Spin observables in photon- and pion-nucleon interactions

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Motivation

- Strict χPT (a systematic expansion in terms of pion mass and momentum) has a limited range of convergence (for $\pi(\gamma)N$ scattering: threshold (or subthreshold (P.Büttiker, U.-G. Meißner)) region).
- Higer energies–Phenomenological models:(Jülich model, Giessen model, ...)
- Chiral "unitarized" approach (U.-G. Meißner and J. A. Oller, Nucl. Phys. A 673, 311 (2000)):
  Explicit treatment of u and t-channel analyticity is important. (A. Gasparyan and M. F. M. Lutz, Nucl. Phys. A **848** (2010) 126)

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The scheme

- 2-channel approximation (πN and γN)$\Longrightarrow$ one is limited by energies $\sqrt{s} \simeq 1300$MeV
- Low energy: tree level amplitude (u and t-channel cuts are taken into account) + one loop to chiral order Q^3 (in HBChPT)
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Partial Wave Dispersion Relation with subtraction at $\sqrt{s} = \mu_M = m_N$

Unitarity and Analyticity:

$$T_{ab}(\sqrt{s}\,) = U_{ab}(\sqrt{s}\,) + \sum_{c,d} \int_{w_{\rm thrs}}^{\infty} \frac{dw}{\pi} \, \frac{\sqrt{s} - \mu_M}{w - \mu_M} \, \frac{T_{ac}(w)\,\rho_{cd}(w)\,T^{*}_{db}(w)}{w - \sqrt{s} - i\epsilon} \,.$$

$U(\sqrt{s})$ contains only left hand cuts

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CDD poles and resonances

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Non-linear integral equation may have multiple solutions!

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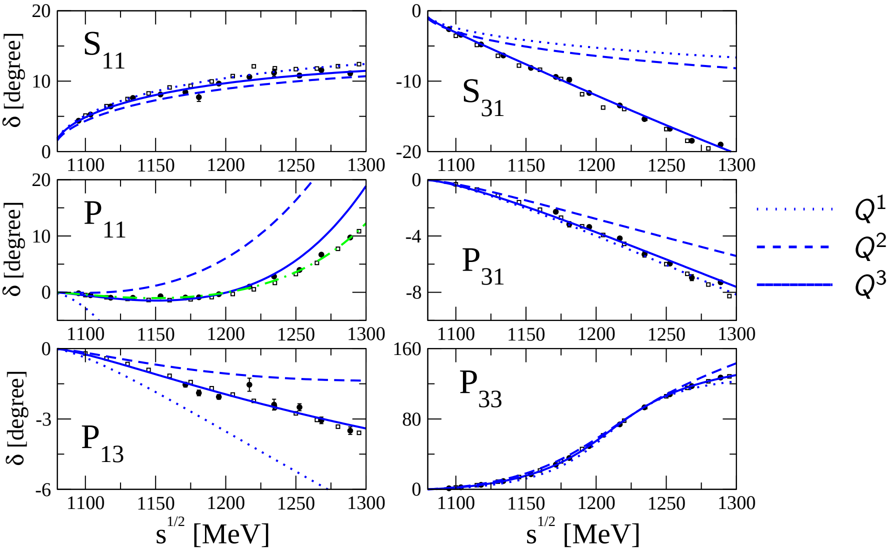
πN phase shifts (S and P waves)



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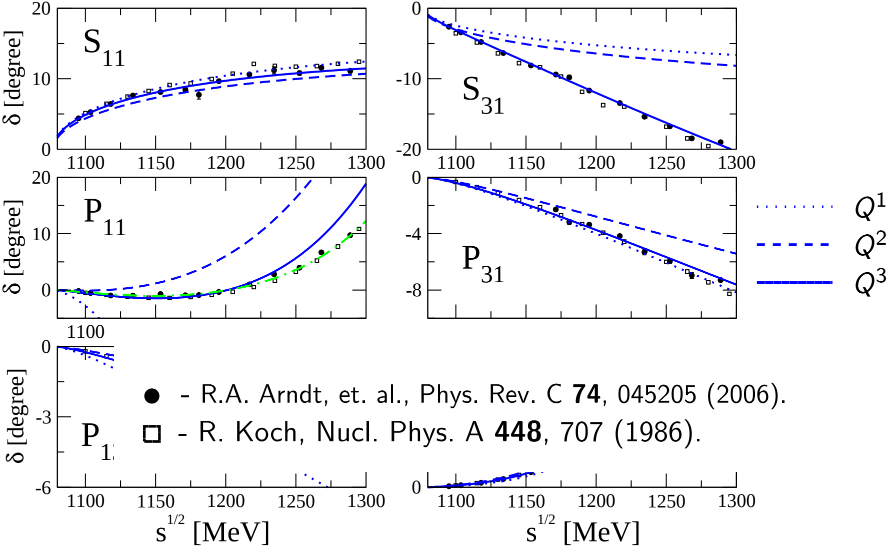
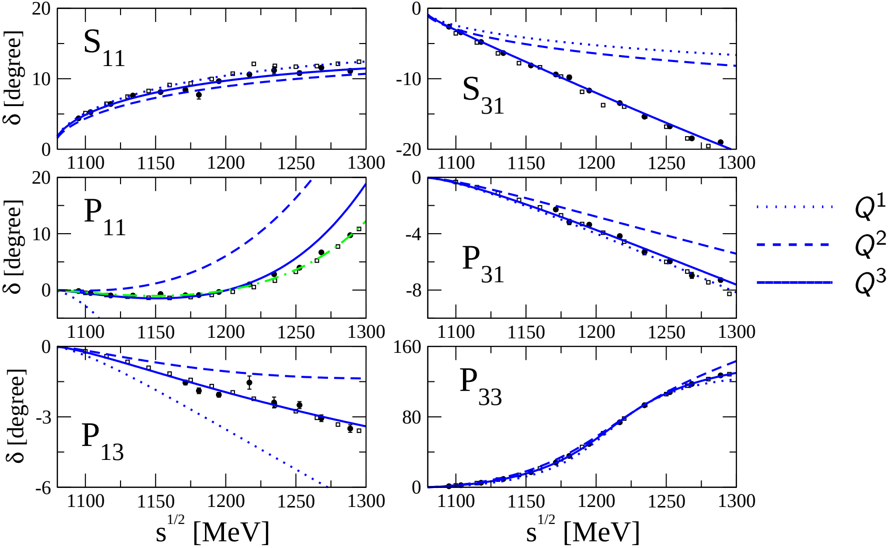


● - R.A. Arndt, et. al., Phys. Rev. C **74**, 045205 (2006).

□ - R. Koch, Nucl. Phys. A **448**, 707 (1986).

πN phase shifts (S and P waves)

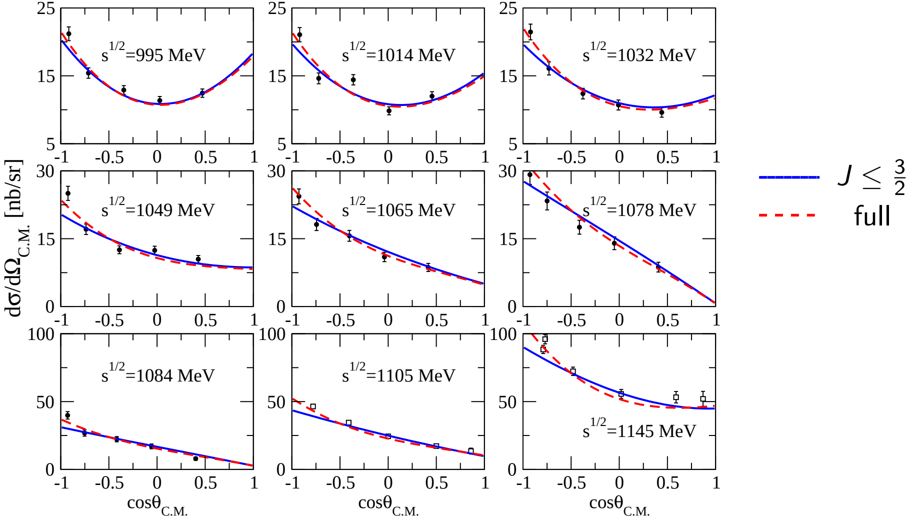


Pion photoproduction

s- and p-waves multipoles
and differential observables are well described
up to $\sqrt{s} = 1300$ MeV (at order Q^3).

Threshold data are not included
in the fit! (Isospin symmetric case)

Differential cross section for Compton scattering off the proton



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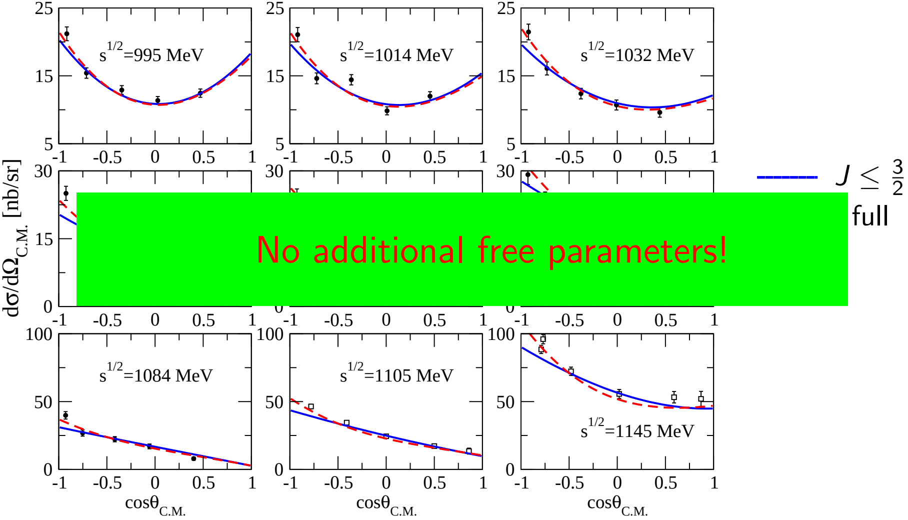
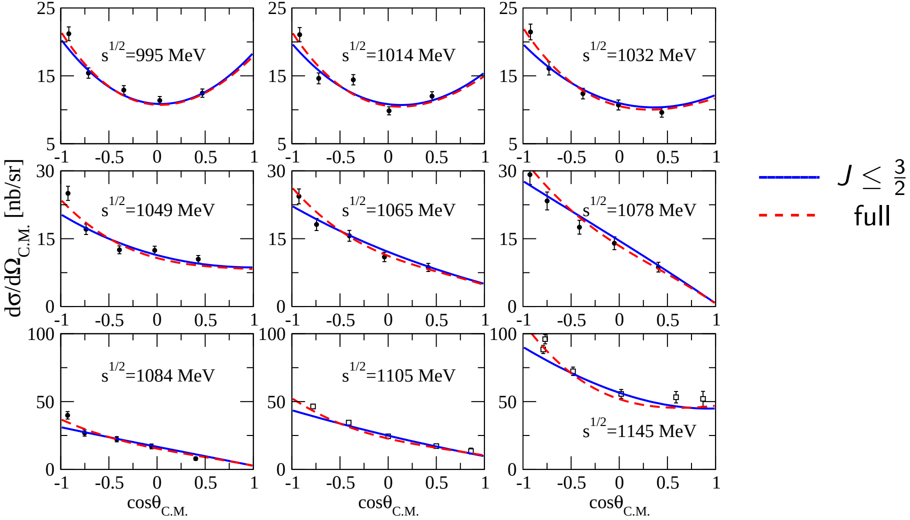


No additional free parameters!

Differential cross section for Compton scattering off the proton



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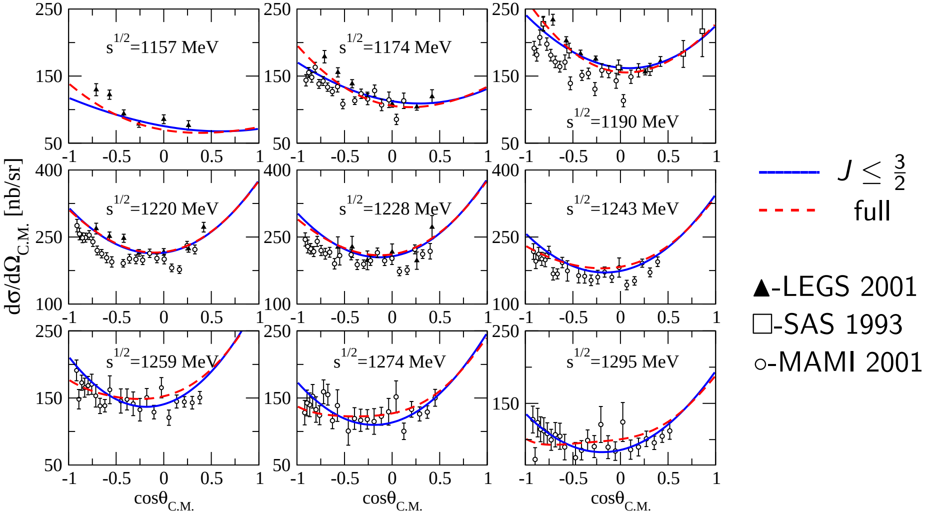
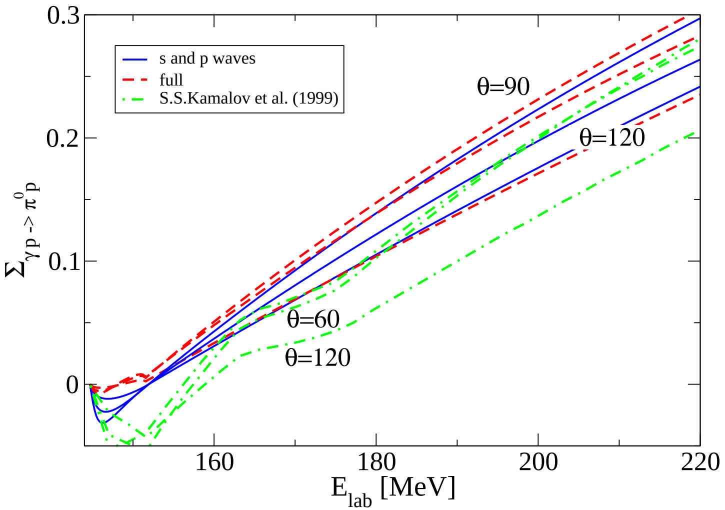


Threshold p-wave multipoles

	our values	HBχPT (Q^3)	Experiment
$\bar{P}_1$ $(\pi^0 p)$ $[10^{-3}/m^2_{\pi^+}]$	10.2	9.4	$9.46 \pm 0.05 \pm 0.28$
$\bar{P}_2$ $(\pi^0 p)$ $[10^{-3}/m^2_{\pi^+}]$	-10.7	-10.0	$-9.5 \pm 0.09 \pm 0.28$
$\bar{P}_3$ $(\pi^0 p)$ $[10^{-3}/m^2_{\pi^+}]$	10.3	10.6	$11.32 \pm 0.11 \pm 0.34$

Energy dependence of the beam asymmetry.



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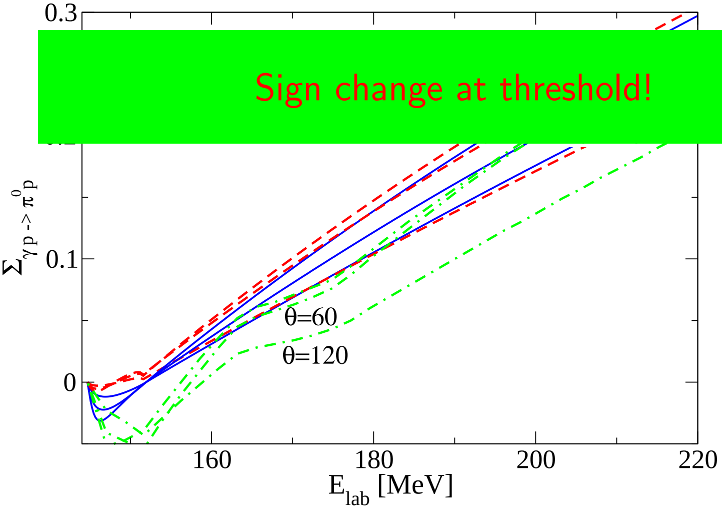

Sign change at threshold!

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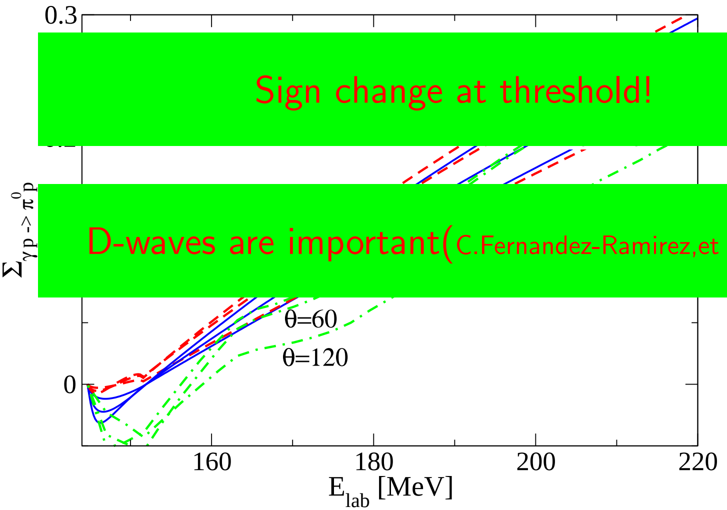

Sign change at threshold!

D-waves are important(C.Fernandez-Ramirez,et al.2009)

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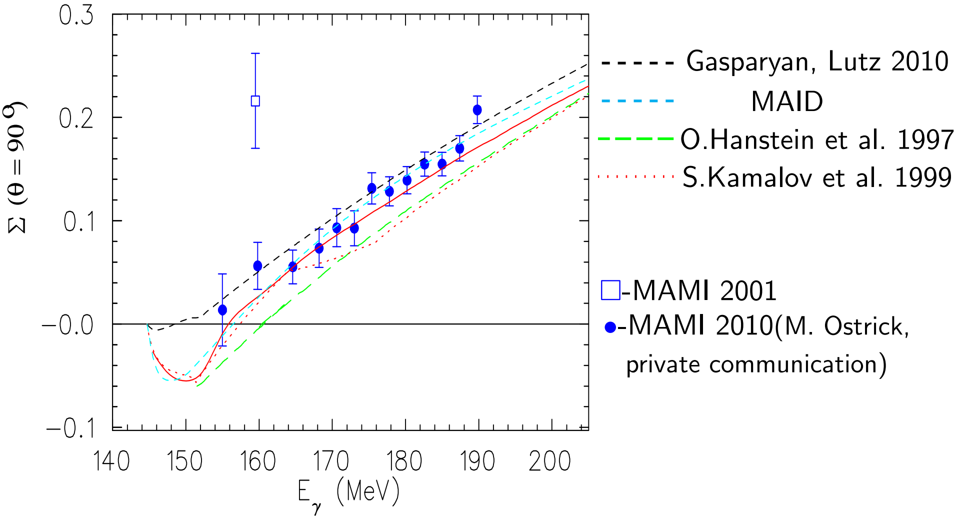
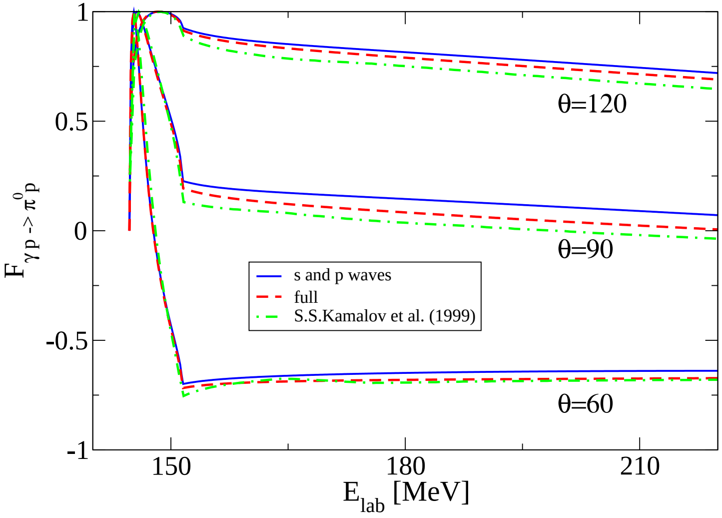


Energy dependence of the double polarization observable F.



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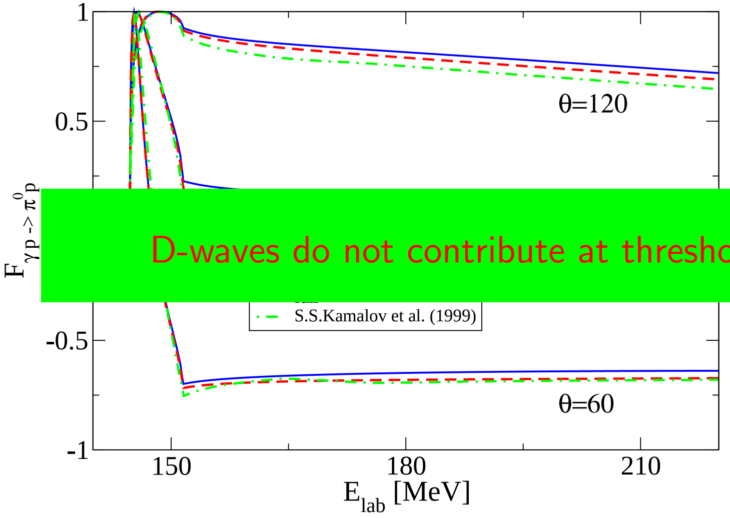
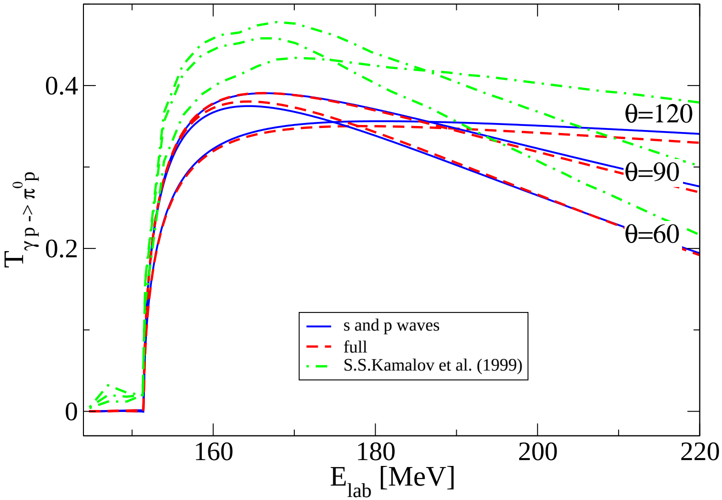
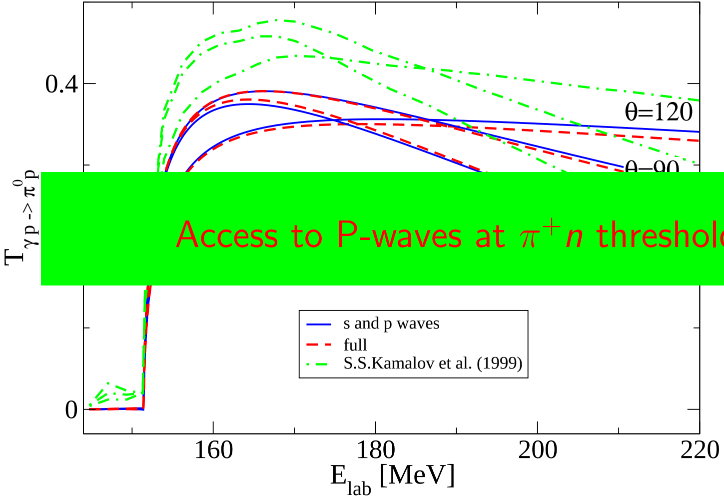


D-waves do not contribute at threshold

Energy dependence of the target asymmetry.



Energy dependence of the target asymmetry.



Access to P-waves at $\pi^+ n$ threshold

No additional parameters need to be adjusted for Compton scattering!

Proton spin polarizabilities in units of 10^{-4} fm^4

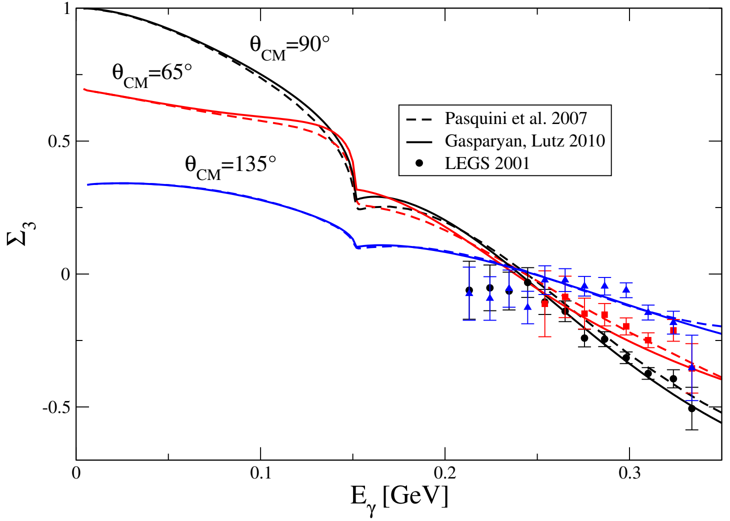
	χPT, Q^3	χPT, Q^4	DR	our values
γ_{E1E1}	-5.93	-1.41	-4.3	-3.68
γ_{M1M1}	-1.19	3.38	2.9	2.47
γ_{E1M2}	1.19	0.23	0.0	1.19
γ_{M1E2}	1.19	1.82	2.1	1.19
γ_0	4.74	-4.02	-0.7	-1.16
γ_π	4.74	6.39	9.3	6.14

Empirical values:

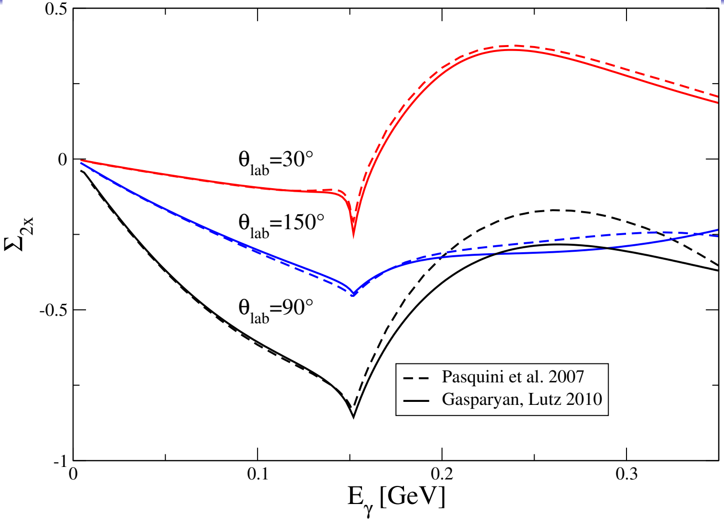
$$\begin{aligned}\gamma_0 &= -\gamma_{E1E1} - \gamma_{M1M1} - \gamma_{E1M2} - \gamma_{M1E2} \\ &= (-1.01 \pm 0.08 \pm 0.13)\,10^{-4}\mathrm{fm}^4\,,\end{aligned}$$

$$\begin{aligned}\gamma_\pi &= -\gamma_{E1E1} + \gamma_{M1M1} - \gamma_{E1M2} + \gamma_{M1E2} \\ &= (8.0 \pm 1.8)\,10^{-4}\mathrm{fm}^4\,.\end{aligned}$$

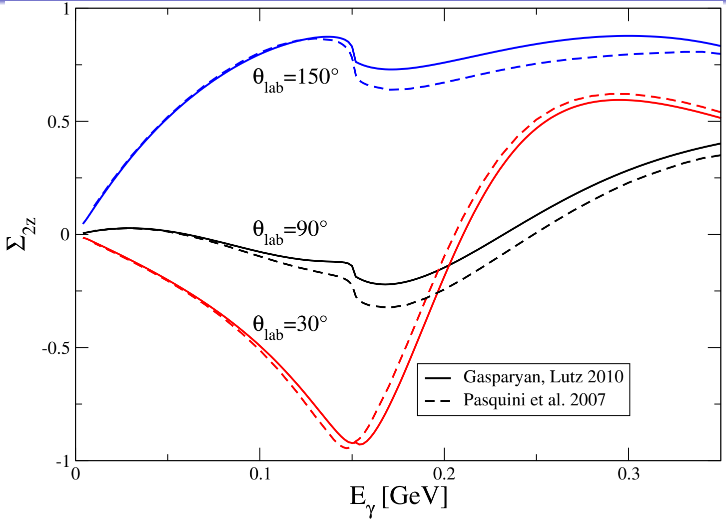
Energy dependence of the beam asymmetry.



Energy dependence of the double polarization asymmetry Σ_{2x}.



Energy dependence of the double polarization asymmetry Σ_{2z}.



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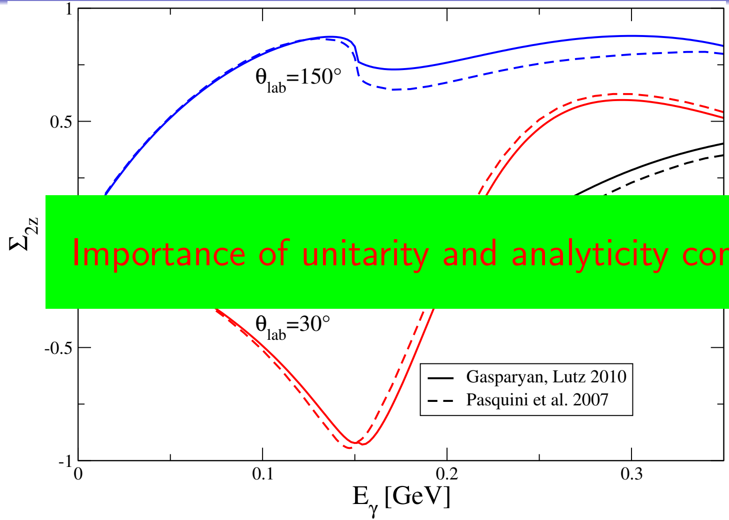

Importance of unitarity and analyticity constraints

Summary

- A method to extrapolate chiral amplitudes beyond threhold region is reviewed.
- Causality and unitarity constraints are utilized to stabilize the extrapolation.
- The processes $\pi N \to \pi N$, $\gamma N \to \pi N$ and $\gamma N \to \gamma N$ are well described up to $\sqrt{s} = 1300$ MeV.
- Predictions for various polarization observables for neutral pion photoproduction and proton Compton scattering as well as for proton spin polarizabilities are presented.