PWA 2011

Sixth International Workshop on Pion-Nucleon Partial-Wave Analysis and the Interpretation of Baryon Resonances

23–27 May, 2011 — Washington, DC, U.S.A.

Reciprocal consistency constraints among photoprocesses

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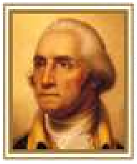
CNS

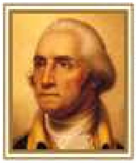
Center for Nuclear Studies

Collaborators: F. Huang, K. Nakayama (UGA); M. Döring, S. Krewald (FZJ)



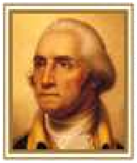
Goal

- **Derive a detailed microscopic description of the nucleon current J^μ**

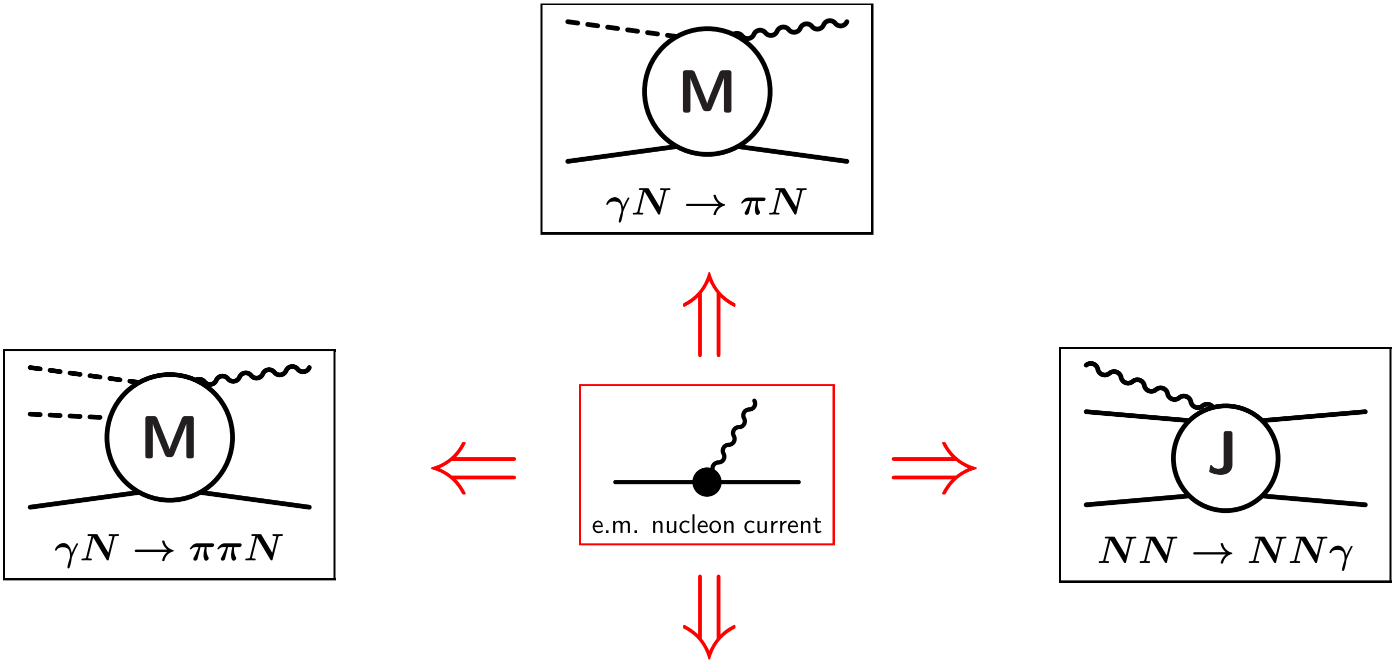


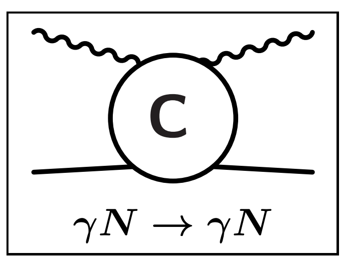
Goal

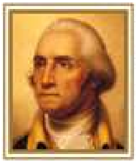
- **Derive a detailed microscopic description of the nucleon current J^μ:**
  - Full implementation of **gauge invariance** in terms of **Generalized Ward–Takahashi identities**
  - Assure **reciprocal consistency** of reaction dynamics among all affected photoprocesses



Introduction

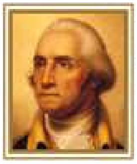






Electromagnetic Current J^μ of the Nucleon

How does one describe the current in a Lorentz-covariant Bethe-Salpeter-type approach?



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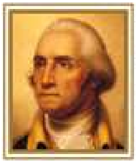
How does one describe the current in a Lorentz-covariant Bethe-Salpeter-type approach?

- The most general Lorentz-covariant structure of J^μ requires **12 form factors**. Bincer, PR118,855(1960)
- Applying gauge invariance, this reduces to **8 form factors**.
- Applying time-reversal invariance, this reduces further to **6 form factors**.



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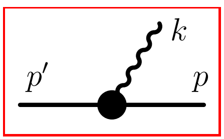
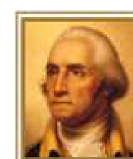
$$\gamma^\mu_{\rm T} = \gamma^\mu - k^\mu \frac{\not k}{k^2}$$

$$J^\mu(p',p) = e\Bigg[\delta_N\gamma^\mu + \delta_N\gamma^\mu_{\rm T}(F_1-1) + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N F_2$$
$$+ \frac{S^{-1}(p')}{2m}\left(\gamma^\mu_{\rm T} f_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N f_2\right) + \left(\gamma^\mu_{\rm T} f_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N f_2\right)\frac{S^{-1}(p)}{2m}$$
$$+ \frac{S^{-1}(p')}{2m}\left(\gamma^\mu_{\rm T} g_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N g_2\right)\frac{S^{-1}(p)}{2m}\Bigg]$$

(Approximation)





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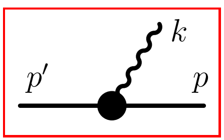
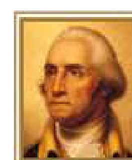


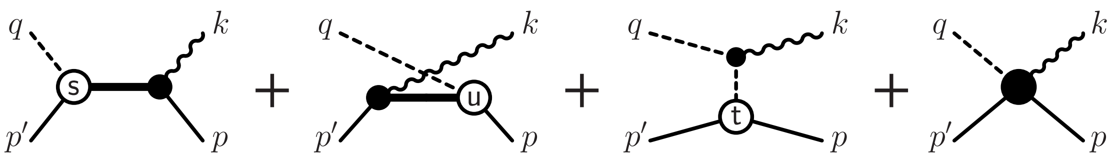
Constraints:

no kinematic singularity: $f_1(k^2) \xrightarrow{k^2=0} 0$ and $g_1(k^2) \xrightarrow{k^2=0} 0$

chiral-symmetry limit : $f_1 \to \dfrac{g_A - G_A(k^2)}{g_A}$ and $f_2 \to 1$



Implications of off-shell structure: Pion photoproduction



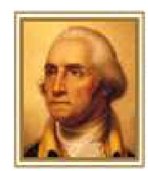
s-channel:

$$F_s S(p+k) J_i^\mu(p+k,p) = F_s S(p+k)\left(e\delta_i\gamma^\mu + \frac{i\sigma^{\mu\nu}k_\nu}{2m}e\kappa_i\right) + F_s \frac{i\sigma^{\mu\nu}k_\nu}{2m}\frac{e\kappa_i}{2m}f_{2i}$$

contact terms

u-channel:

$$J_f^\mu(p',p'-k) S(p'-k) F_u = \left(e\delta_f\gamma^\mu + \frac{i\sigma^{\mu\nu}k_\nu}{2m}e\kappa_f\right) S(p'-k) F_u + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\frac{e\kappa_f}{2m}f_{2f}F_u$$



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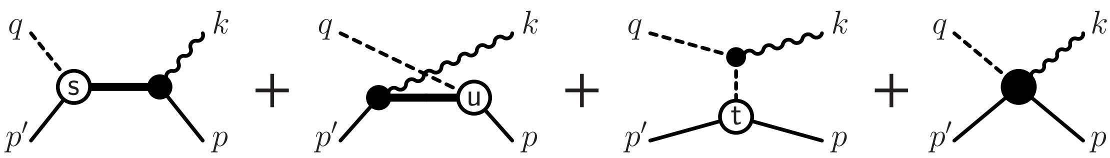

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One cannot make the connection to low-energy χPT results without such contact terms.

u-channel:

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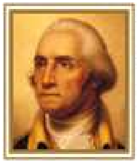
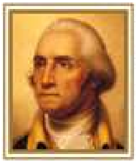


A Word about "Off-shell Effects"

It is often stated that "off-shell effects are not measurable" and that, therefore, any such effects should be summarily banished from any theory.



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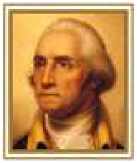
Franz Gross on off-shell effects

Panel discussion at the "17th European Conference on Few-Body Problems in Physics," Evora, Portugal, September 11–16, 2000, NPA689 (2001)

It is commonly stated that "off-shell effects" are unobservable. This is of course true, but so are wave functions, potentials, and most of the theoretical tools we use to describe physics. A better point is that off-shell effects are *meaningless without a theory or model to define them*. Almost all models provide such a definition, and off-shell effects should be discussed only in the context of a particular model that defines these effects *uniquely*.



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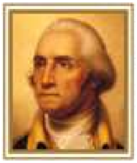
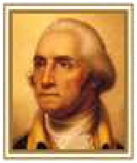
$\Longrightarrow$ Within the Bethe-Salpeter-type equations that originate from effective Lagrangian formulations, the off-shell structure of the nucleon current arises naturally as an integral part of the description of the reaction dynamics.



Electromagnetic Current J^μ of the Nucleon

For photoprocesses...

- the generic structural description of the nucleon current, in general, is not good enough
- more details of the current's internal explicit reaction dynamics are required



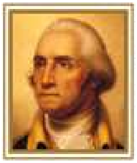
Electromagnetic Current J^μ of the Nucleon

For photoprocesses. . .

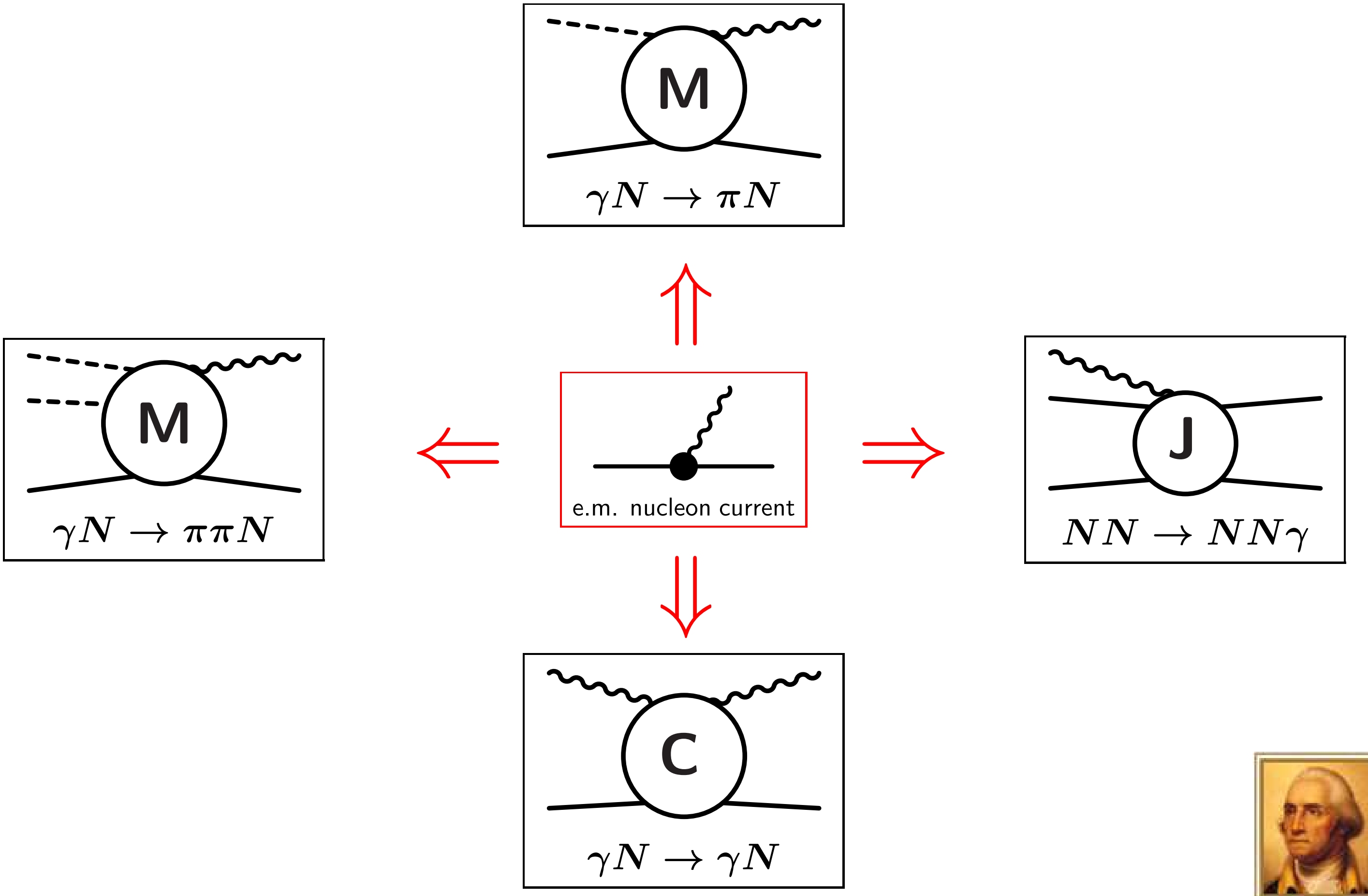
- the generic structural description of the nucleon current is, in general, not good enough;
- more details of the current's internal explicit reaction dynamics are required.

$$\Downarrow$$

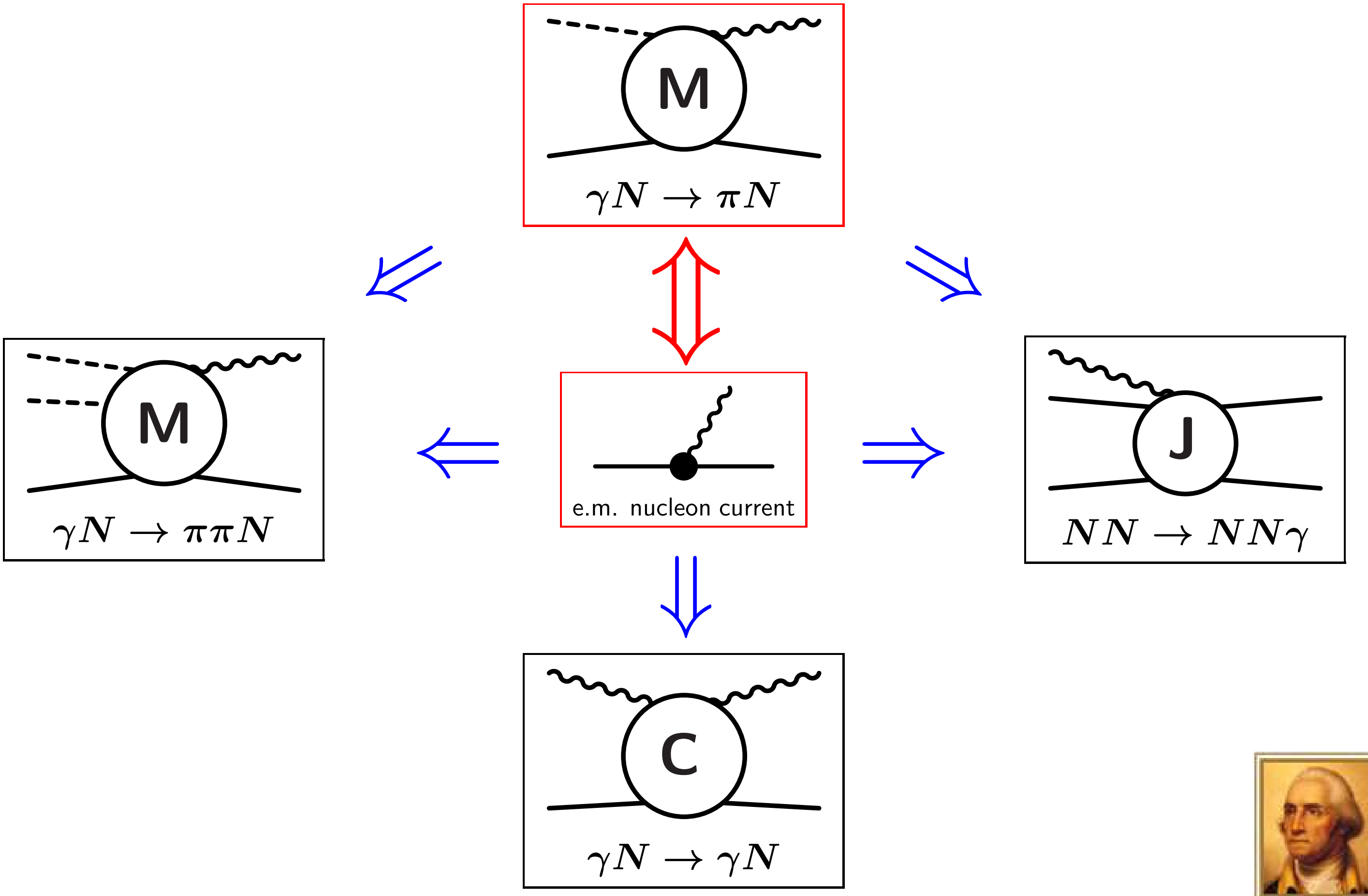
- Require reciprocal consistency among the various photoprocesses to determine the dynamical structures of the current J^μ.



Introduction



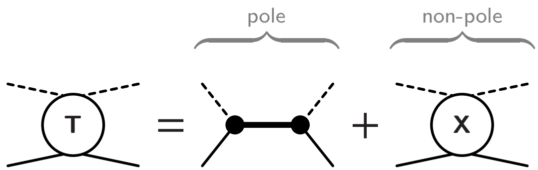
Dynamical Links between Photoprocesses



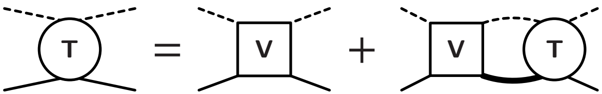
Pions, Nucleons, and Photons

HH, PRC **56**, 2041 (1997)

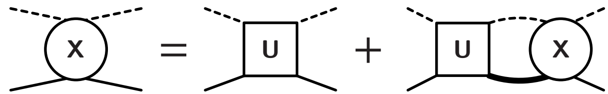
$\pi N\ T$ matrix



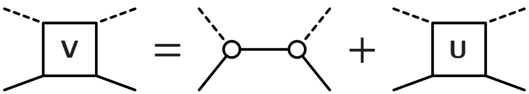
(a)



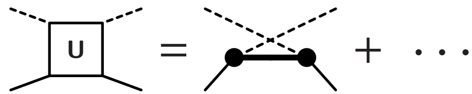
(b)



(d)

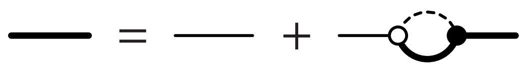


(c)



(e)

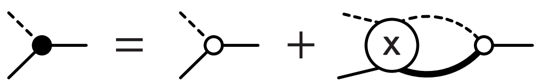
dressed nucleon propagator



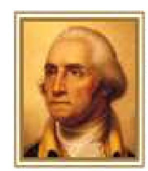
(a)

propagator determines current

dressed πNN vertex



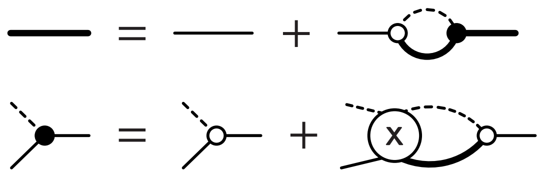
(b)



- Tower of *nonlinear* Dyson-Schwinger-type equations

Nucleon Current J^μ

HH, PRC **56**, 2041 (1997)

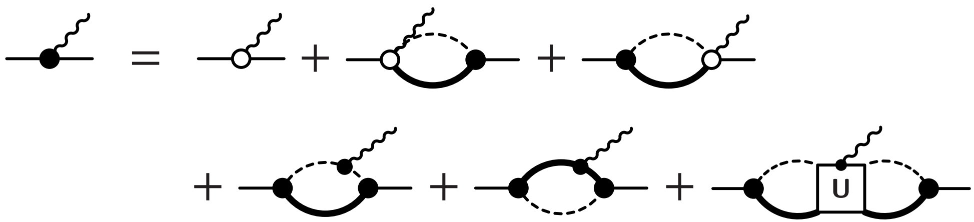


(a)

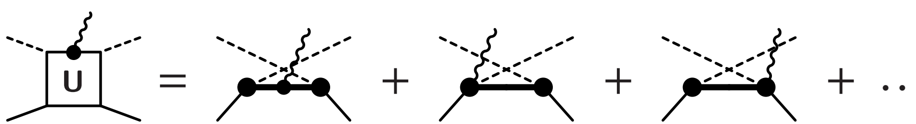
(b)

propagator
determines current

- **Couple photon to dressed propagator:**

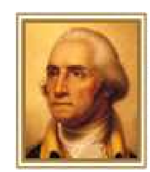


(a)

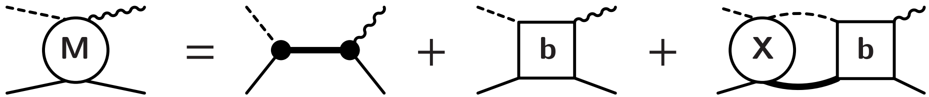


(b)

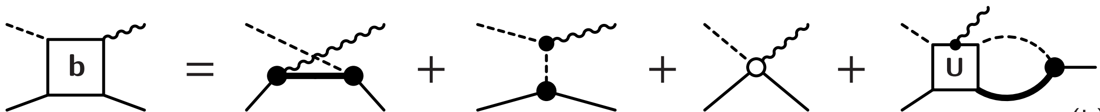
- Tower of *nonlinear* Dyson-Schwinger-type equations



- **Pion-production current M^μ:**



(a)



(b)

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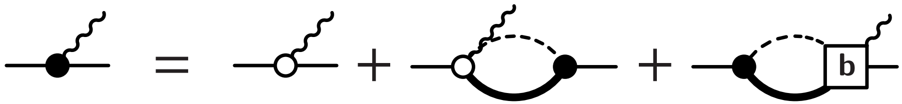
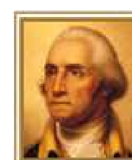


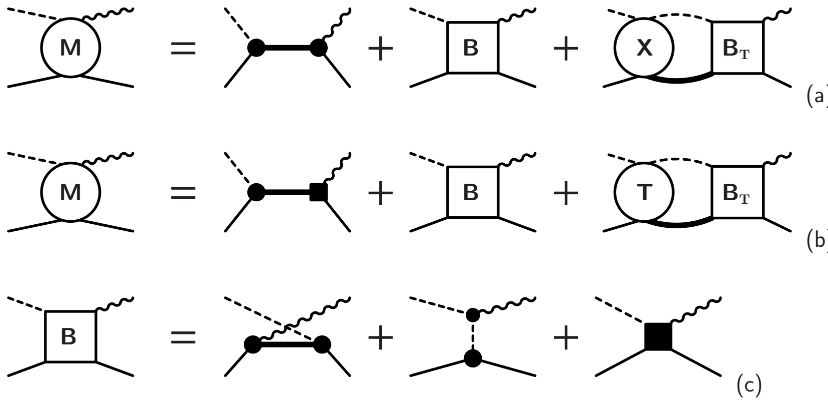
$\Rightarrow$ The internal structures of the dressed nucleon current can be understood by the dynamics of the pion production current.



- Tower of *nonlinear* Dyson-Schwinger-type equations

Rewriting the Production Current

- **Pion-production current M^μ:**

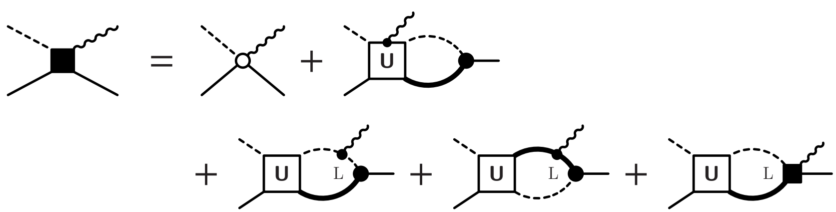


X

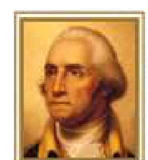
equivalent

T

- **Contact-type current M_c^μ:**

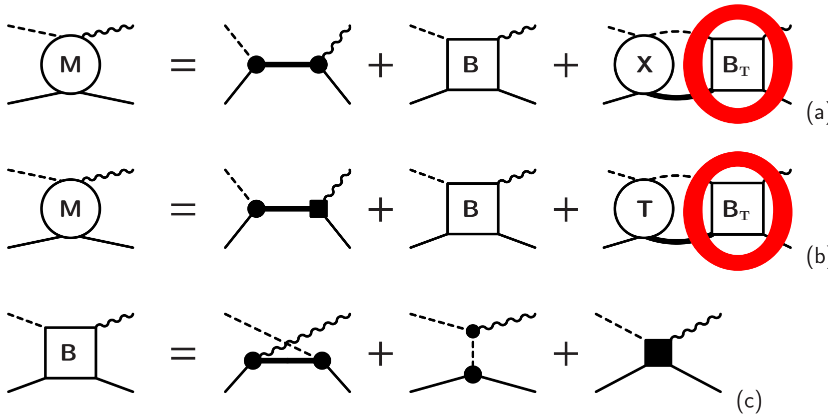


- Tower of *nonlinear* Dyson-Schwinger-type equations



Rewriting the Production Current

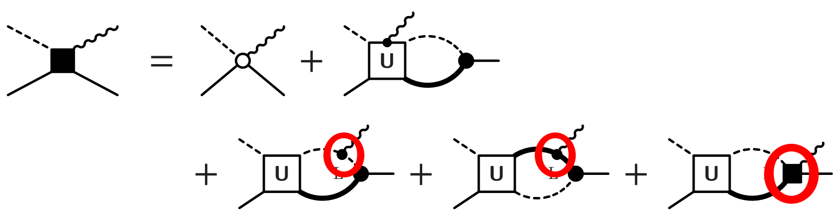
- **Pion-production current M^{μ}:**



transverse

(irrelevant for gauge invariance)

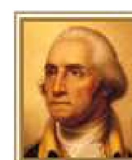
- **Contact-type current M_c^{μ}:**



partial integral equation

longitudinal

- Tower of *nonlinear* Dyson-Schwinger-type equations



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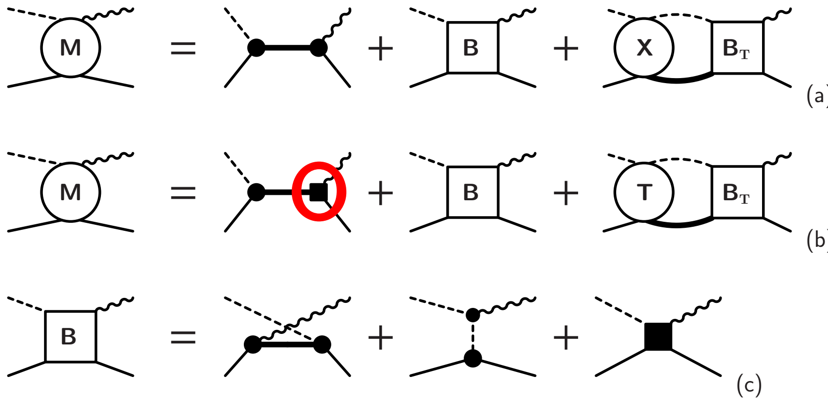
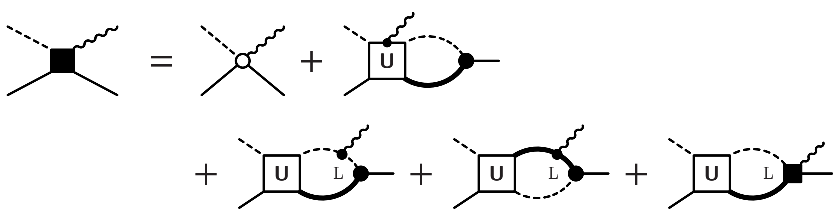


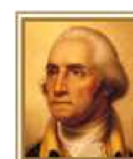
J^μ_s

not the full nucleon current

- **Contact-type current M^μ_c:**

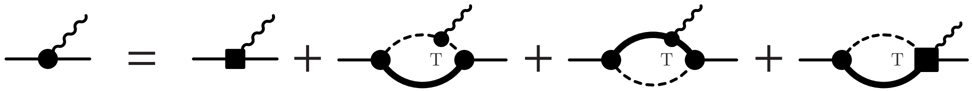


- Tower of *nonlinear* Dyson-Schwinger-type equations

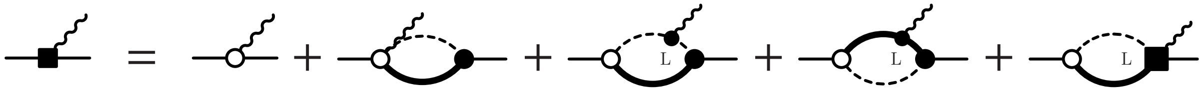


Nucleon Current J^μ

HH, F. Huang, K. Nakayama, arXiv:1103.2065 [nucl-th] (2011)

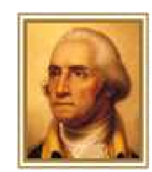


(a)



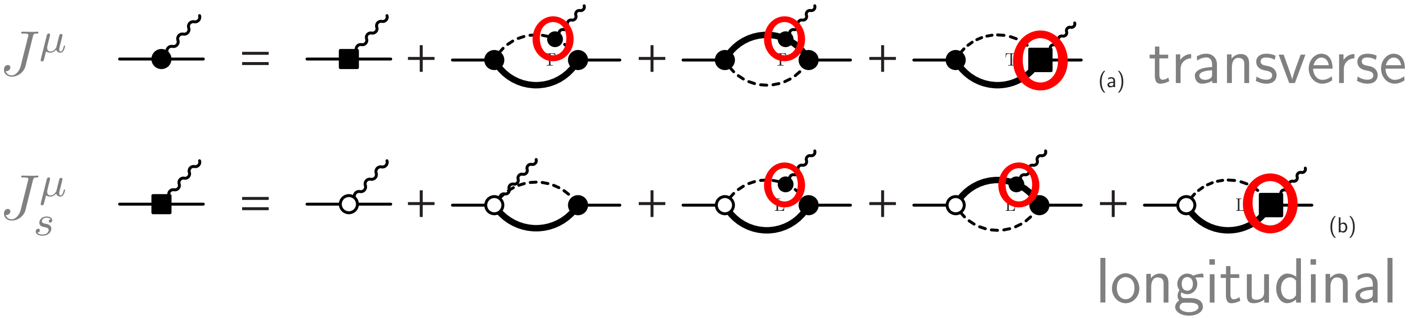
(b)

- Tower of *nonlinear* Dyson-Schwinger-type equations

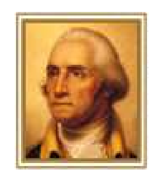


Nucleon Current J^μ

HH, F. Huang, K. Nakayama, arXiv:1103.2065 [nucl-th] (2011)

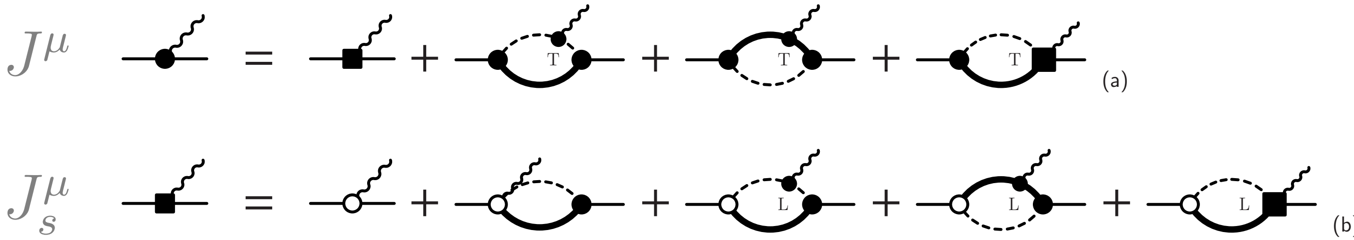


- Tower of *nonlinear* Dyson-Schwinger-type equations



Nucleon Current J^μ

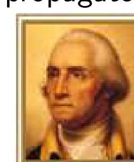
HH, F. Huang, K. Nakayama, arXiv:1103.2065 [nucl-th] (2011)



Gauge Invariance: Ward-Takahashi Identity (WTI)

$$k_\mu J^\mu(p',p) = k_\mu J_s^\mu(p',p) = S^{-1}(p')Q_N - Q_N S^{-1}(p)$$

S: dressed nucleon propagator

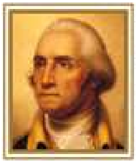


Problems?

- Everything is exact!

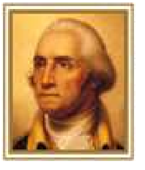
- Everything is nonlinear!

- Everything is hideously complicated!



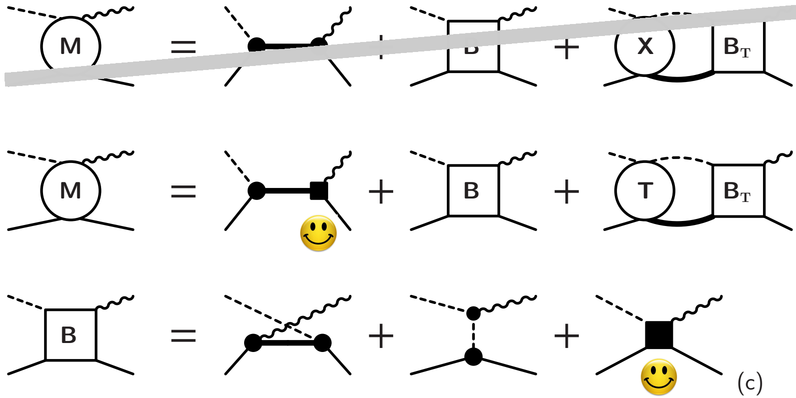
- Everything is exact!
- Everything is nonlinear!
- Everything is hideously complicated!

But...



Let's cut the Gordian knot!

HH, F. Huang, K. Nakayama, arXiv:1103.2065 [nucl-th] (2011)

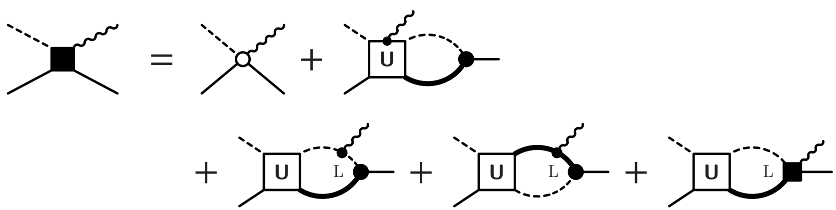


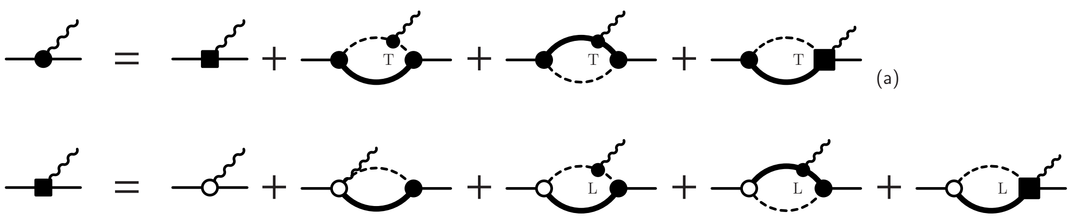
(a)

(b)

(c)

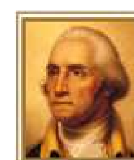
Do not use X.
Work with full T.





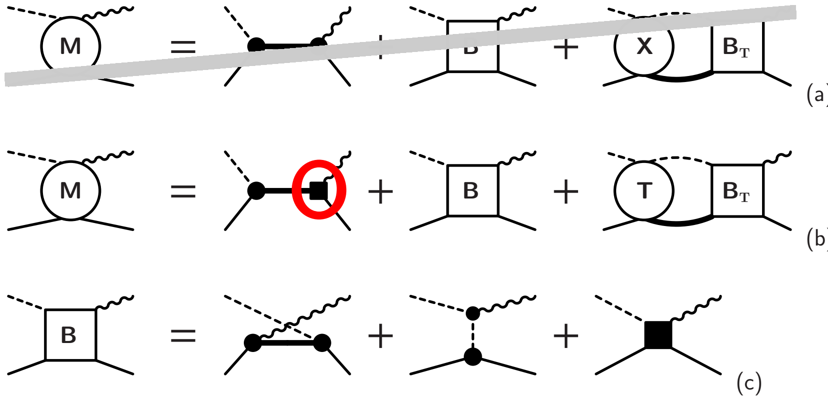
(a)

(b)



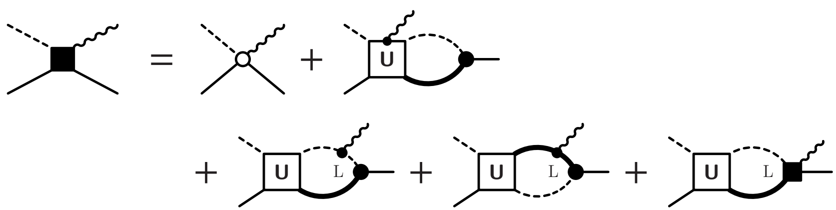
Cutting the Gordian Knot

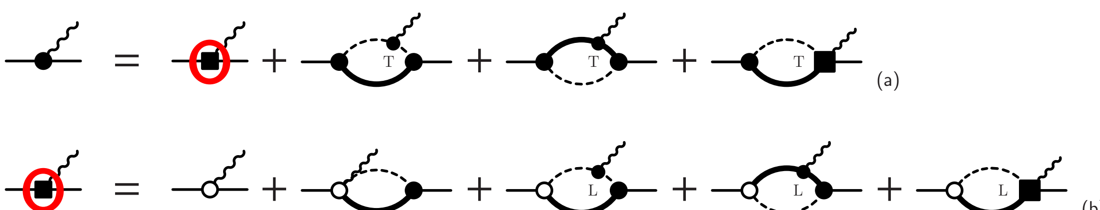
HH, F. Huang, K. Nakayama, arXiv:1103.2065 [nucl-th] (2011)



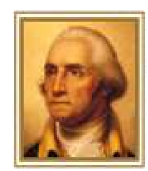
J^{μ}_{s}

not the full nucleon current



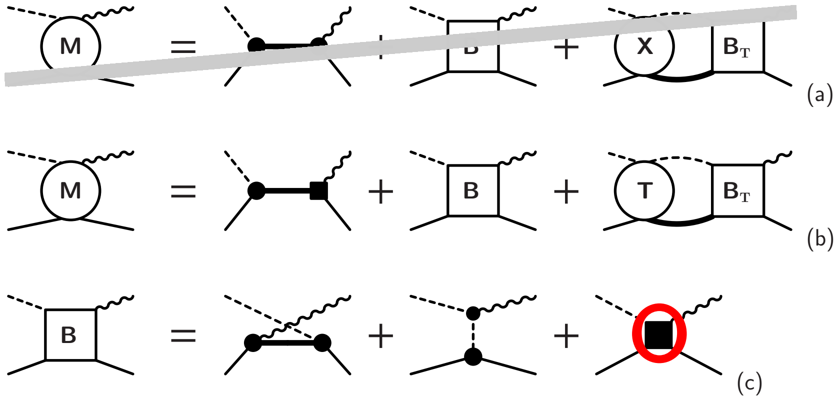


determine approximation by WTI for the nucleon current J^{μ}

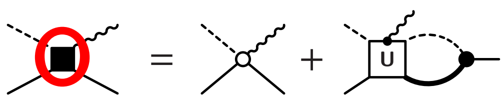


Cutting the Gordian Knot

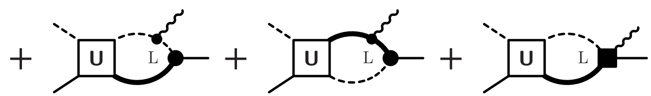
HH, F. Huang, K. Nakayama, arXiv:1103.2065 [nucl-th] (2011)

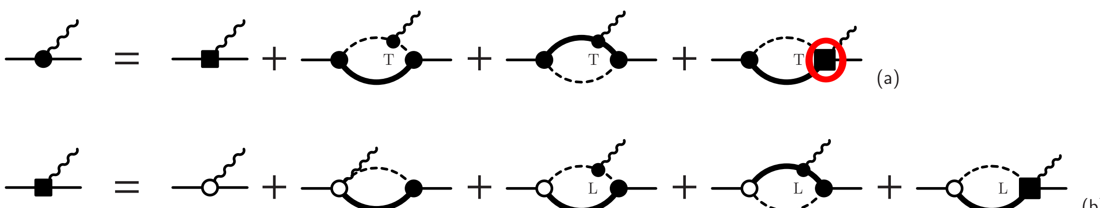


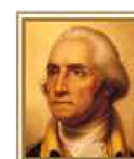
$$M_c^\mu$$



determine approximation of M_c^μ by generalized WTI for the photoproduction current M^μ

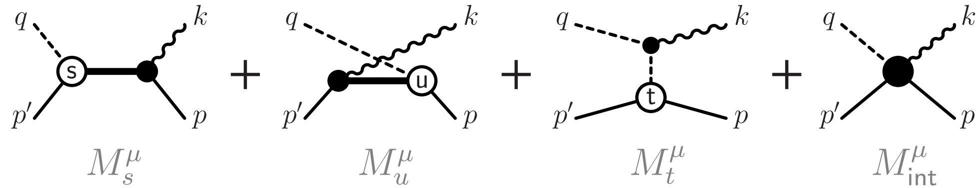






Reminder: Generalized Ward–Takahashi Identity

$M^{\mu} =$

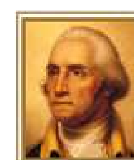


- **Generalized WTI for the full current M^{μ}:**

$$k_\mu M^\mu = -F_s S(p+k) Q_i S^{-1}(p) + S^{-1}(p') Q_f S(p'-k) F_u + \Delta_\pi^{-1}(q) Q_\pi \Delta_\pi(q-k) F_t$$

- **Equivalent Generalized WTI for the interaction current M_{int}^{μ}:**

$$k_\mu M_{\text{int}}^\mu = -F_s Q_i + Q_f F_u + Q_\pi F_t$$



Reminder: Generalized Ward–Takahashi Identity

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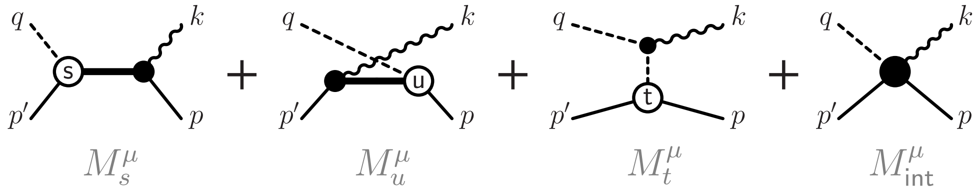

- **Generalized WTI for the full current M^μ:**

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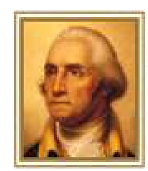
- **Equivalent Generalized WTI for the interaction current M_{int}^μ:**

$$k_\mu M_{\text{int}}^\mu = -F_s Q_i + Q_f F_u + Q_\pi F_t$$

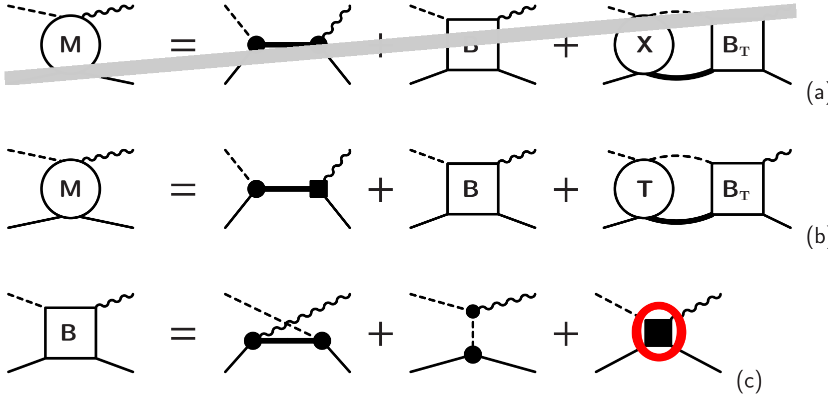
Here: $k_\mu M_{\text{int}}^\mu = k_\mu M_c^\mu$

Off-shell constraints!



Approximating M_c^μ

HH, F. Huang, K. Nakayama, arXiv:1103.2065 [nucl-th] (2011)



M_c^μ

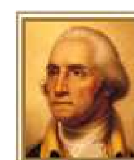
- **Lowest-order approximation in terms of phenomenological form factors:**

$$M_c^\mu = ge\gamma_5 \frac{i\sigma^{\mu\nu}k_\nu}{4m^2}\tilde{\kappa}_N - (1-\lambda)g\frac{\gamma_5\gamma^\mu}{2m}\tilde{F}_t e_\pi - G_\lambda \Bigg[e_i \frac{(2p+k)^\mu}{s-p^2}\left(\tilde{F}_s - \hat{F}\right)$$

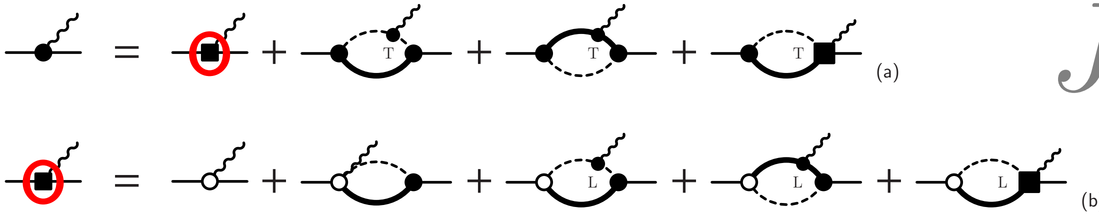
$$+ e_f \frac{(2p'-k)^\mu}{u-p'^2}\left(\tilde{F}_u - \hat{F}\right)$$

$$+ e_\pi \frac{(2q-k)^\mu}{t-q^2}\left(\tilde{F}_t - \hat{F}\right)\Bigg]$$

Don't try to read the details. What is important is that this is a simple expression, easy to evaluate, and that it helps preserve gauge invariance of the entire production current.



Approximating J_s^μ



J_s^μ

determine approximation by WTI for the nucleon current J^μ

- **Approximate J_s^μ by the minimal current that reproduces the WTI:**

$S^{-1}(p) = \not{p}A(p^2) - mB(p^2)$

$$J_s^\mu(p',p) = (p'+p)^\mu \frac{S^{-1}(p')Q_N - Q_N S^{-1}(p)}{p'^2 - p^2} + \left[\gamma^\mu - \frac{(p'+p)^\mu}{p'^2-p^2}\not{k}\right] Q_N \frac{A(p'^2)+A(p^2)}{2}$$

Ball–Chiu:
Satisfies WTI
Nonsingular
Minimal
Unique!

- **Half on-shell:**

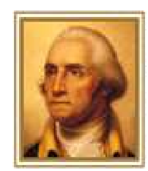
$$S J_s^\mu u = \left(\frac{1}{\not{p}+\not{k}-m} j_1^\mu + \frac{2m}{s-m^2} j_2^\mu\right) Q_N\, u(p)\ , \qquad \text{with} \qquad s=(p+k)^2$$

Exact!

- Auxiliary currents:

$$j_1^\mu = \gamma^\mu\ (1-\kappa_1) + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_1 \qquad\qquad j_2^\mu = \frac{(2p+k)^\mu}{2m}\kappa_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_2$$

Two parameters!

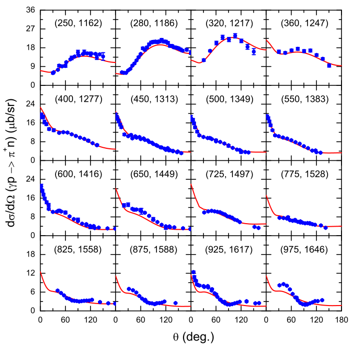


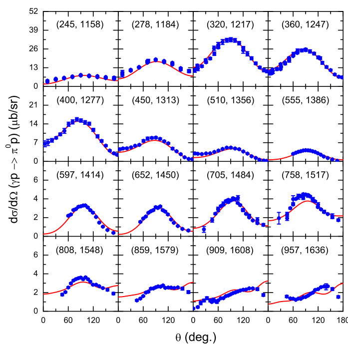
Does it work? — Yes!

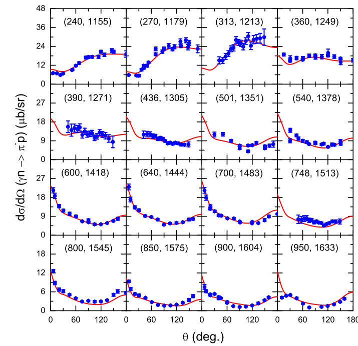
- **Preliminary results for $\gamma N \to \pi N$**

Fei Huang, this afternoon

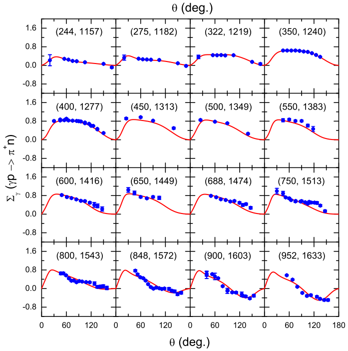
$\frac{d\sigma}{d\Omega}$

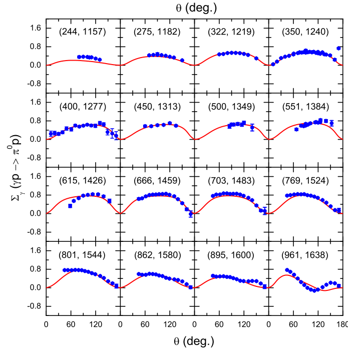


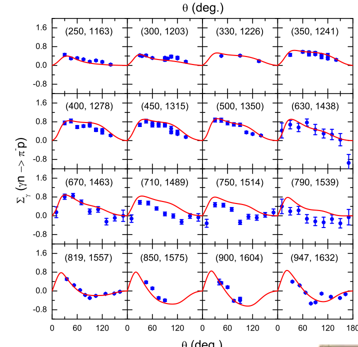




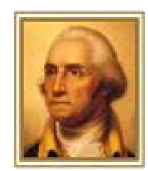
Σ







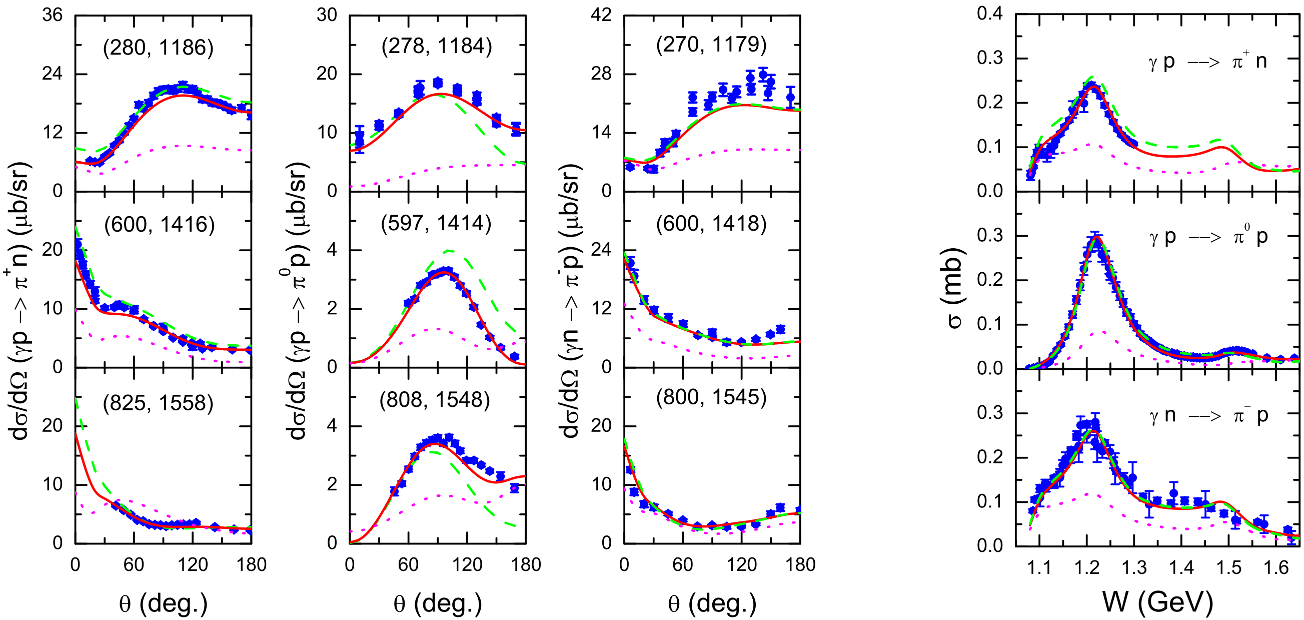
$\gamma p \to \pi^+ n$ $\qquad$ $\gamma p \to \pi^0 p$ $\qquad$ $\gamma n \to \pi^- p$

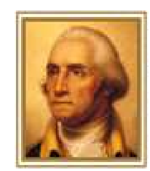


F. Huang, M. Döring, H. Haberzettl, J. Haidenbauer, C. Hanhart, S. Krewald, U.-G. Meißner, and K. Nakayama, *in preparation*

On the importance of maintaining gauge invariance

- **Preliminary results for $\gamma N \to \pi N$:**

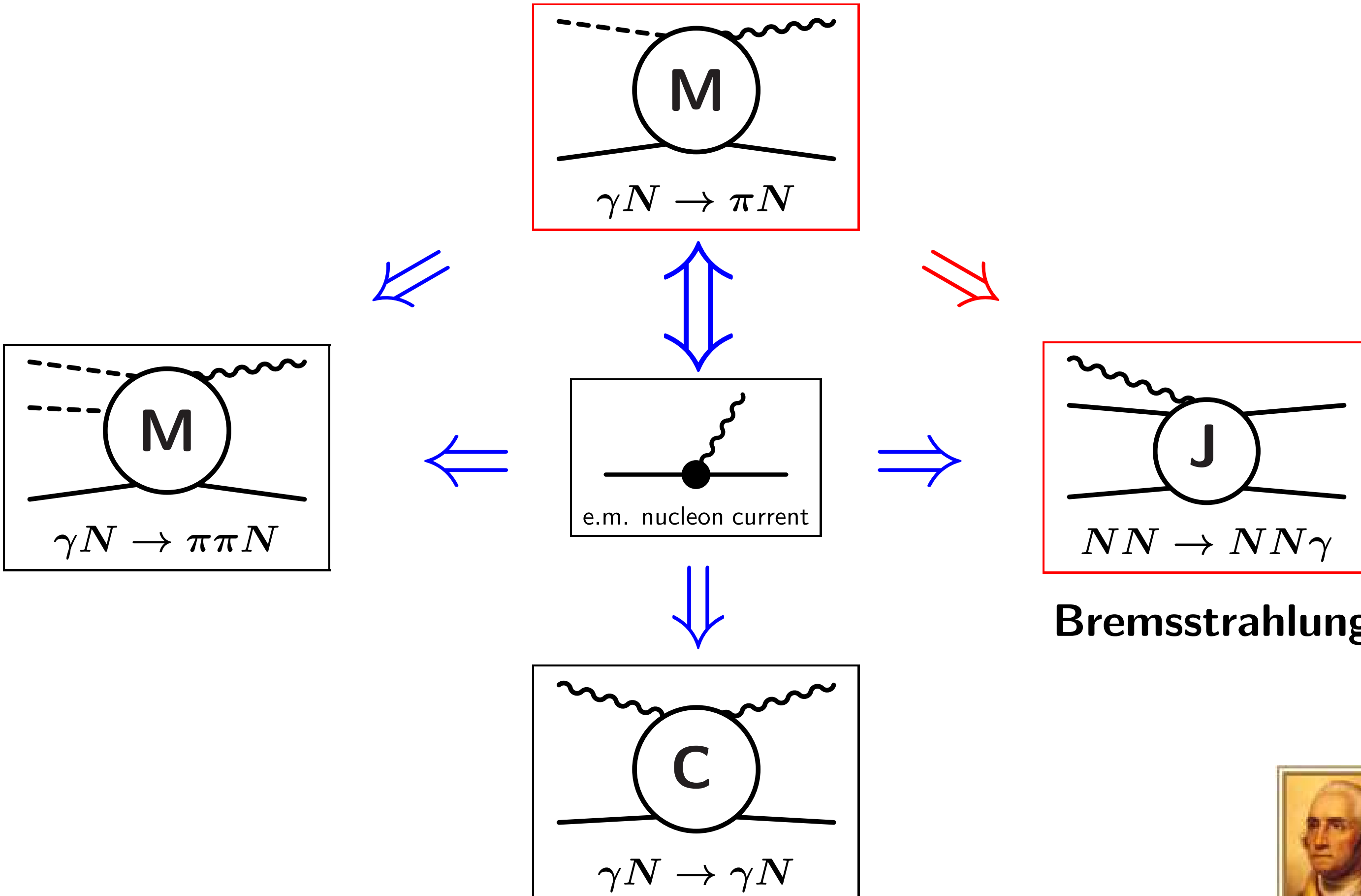




Dashed green curves: w/o M_c^μ

F. Huang, M. Döring, H. Haberzettl, J. Haidenbauer, C. Hanhart, S. Krewald, U.-G. Meißner, K. Nakayama, *to be published* (2011)

Dynamical Links between Photoprocesses — Bremsstrahlung



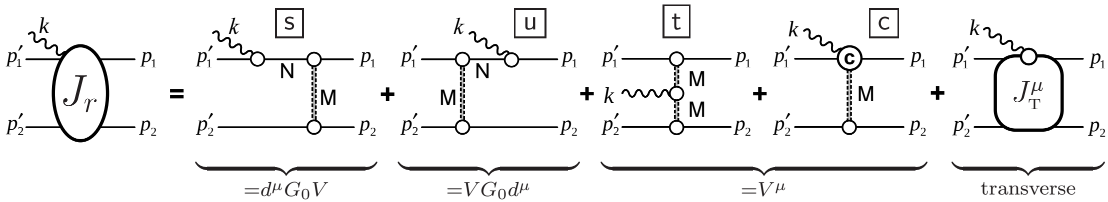
Bremsstrahlung $NN \to NN\gamma$

K. Nakayama, HH, PRC +bf80, 051001(R) (2009)

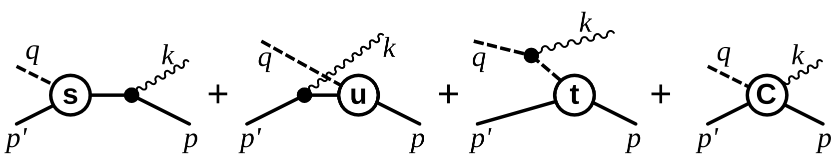
- **Bremsstrahlung Current:**

$$J^\mu_{\mathrm{B}} = (TG_0 + 1) J^\mu_r (1 + G_0 T)$$

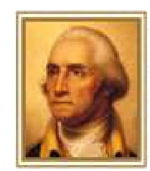
T: NN T-matrix



- **Compare the photon processes along the top nucleon line above to the meson production diagrams below.**



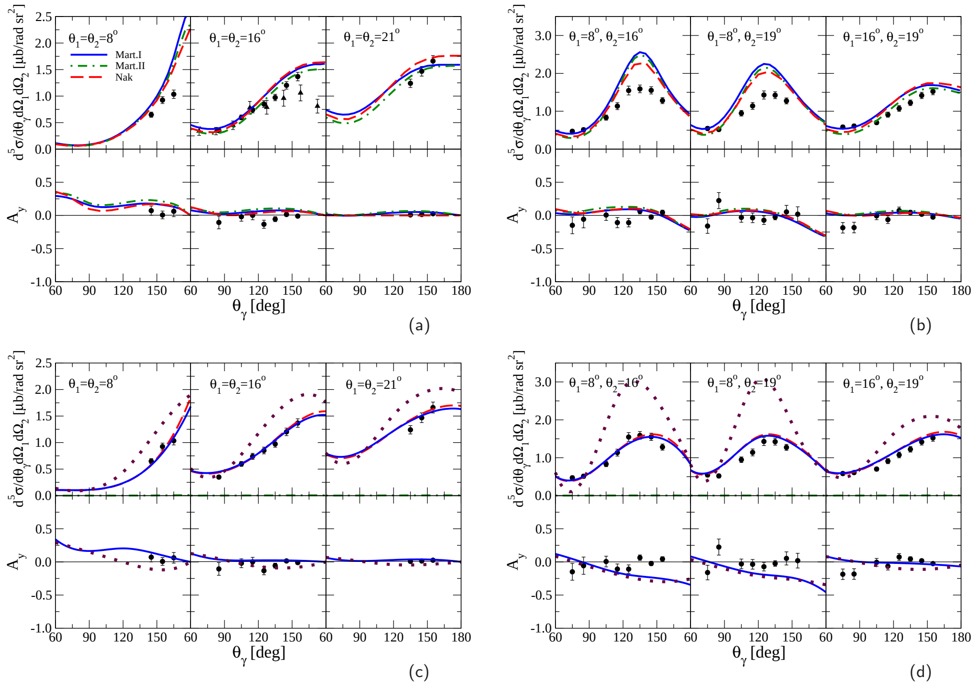
$\Rightarrow$ Essential parts of the process can be described as a meson capture process — i.e., as an inverse photoproduction process — in the presence of a spectator nucleon.



Bremsstrahlung $NN \to NN\gamma$

K. Nakayama, HH, PRC **80**, 051001(R) (2009)

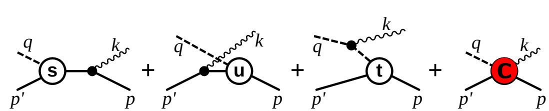
- **Application to KVI data. — Or: Resolving a longstanding problem:**

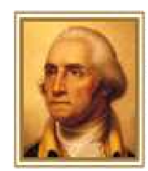


Old

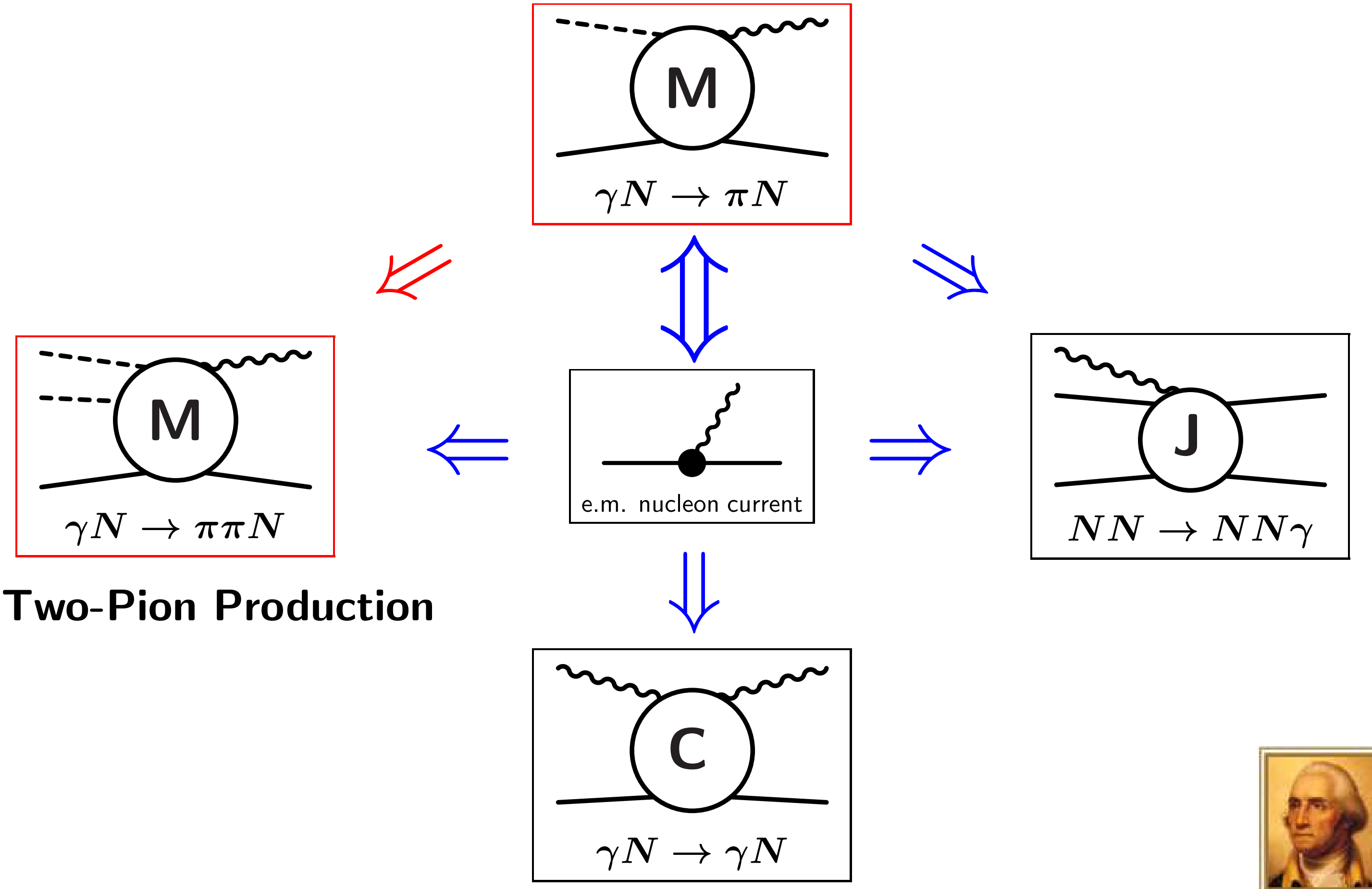
New

- Inclusion of the four-point interaction current from meson photoproduction brings about a dramatic improvement.

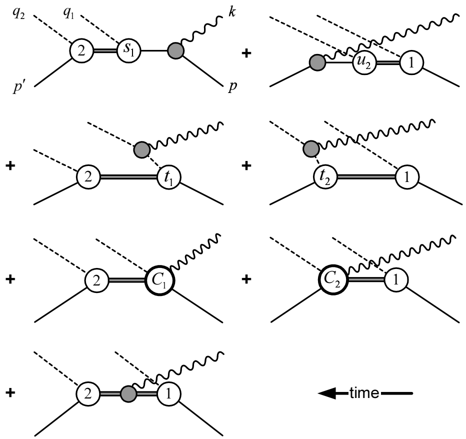




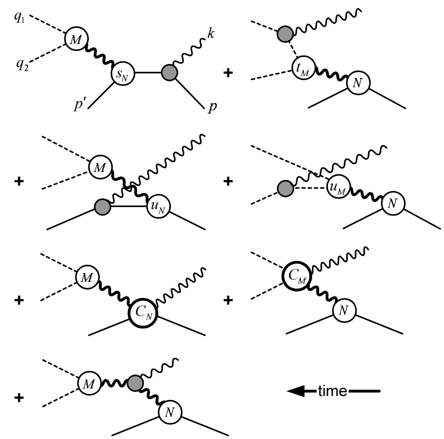
Dynamical Links between Photoprocesses — Two-Pion Production



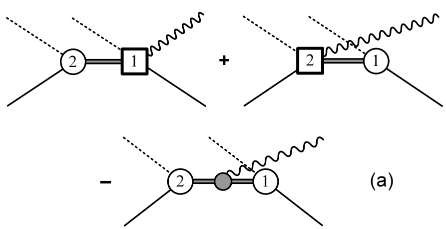
Basic Two-pion Production Mechanisms



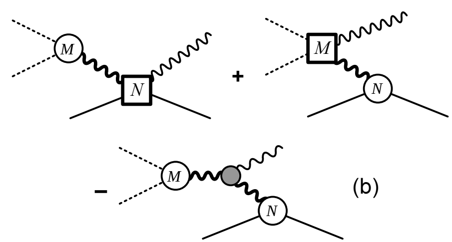
(a)



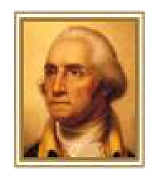
(b)



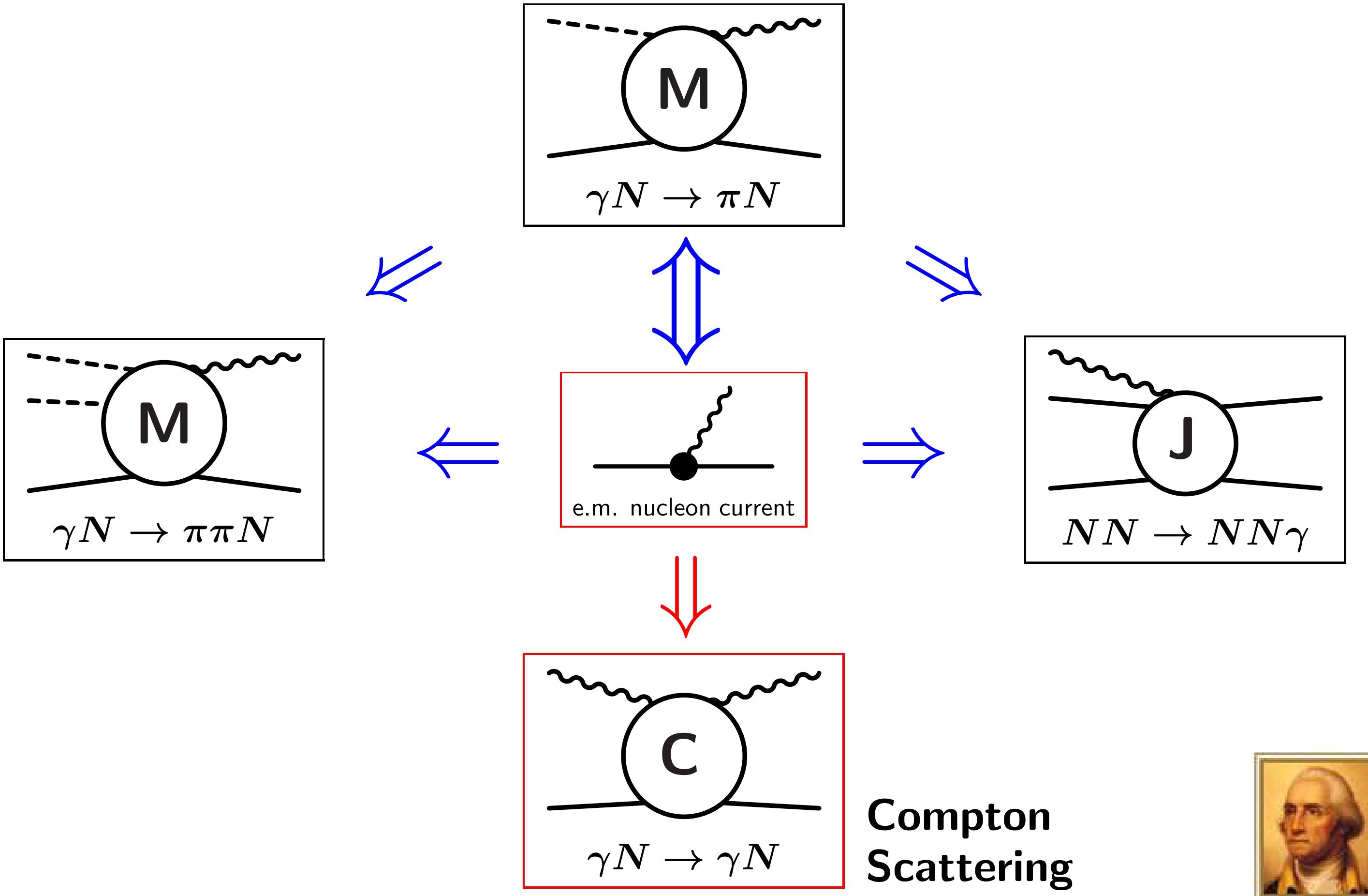
(a)



(b)

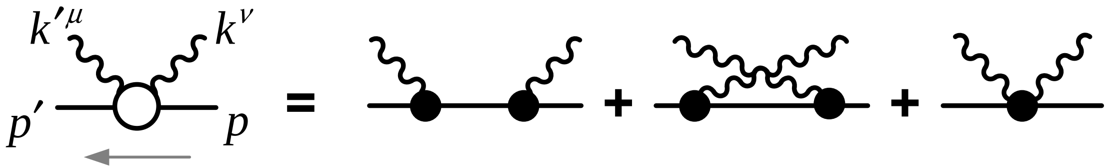


Dynamical Links between Photoprocesses — Compton Scattering

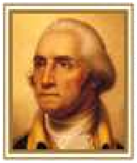


Compton Scattering

Compton Scattering $\gamma N \to \gamma N$

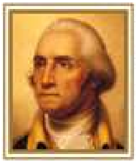


- s- and u-channel terms employ dressed current just described.
- Contact term constrained by gauge invariance.



Conclusions

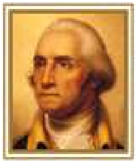
- There exists a very close relationship between the dressed nucleon current and the pion photoproduction current.
- Exploiting this relationship suggests physically meaningful approximations that work, despite the enormous complexity of the exact formalism.
- Maintaining full gauge invariance (as opposed to mere current conservation) is not a luxury but a necessity for the correct microscopic description of the reaction dynamics.
- Requiring gauge invariance in the form of *off-shell* (generalized) Ward-Takahashi identities for each subprocess provides a powerful tool for constraining the contributing mechanisms *and* ensuring overall gauge invariance as a matter of course.
- **Note:** Gauge invariance (as an off-shell condition) cannot be maintained in a non-covariant phenomenological Lagrange-type formalism. At best, one can have non-unique non-relativistic types of current conservation.



Goal ✓

- **Derive a detailed microscopic description of the nucleon current J^μ:**

  - ☑ Full implementation of **gauge invariance** in terms of **Generalized Ward–Takahashi identities**
  - ☑ Assure **reciprocal consistency** of reaction dynamics among all affected photoprocesses



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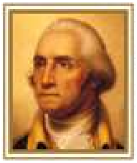
- **Derive a detailed microscopic description of the nucleon current J^μ:**

  - ☑ Full implementation of **gauge invariance** in terms of **Generalized Ward–Takahashi identities**
  - ☑ Assure **reciprocal consistency** of reaction dynamics among all affected photoprocesses
  - ☑ As a bonus, this provides a novel* description of the pion photoproduction process that has many features that make it particularly well suited for practical applications

Thank you!



*) In the spirit of HH, Nakayama, Krewald, PRC **74**, 045202 (2006), but decisively different in detail.