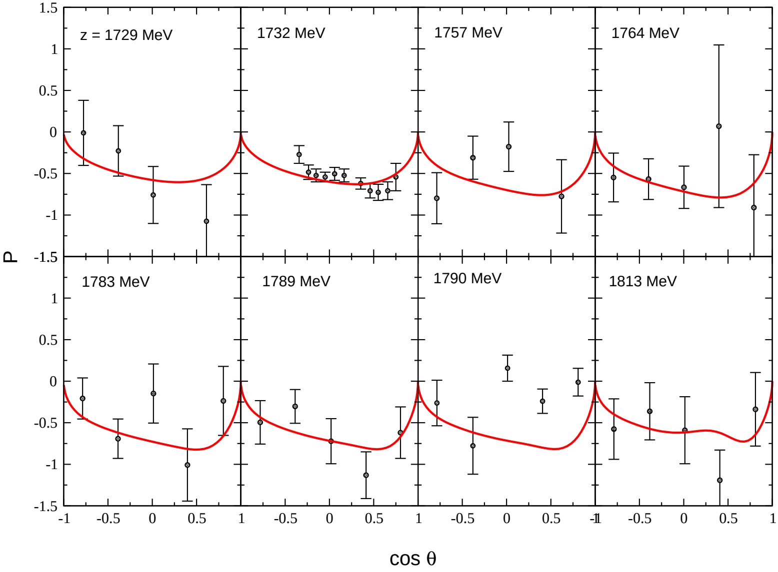


Differential cross section of $\pi^+ p \to K^+ \Sigma^+$

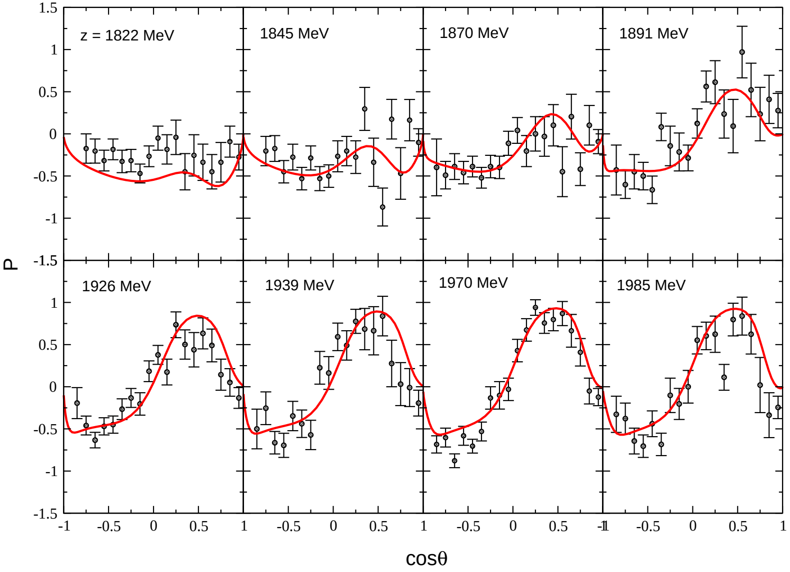
◂ back



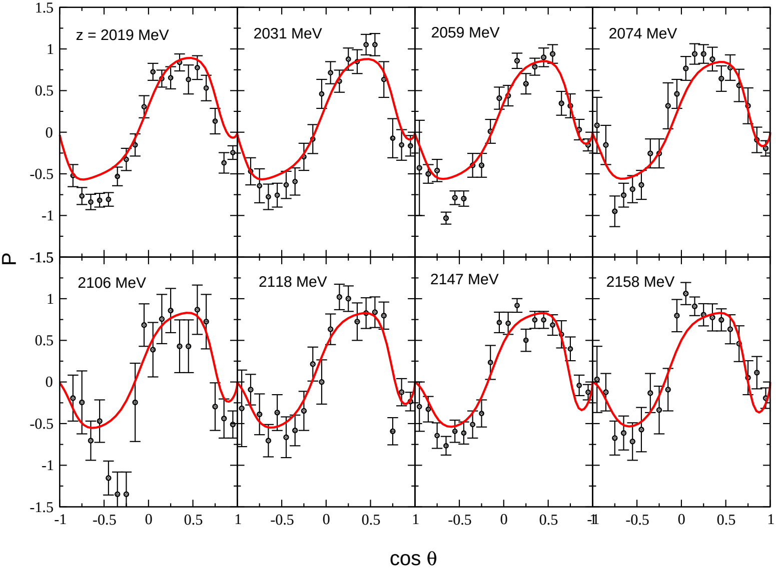
Polarization of $\pi^+ p \to K^+ \Sigma^+$



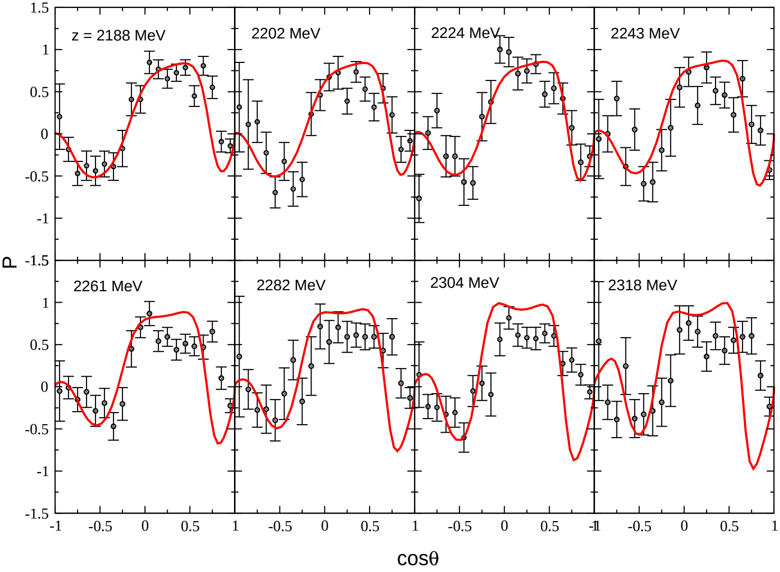
Polarization of $\pi^+ p \to K^+ \Sigma^+$



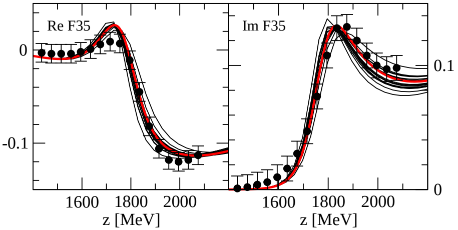
Polarization of $\pi^+ p \to K^+ \Sigma^+$



Polarization of $\pi^+ p \to K^+ \Sigma^+$



- Determination of the non-linear parameter error
  - $\chi^2 + 1$ criterion.
  - Varying 39 of 40 parameters to get parameter error.
- Get error on derived quantities like pole positions and residues.
- So far, simplified consideration (error from πN not available, because energy dependent GWU/SAID solution is fitted [PRC74 (2006)]).



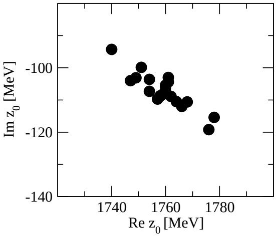


Table: Error estimates of bare mass m_b and bare coupling f for the $\Delta(1905)F_{35}$ resonance.

m_b [MeV]	πN	ρN	$\pi\Delta$	ΣK
2258^{+44}_{-43}	$0.0500^{+0.0011}_{-0.0012}$	$-1.62^{+1.29}_{-1.61}$	$-1.15^{+0.030}_{-0.022}$	$0.120^{+0.0065}_{-0.0059}$

Table: Error estimates of pole position and residues for the $\Delta(1905)F_{35}$ resonance.

			$\pi N \to \pi N$	$\pi N \to K\Sigma$
Re z_0 [MeV]	1764^{+18}_{-20}	$\lvert r\rvert$ [MeV]	$11^{+1.7}_{-1.4}$	$1.4^{+0.24}_{-0.21}$
Im z_0 [MeV]	-109^{+13}_{-12}	θ [^{0}]	$-45^{+3.8}_{-11}$	$-313^{+4.2}_{-10}$

Resonance content from $\pi N \to \pi N$ plus $\pi^+ p \to K^+\Sigma^+$

back

	Re z_0 [MeV]	$\|r\|$ [MeV]	$(\Gamma_{\pi N}^{1/2}\Gamma_{K\Sigma}^{1/2})/\Gamma_{\rm tot}$ [%]		
	-2 Im z_0 [MeV]	θ [^{0}]	This study	Candlin (1983)	Gießen (2004)
$\Delta(1905)F_{35}$	1764	1.4	1.23	1.5(3)	$<$1
$5/2^+$ ****	218	-313			
$\Delta(1910)P_{31}$	1721	5.5	2.98	$<$3	1.1
$1/2^+$ ****	323	-6			
$\Delta(1920)P_{33}$	1884	5.9	5.07	5.2(2)	2.1(3)
$3/2^+$ ***	229	-38			
$\Delta(1930)D_{35}$	1865	1.6	2.14	$<$1.5	
$5/2^-$ ***	147	-43			
$\Delta(1950)F_{37}$	1873	2.7	2.54	5.3(5)	—
$7/2^+$ ****	206	-255			

Haberzettl, PRC56 (1997), Haberzettl, Nakayama, Krewald, PRC74 (2006),

back

Hadronic scattering:

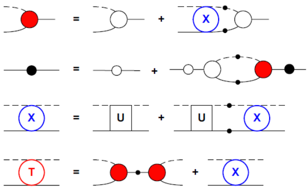
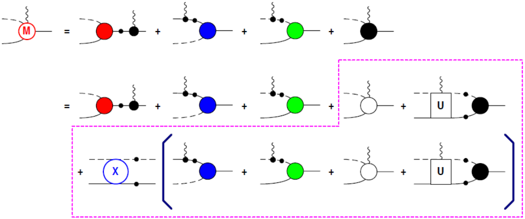


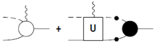
Photo-production:



Gauge invariance: Generalized Ward-Takahashi identity (WTI)

(Note the condition of current conservation $k_\mu M^\mu = 0$ is necessary but not sufficient!)

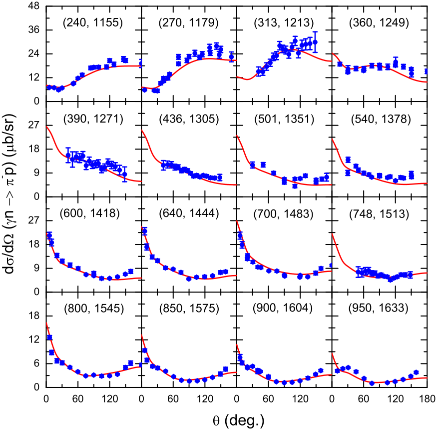
$$k_\mu M^\mu = -|F_s\tau\rangle S_{p+k} Q_i S_p^{-1} + S_{p'}^{-1} Q_f S_{p'-k} |F_u\tau\rangle + \Delta_{p-p'+k}^{-1} Q_\pi \Delta_{p-p'} |F_t\tau\rangle$$

Strategy: Replace  by phenomenological contact term such that the generalized WTI is satisfied

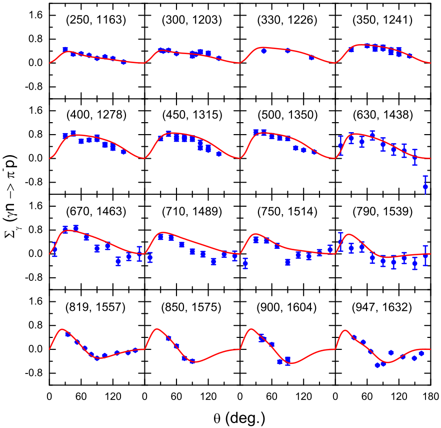
$d\sigma/d\Omega$ and Σ_γ for $\gamma n \to \pi^- p$

preliminary

back



Differential cross section for $\gamma n \to \pi^- p$

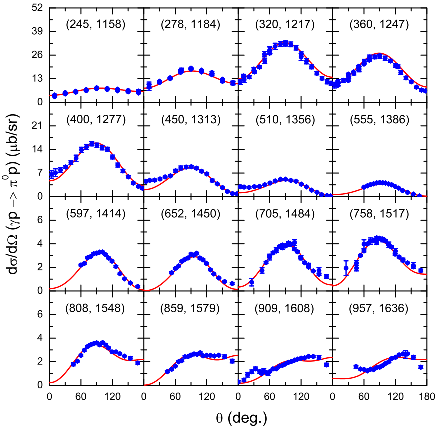


Photon spin asymmetry for $\gamma n \to \pi^- p$

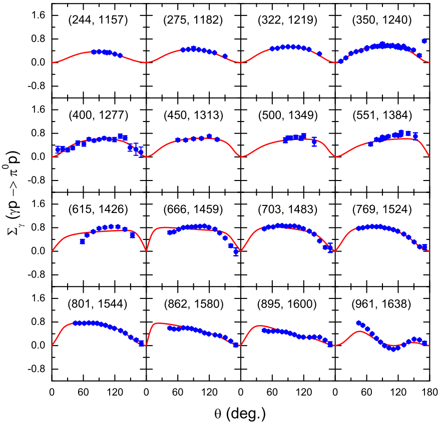
$d\sigma/d\Omega$ and Σ_γ for $\gamma p \to \pi^0 p$

preliminary

back



Differential cross section for $\gamma p \to \pi^0 p$



Photon spin asymmetry for $\gamma p \to \pi^0 p$

Dynamical coupled channels models in a box

Prediction & analysis of lattice data [M.D., J. Haidenbauer, A. Rusetsky, U.-G. Meißner, E. Oset, in preparation]

Discretization of momenta in the scattering equation:

$$T(q'', q') = V(q'', q') + \int_0^\infty dq\, q^2\, V(q'', q) \frac{1}{z - E_1(q) - E_2(q) + i\epsilon} T(q, q')$$

$$\int \frac{\vec{d}^{\,3} q}{(2\pi)^3} f(|\vec{q}\,|^2) \to \frac{1}{L^3} \sum_{\vec{n}_i} f(|\vec{q}_i|^2), \quad \vec{q}_i = \frac{2\pi}{L} \vec{n}_i, \quad \vec{n}_i \in \mathbb{Z}^3$$

$$T(q'', q') = V(q'', q') + \frac{2\pi^2}{L^3} \sum_{i=0}^{\infty} \vartheta(i)\, V(q'', q_i) \frac{1}{z - E_1(q_i) - E_2(q_i)} T(q_i, q'),$$

- Can be also expressed in terms of the Lüscher $\mathcal{Z}_{00}$ function up to e^{-L} relativistic corrections.
- Takes into account discretization effects of the potentials themselves.
- Twisted boundary conditions, e.g.

$$u(\mathbf{x} + L\mathbf{e}_i) = u(\mathbf{x}),\ d(\mathbf{x} + L\mathbf{e}_i) = d(\mathbf{x}),\ s(\mathbf{x} + L\mathbf{e}_i) = e^{i\theta} s(\mathbf{x}),$$

especially suited for coupled-channels problem (enables to move thresholds) [V. Bernard, M. Lage, U.-G. Meißner, A. Rusetsky, JHEP (2011)].