Resonances in chiral effective field theory

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Outline

- Magnetic moment of the Roper
- Vector form factor of the pion
- Some aspects of complex-mass scheme
- Imaginary parts of loop integrals
- Perturbative unitarity in CMS
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Magnetic moment of the Roper

Effective Lagrangian relevant for the magnetic moment of the Roper

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\pi + \mathcal{L}_R + \mathcal{L}_{NR} + \mathcal{L}_{\Delta R}.$$

Here

$$\begin{aligned}\mathcal{L}_0 &= \bar{N}\,(i\not{D} - m_{N0})N + \bar{R}(i\not{D} - m_{R0})R \\ &- \bar{\Psi}_\mu \xi^{\frac{3}{2}}[(i\not{D} - m_{\Delta 0})\,g^{\mu\nu} - i\,(\gamma^\mu D^\nu + \gamma^\nu D^\mu) + i\,\gamma^\mu \not{D} \gamma^\nu + m_{\Delta 0}\,\gamma^\mu\gamma^\nu]\xi^{\frac{3}{2}}\Psi_\nu\end{aligned}$$

N and R denote nucleon and Roper isospin doublets. Ψ_ν are the Rarita-Schwinger fields of the Δ resonance, $\xi^{\frac{3}{2}}$ is the isospin-3/2 projector.

Covariant derivatives are defined as follows:

$$\begin{aligned}D_\mu H &= \left(\partial_\mu + \Gamma_\mu - i\,v_\mu^{(s)}\right) H\,, \\ (D_\mu \Psi)_{\nu,i} &= \partial_\mu \Psi_{\nu,i} - 2\,i\,\epsilon_{ijk}\Gamma_{\mu,k}\Psi_{\nu,j} + \Gamma_\mu \Psi_{\nu,i} - i\,v_\mu^{(s)}\Psi_{\nu,i}\,, \\ \Gamma_\mu &= \frac{1}{2}\left[u^\dagger \partial_\mu u + u \partial_\mu u^\dagger - i\,\left(u^\dagger v_\mu u + u v_\mu u^\dagger\right)\right] = \tau_k \Gamma_{\mu,k}.\end{aligned}$$

Lowest-order Goldstone boson Lagrangian:

$$\mathcal{L}_\pi^{(2)} = \frac{F^2}{4}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{F^2 M^2}{4}\mathrm{Tr}\left(U^\dagger + U\right) + i\frac{F^2}{2}\mathrm{Tr}\left[\left(\partial_\mu U U^\dagger + \partial_\mu U^\dagger U\right) v_\mu\right] + \cdots .$$

$v^\mu = -e\,\frac{\tau_3}{2}\,\mathcal{A}^\mu$; Pion fields are contained in U; F is the pion-decay constant in the chiral limit; $M^2 = 2B\widehat{m}$, where B is related to the quark condensate $\langle\bar{q}q\rangle_0$ in the chiral limit.

Leading order pion-Roper Lagrangian:

$$\mathcal{L}_R^{(1)} = \frac{g_R}{2}\,\bar{R}\gamma^\mu\gamma_5 u_\mu R\,,$$

where g_R is an unknown coupling constant and

$$u_\mu = i\left[u^\dagger\partial_\mu u - u\partial_\mu u^\dagger - i\,\left(u^\dagger v_\mu u - u v_\mu u^\dagger\right)\right],$$

where $u = \sqrt{U}$.

Second and third order Roper Lagrangians:

$$\mathcal{L}_R^{(2)} = \bar{R}\left[\frac{c_6^*}{2}\, f_{\mu\nu}^{+} + \frac{c_7^*}{2}\, v_{\mu\nu}^{(s)}\right] \sigma^{\mu\nu} R + \cdots \,,$$

$$\mathcal{L}_R^{(3)} = \frac{i}{2}\, d_6^* \bar{R}\left[D^{\mu}, f_{\mu\nu}^{+}\right] D^{\nu} R + \text{h.c.} + 2\, i\, d_7^* \bar{R}\left(\partial^{\mu} v_{\mu\nu}^{(s)}\right) D^{\nu} R + \text{h.c.} + \cdots \,,$$

where

$$v_{\mu\nu}^{(s)} = \partial_\mu v_\nu^{(s)} - \partial_\nu v_\mu^{(s)} \,,$$

$$v_\mu^{(s)} = -\frac{e\, \mathcal{A}_\mu}{2} \,,$$

$$f_{\mu\nu}^{+} = u f_{\mu\nu} u^{\dagger} + u^{\dagger} f_{\mu\nu} u \,,$$

$$f_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu]$$

and c_6^*, c_7^*, d_6^* and d_7^* are unknown coupling constants.

Leading order interaction between the nucleon and the Roper:

$$\mathcal{L}_{NR}^{(1)} = \frac{g_{NR}}{2}\,\bar{R}\gamma^{\mu}\gamma_5 u_{\mu} N + \text{h.c.}$$

with an unknown coupling constant g_{NR}.

Leading-order interaction between the delta and the Roper:

$$\mathcal{L}_{\Delta R}^{(1)} = -g_{\Delta R}\,\bar{\Psi}_{\mu}\,\xi^{\frac{3}{2}}\,(g^{\mu\nu} + \tilde{z}\,\gamma^{\mu}\gamma^{\nu})\,u_{\nu}\,R + \text{h.c.}\,,$$

where $g_{\Delta R}$ is a coupling constant and we take $\tilde{z} = -1$.

We apply the complex-mass scheme (CMS):

R. G. Stuart, in Z^0 *Physics*, ed. J. Tran Thanh Van (Editions Frontiers, Gif-sur-Yvette, 1990), p.41.

A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, Nucl. Phys. **B560**, 33 (1999).

A. Denner, S. Dittmaier, M. Roth and L. H. Wieders, Nucl. Phys. **B724**, 247 (2005).

Generalization of the on-mass-shell scheme to unstable particles.

Well suited for unstable particles in perturbation theory.

Bare parameters of the Lagrangian are split into complex (in general) renormalized parameters and complex (in general) counterterms.

Renormalized masses are chosen as poles of dressed propagators in chiral limit:

$$
\begin{aligned}
m_{R0} &= z_\chi + \delta z_\chi \,, \\
m_{N0} &= m_\chi + \delta m \,, \\
m_{\Delta 0} &= z_{\Delta\chi} + \delta z_{\Delta\chi} \,.
\end{aligned}
\tag{1}
$$

z_χ - complex pole of the Roper propagator in the chiral limit.
m_χ - mass of the nucleon in the chiral limit.
$z_{\Delta\chi}$ - pole of the delta propagator in the chiral limit.

Renormalized parameters z_χ, m, and $z_{\Delta\chi}$ are included in the propagators and the counterterms are treated perturbatively.

Power counting:

Interaction vertex obtained from an $\mathcal{O}(q^n)$ Lagrangian $\sim q^n$,

Pion propagator $\sim q^{-2}$,

Nucleon propagator $\sim q^{-1}$,

Δ propagator $\sim q^{-1}$,

Roper propagator $\sim q^{-1}$,

Loop integration $\sim q^4$.

Within CMS, such a power counting is respected in the range of energies close to the Roper mass.

Dressed propagator of the Roper

$$iS_R(p) = \frac{i}{\not{p} - z_\chi - \Sigma_R(\not{p})}\,,$$

where $-i\Sigma_R(\not{p})$ denotes the self-energy of the Roper.

the pole of the dressed propagator S_R is obtained by solving

$$z - z_\chi - \Sigma_R(z) = 0\,. \qquad (2)$$

The pole mass and the width:

$$z = m_R - i\,\frac{\Gamma_R}{2}\,. \qquad (3)$$

Dressed propagator has a pole only on the second Riemann sheet.

$$\Sigma_R(z_\chi) = 0$$

on the second Riemann sheet.

$$\Sigma_R(z_\chi) \neq 0$$

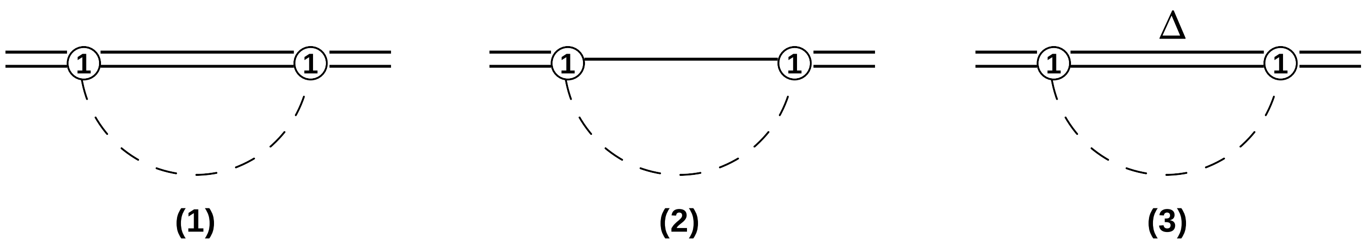
on the first Riemann sheet.

Roper propagator close to the pole

$$iS_R(p) = \frac{i\,Z}{\not{p} - z} + \text{n.p.}\,.$$

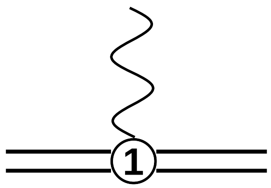
Physical quantities characterizing unstable particles have to be extracted at pole positions using the complex-valued Z.

Up to order $\mathcal{O}(q^3)$, Z is obtained by calculating the Roper self-energy diagrams shown in Figure.

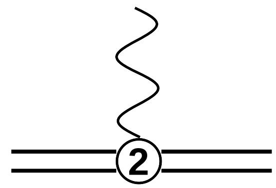


Diagrams contributing in Roper form factors

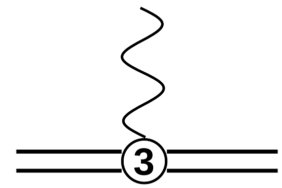
Tree diagrams:



(T1)

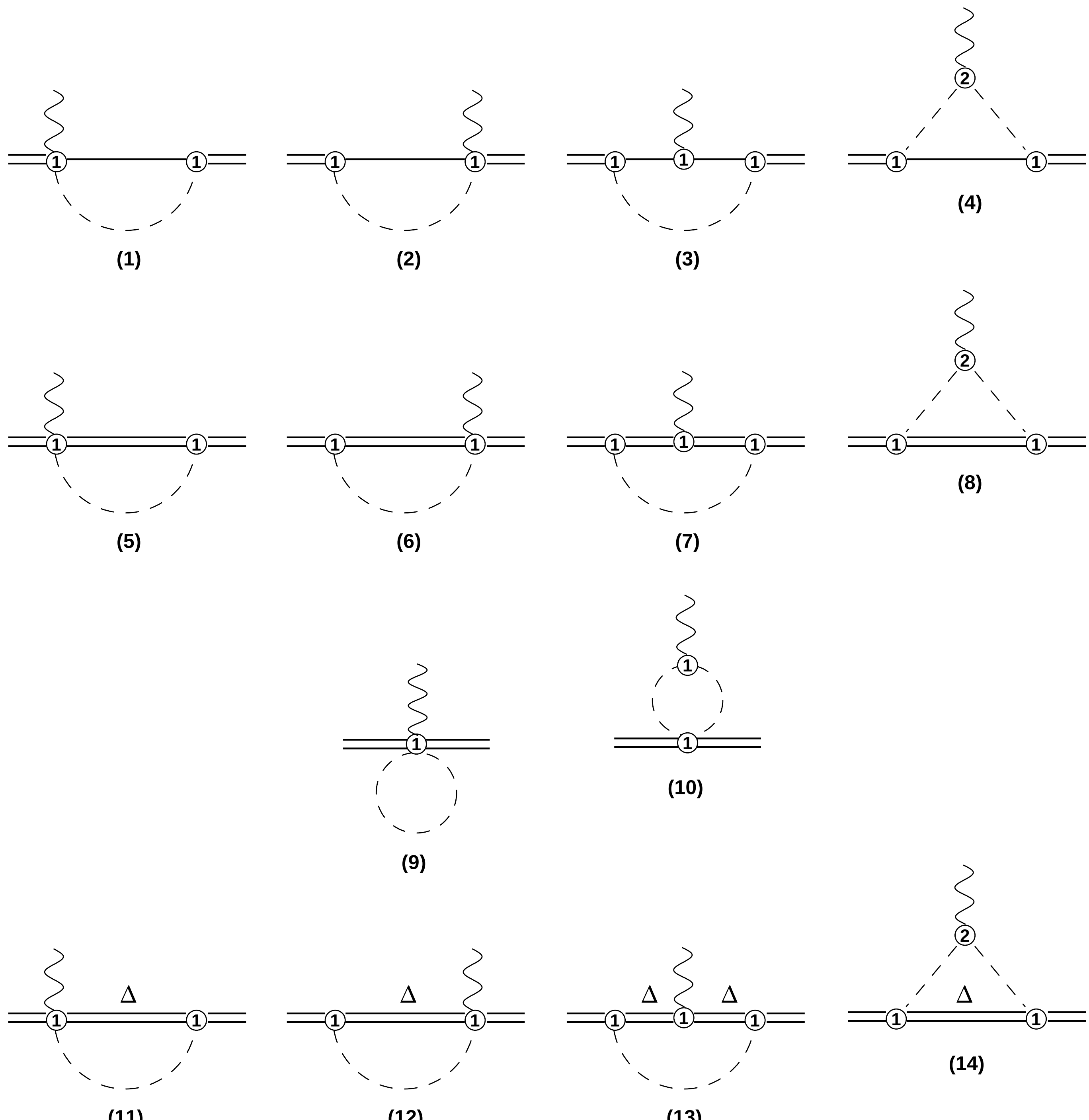


(T2)



(T3)

Loop Diagrams:



Parameterize the renormalized vertex function for $p_f^2 = p_i^2 = z^2$:

$$\sqrt{Z}\,\bar{w}^i(p_f)\Gamma^\mu(p_f, p_i)w^j(p_i)\sqrt{Z} = \bar{w}^i(p_f)\left[\gamma^\mu F_1(q^2) + \frac{i\,\sigma^{\mu\nu}\,q_\nu}{2\,m_N}F_2(q^2)\right]w^j(p_i).$$

$F_1(q^2)$ and $F_2(q^2)$ are complex valued functions.

$Z\times$ tree diagrams subtracts all power counting violating loop contributions in $F_1(q^2)$. We obtain $F_1(0) = (1+\tau_3)/2$.

Power counting violating loop contributions in magnetic form factor are absorbed in the renormalization of c_6^* and c_7^*.

Subtracted loop contributions satisfy the power counting.

Magnetic moment:

$$\mu_R = F_2(0).$$

$$F_2(t) = m_N[\,G_1(t) + \tau_3 G_2(t)].$$

Tree order result:

$$\begin{aligned} G_1^{\text{tree}}(0) &= m_N\, c_7^*\,, \\ G_2^{\text{tree}}(0) &= 2\, m_N\, c_6^*\,. \end{aligned}$$

Loop contributions:

$$\begin{aligned} G_1^{\text{loop}}(0) &= \frac{3g_R^2}{16F^2 z_\chi \left(M^2 - 4z_\chi^2\right)\pi^2}\Big\{[z_\chi^2 - A_0\left(z_\chi^2\right) \\ &- \left(M^2 - 3z_\chi^2\right) B_0\left(z_\chi^2, M^2, z_\chi^2\right)]M^2 + \left(M^2 - 2z_\chi^2\right) A_0\left(M^2\right)\cdots\Big\} \end{aligned}$$

$$\begin{aligned} G_2^{\text{loop}}(0) &= \frac{g_R^2}{16F^2 z_\chi \left(M^2 - 4z_\chi^2\right)\pi^2}\Big\{-A_0\left(z_\chi^2\right) M^2 + \left(3M^2 - 10z_\chi^2\right) A_0\left(M^2\right) \\ &+ z_\chi^2\left[M^2 - 2\left(M^2 - 4z_\chi^2\right) B_0\left(z_\chi^2, 0, z_\chi^2\right)\right] + \cdots\Big\}. \end{aligned}$$

Loop functions are given as

$$A_0\left(m^2\right) = \frac{(2\pi\,\mu)^{4-n}}{i\,\pi^2}\int\frac{d^nk}{k^2-m^2+i\epsilon} = -32\pi^2\lambda\,m^2 - 2\,m^2\ln\frac{m}{\mu}\,,$$

$$\begin{aligned} B_0\left(p^2,m_1^2,m_2^2\right) &= \frac{(2\pi\,\mu)^{4-n}}{i\,\pi^2}\int\frac{d^nk}{\left[k^2-m_1^2+i\epsilon\right]\left[(p+k)^2-m_2^2+i\epsilon\right]} \\ &= -32\pi^2\lambda + 2\ln\frac{\mu}{m_2} - 1 - \frac{\omega}{2}\;{}_2F_1\left(1,2;3;\omega\right) \\ &\quad -\frac{1}{2}\left(1+\frac{m_2^2}{m_1^2(\omega-1)}\right)\;{}_2F_1\left(1,2;3;1+\frac{m_2^2}{m_1^2(\omega-1)}\right)\,, \\ \omega &= \frac{m_1^2-m_2^2+p^2+\sqrt{\left(m_1^2-m_2^2+p^2\right)^2-4m_1^2p^2}}{2m_1^2}\,, \end{aligned}$$

where ${}_2F_1\left(a,b;c;z\right)$ is the standard hypergeometric function, μ is the scale parameter of the dimensional regularization and

$$\lambda = \frac{1}{16\,\pi^2}\left\{\frac{1}{n-4} - \frac{1}{2}\left[\ln(4\pi) + \Gamma'(1) + 1\right]\right\}\,.$$

Vector form factor of the pion

Effective Lagrangian relevant for the pion form factor calculation (up to higher order terms)

$$\begin{aligned}
\mathcal{L} &= \frac{F^2}{4}\mathrm{Tr}\left[D_\mu U\left(D^\mu U\right)^\dagger\right] + \frac{F^2}{4}\mathrm{Tr}\left[\chi U^\dagger + U\chi^\dagger\right] \\
&+ \frac{M^2 + c_x \mathrm{Tr}\left[\chi_+\right]/4}{g^2}\mathrm{Tr}\left[\left(g\rho^\mu - i\Gamma^\mu\right)\left(g\rho_\mu - i\Gamma_\mu\right)\right] \\
&- \frac{1}{2}\mathrm{Tr}\left[\rho_{\mu\nu}\rho^{\mu\nu}\right] + i\, d_x \mathrm{Tr}\left[\rho_{\mu\nu}\Gamma^{\mu\nu}\right] \\
&- \frac{\sqrt{2}}{2} f_V \mathrm{Tr}\left\{\rho_{\mu\nu} f_+^{\mu\nu}\right\}, \qquad (4)
\end{aligned}$$

Here

$$
\begin{aligned}
U(x) &= u^2(x) = \exp\left(\frac{i\Phi(x)}{F}\right), \\
D_\mu A &= \partial_\mu A - i r_\mu A + i A l_\mu \,, \\
\chi_+ &= M^2(U^\dagger + U)\,, \\
\Gamma_\mu &= \frac{1}{2}\left[u^\dagger\partial_\mu u + u\partial_\mu u^\dagger - i\,\left(u^\dagger r_\mu u + u l_\mu u^\dagger\right)\right], \\
\rho^{\mu\nu} &= \partial^\mu\rho^\nu - \partial^\nu\rho^\mu - ig\,[\rho^\mu, \rho^\nu]\,, \\
\Gamma_{\mu\nu} &= \partial_\mu\Gamma_\nu - \partial_\nu\Gamma_\mu + [\Gamma_\mu, \Gamma_\nu], \\
f_\pm^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u\,, \\
r_\mu &= v_\mu + a_\mu\,, \qquad l_\mu = v_\mu - a_\mu\,,
\end{aligned}
$$

v_μ and a_μ are external vector and axial vector fields.

Renormalization using CMS.

Power counting rules:

Pion propagator $\sim \mathcal{O}(q^{-2})$ if it does not carry large external momenta and $\sim \mathcal{O}(q^0)$ if it does.

Vector-meson propagator $\sim \mathcal{O}(q^0)$ without large external momenta and $\sim \mathcal{O}(q^{-1})$ - otherwise.

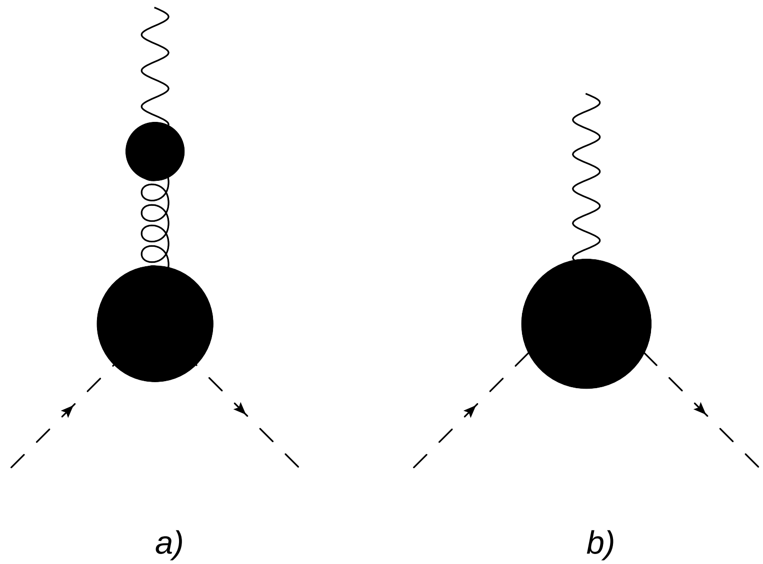
Pion mass $\sim \mathcal{O}(q^1)$, ρ-meson mass $\sim \mathcal{O}(q^0)$, and the width $\sim \mathcal{O}(q^1)$.

Vertices generated by $\mathcal{L}_\pi^{(n)}$ count $\sim \mathcal{O}(q^n)$.

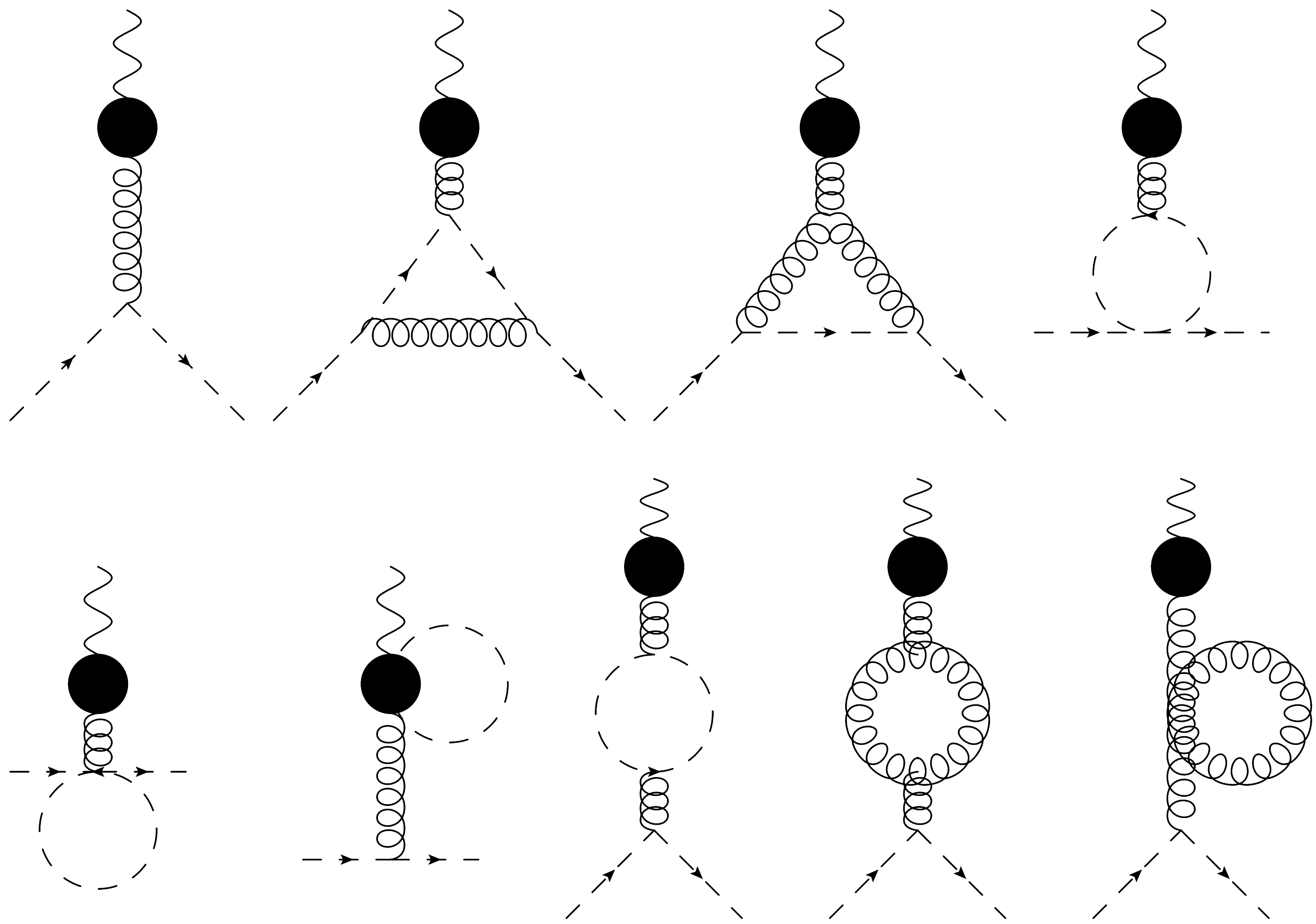
Derivatives acting on heavy vector mesons $\sim \mathcal{O}(q^0)$.

Investigate all possible flows of external momenta through loop diagrams and determine the chiral order for each flow.

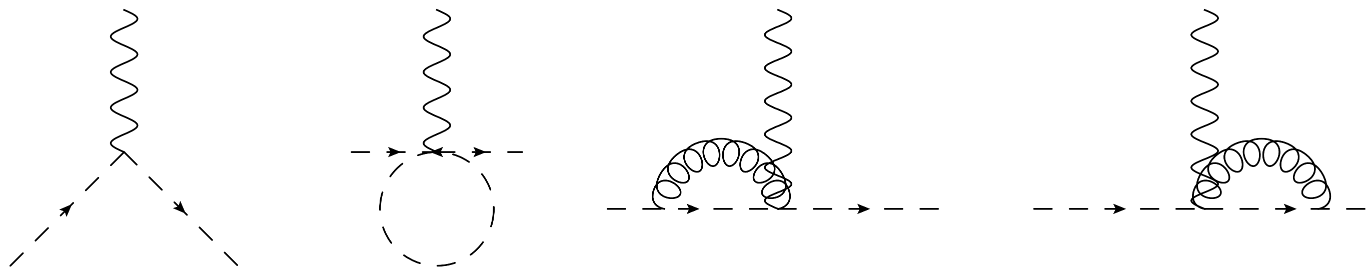
Smallest order resulting from various assignments = chiral order of the diagram.



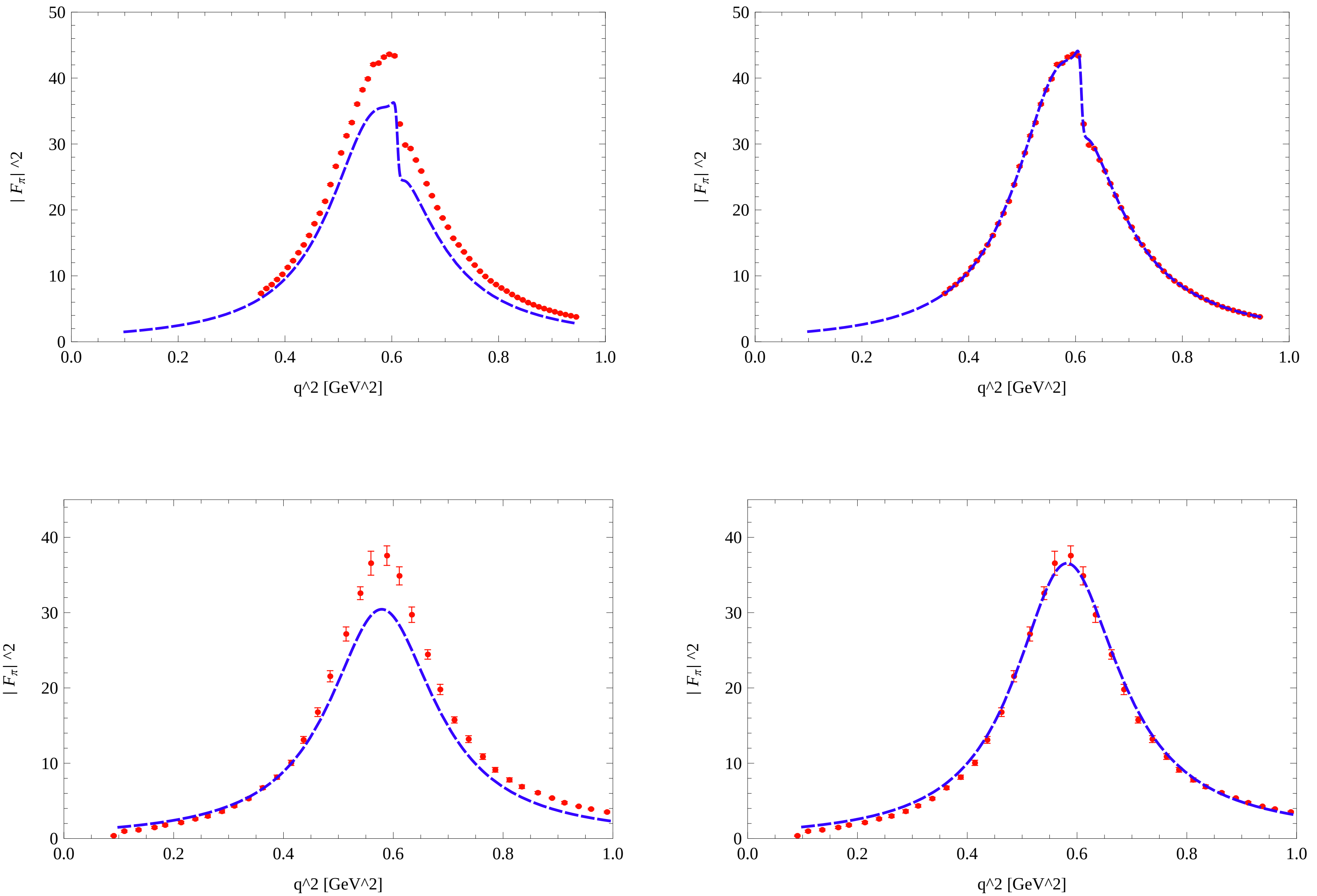
Diagrams contributing to the pion form factor.



Diagrams of group a).



Diagrams of group b).



Pion form factor: Tree order - left; Tree + loop - right; Data - red dots;

Data:

Kloe Collab., Phys. Lett. **B 670** (2009);

S. Schael *et al.* [ALEPH Collaboration], Phys. Rept. **421**, 191 (2005).

Some aspects of complex-mass scheme

CMS leads to complex-valued renormalized parameters.

Issue of perturbative unitarity of the S-matrix in the CMS is still open.

Lagrangian does not change $\longrightarrow$ Unitarity is not violated in exact theory.

Perturbation theory is based on order-by-order approximation to the exact results $\longrightarrow$ Not obvious that the approximate expressions to the S-matrix satisfy unitarity.

Branching points are determined by poles of full propagators.
Branching points corresponding to unstable particles are complex.
Cuts can be placed arbitrarily.

Sometimes branching points, corresponding to unstable states, are placed on real axis.
If poles are not very far from the real axis this is a reasonable approximation.

CMS places the poles and branching points at exact (complex) positions already at the leading order.

Imaginary parts of loop integrals with complex masses

Standard causal propagator of a scalar particle

$$S(p) \;=\; \frac{1}{p^2 - m^2 + i\,\epsilon} = \frac{p^2 - m^2}{\left(p^2 - m^2\right)^2 + \epsilon^2} - i\,\pi\,\delta\left(p^2 - m^2\right)\;.$$

$\epsilon \to 0$ limit is assumed.

Corresponding advanced and retarded propagators:

$$
\begin{aligned}
S_A(p) &= \frac{1}{2\,E_p}\left[\frac{1}{p_0 - E_p - i\,\epsilon} - \frac{1}{p_0 + E_p - i\,\epsilon}\right] \\
&= \frac{p^2 - m^2}{\left(p^2 - m^2\right)^2 + \epsilon^2} + i\,\pi\,\delta\left(p^2 - m^2\right)\sigma_p, \\
S_R(p) &= \frac{1}{2\,E_p}\left[\frac{1}{p_0 - E_p + i\,\epsilon} - \frac{1}{p_0 + E_p + i\,\epsilon}\right] \\
&= \frac{p^2 - m^2}{\left(p^2 - m^2\right)^2 + \epsilon^2} - i\,\pi\,\delta\left(p^2 - m^2\right)\sigma_p,
\end{aligned}
$$

where $\sigma_p = \mathrm{sign}(p_0)$ and $E_p = \sqrt{\vec{p}^{\,2} + m^2}$.

Propagator of an unstable scalar particle in CMS

$$S'(p) = \frac{1}{p^2 - M^2 + i\,(M\Gamma + \epsilon)}$$
$$= \frac{p^2 - M^2}{\left(p^2 - M^2\right)^2 + (M\,\Gamma + \epsilon)^2} - \frac{i\,(M\Gamma + \epsilon)}{\left(p^2 - M^2\right)^2 + (M\Gamma + \epsilon)^2}\,.$$

ϵ can be neglected for finite Γ, but important for $\Gamma \to 0$.

"Advanced" and "retarded" propagators:

$$
\begin{aligned}
S'_A(p) &= \frac{1}{w(p)+w(p)^*}\left[\frac{1}{p_0-w(p)^*}-\frac{1}{p_0+w(p)}\right], \\
&= \frac{p^2-M^2-M^2\Gamma^2/(2\,x(p)^2)}{\left(p^2-M^2\right)^2+M^2\Gamma^2}+\frac{i\,M\,\Gamma}{\left(p^2-M^2\right)^2+M^2\Gamma^2}\,\frac{p_0}{x(p)}, \\
S'_R(p) &= \frac{1}{w(p)+w(p)^*}\left[\frac{1}{p_0-w(p)}-\frac{1}{p_0+w(p)^*}\right] \\
&= \frac{p^2-M^2-M^2\Gamma^2/(2\,x(p)^2)}{\left(p^2-M^2\right)^2+M^2\Gamma^2}-\frac{i\,M\,\Gamma}{\left(p^2-M^2\right)^2+M^2\Gamma^2}\,\frac{p_0}{x(p)}, \\
w(p) &= x(p)-i\,y(p)\,, \\
x(p) &= \frac{1}{\sqrt{2}}\sqrt{\left(\mathcal{E}_p^4+M^2\Gamma^2\right)^{1/2}+\mathcal{E}_p^2}=\mathcal{E}_p+\mathcal{O}(\Gamma^2)\,, \\
y(p) &= \frac{1}{\sqrt{2}}\sqrt{\left(\mathcal{E}_p^4+M^2\Gamma^2\right)^{1/2}-\mathcal{E}_p^2}=M\,\Gamma/(2\,\mathcal{E}_p)+\mathcal{O}(\Gamma^3)\,, \\
\mathcal{E}_p &= \sqrt{\vec{p}^{\;2}+M^2}\,.
\end{aligned}
$$

Consider one-loop integral

$$I_1 = i \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} (2\pi)^4 \delta^4(k+q-p) S(k) S'(q). \tag{5}$$

Imaginary part of this integral is given by

$$\begin{aligned} \mathrm{Im}[I_1] &= \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} (2\pi)^4 \delta^4(k+q-p) \\ &\times \Big[\frac{k^2 - m^2}{\left(k^2 - m^2\right)^2 + \epsilon^2} \frac{q^2 - M^2}{\left(q^2 - M^2\right)^2 + M^2 \Gamma^2} \\ &- \pi \, \delta\left(k^2 - m^2\right) \frac{M\,\Gamma}{\left(q^2 - M^2\right)^2 + M^2\Gamma^2} \Big]. \end{aligned} \tag{6}$$

To rewrite the first term consider the equality

$$0 = i \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} (2\pi)^4 \delta^4(k+q-p) S_A(k) S'_R(q) \tag{7}$$

and take the imaginary parts of both sides

$$
\begin{aligned}
0 &= \int \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}(2\pi)^4\delta^4(k+q-p) \\
&\times \Big[\frac{k^2-m^2}{\left(k^2-m^2\right)^2+\epsilon^2}\,\frac{q^2-M^2}{\left(q^2-M^2\right)^2+M^2\Gamma^2} \\
&+ \pi\,\delta\left(k^2-m^2\right)\,\sigma_k\,\frac{q_0}{\mathcal{E}_q}\,\frac{M\,\Gamma}{\left(q^2-M^2\right)^2+M^2\Gamma^2}+\mathcal{O}(\Gamma^2)\Big]\,. \qquad (8)
\end{aligned}
$$

Subtracting Eq. (8) from Eq. (6) obtain

$$
\begin{aligned}
\mathrm{Im}[I_1] &= -\pi\int\frac{d^4k}{(2\pi)^4}\,\delta\left(k^2-m^2\right)\frac{(M\,\Gamma+\epsilon)\,\left[1+\sigma_{p-k}\,\sigma_k\right]}{\left[(p-k)^2-M^2\right]^2+(M\,\Gamma+\epsilon)^2}+\mathcal{O}(\Gamma^2), \\
{}_{\Gamma\to 0} &\Rightarrow -\pi\int\frac{d^4k}{(2\pi)^4}\,\delta\left(k^2-m^2\right)\delta((p-k)^2-M^2)\left[1+\sigma_{p-k}\,\sigma_k\right],
\end{aligned}
$$

where the second line corresponds to the standard cutting formula for loop integrals with real masses.

Generalization to any one-loop integral with complex masses leads to:

- For each cut of the line of an unstable particle one obtains one overall factor of Γ.
- For each cut of the line of the stable particle one gets a delta-function.
- For the integrals containing only propagators of stable particles the usual cutting rules apply.

The Model

Consider a model of an unstable vector boson interacting with a fermion

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}\, F_{0\mu\nu}F_0^{\mu\nu} + \frac{M_0^2}{2}\, B_{0\mu}B_0^{\mu} \\ &+ \bar{\psi}_0\,(i\partial\!\!\!/ - m_0)\,\psi_0 + g_0\,\bar{\psi}_0\,\gamma_\mu\psi_0\,B_0^{\mu},\end{aligned}$$

where $F^0_{\mu\nu} = \partial_\mu B^0_\nu - \partial_\nu B^0_\mu$.

Vector boson decays into a fermion-antifermion pair.

Renormalization in two steps: first get rid off the divergences by applying the dimensional regularization with the $\overline{MS}$ scheme.

Next express the renormalized masses of $\overline{MS}$ scheme in terms of poles of the dressed propagators.

Substitutions into the Lagrangian

$$
\begin{aligned}
B_0^\mu &\to B^\mu, \qquad\qquad \psi_0 \to \psi, \\
m_0 &\to m + \delta m, \qquad g_0 \to g, \\
M_0^2 &\to M^2 - i\, M\Gamma + \delta z \equiv z + \delta z\,, \qquad (9)
\end{aligned}
$$

result in

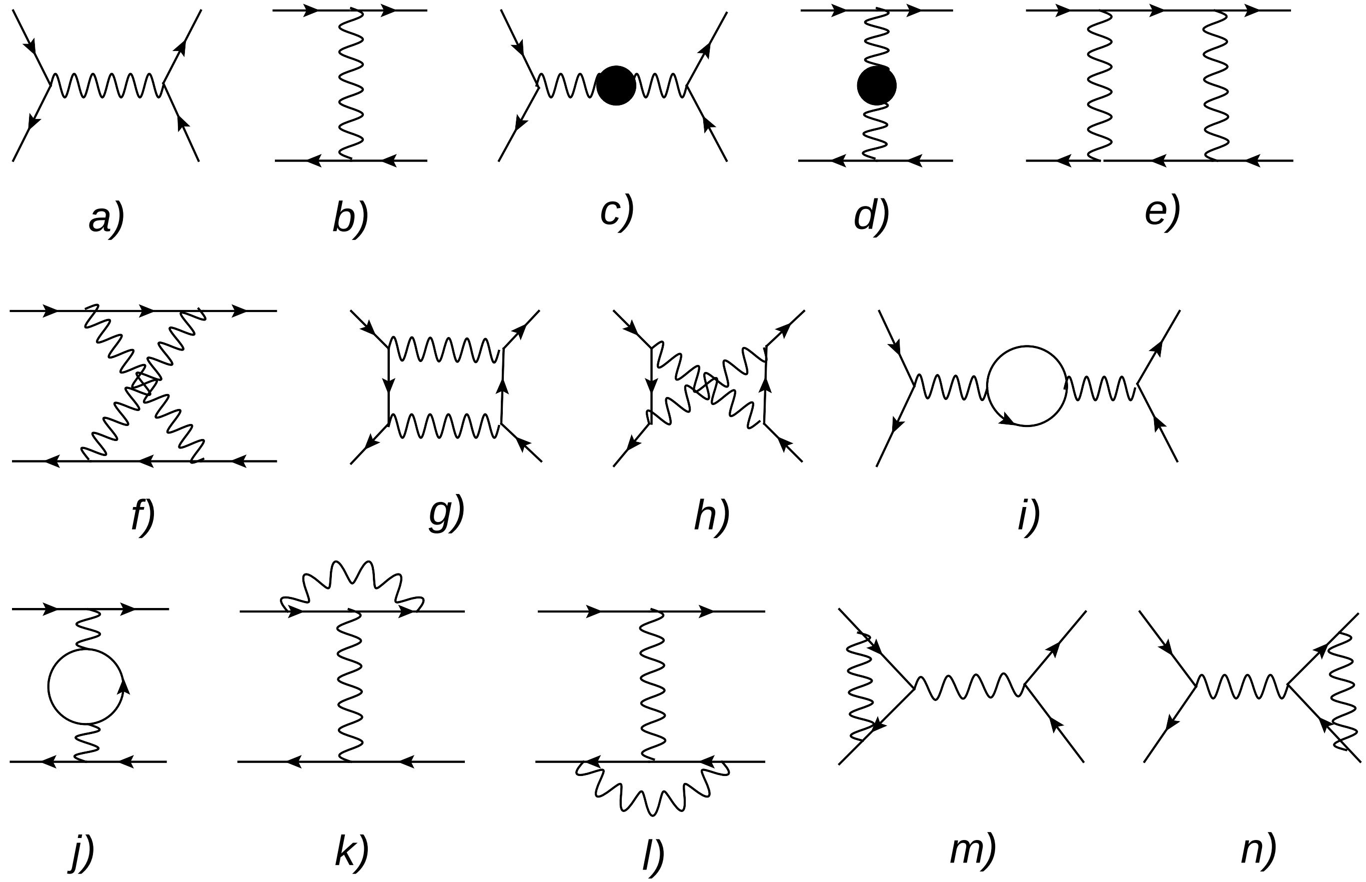
$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_{\text{main}} + \mathcal{L}_{\text{ct}}\,, \\
\mathcal{L}_{\text{main}} &= -\frac{1}{4}\, F_{\mu\nu}F^{\mu\nu} + \frac{z}{2}\, B_\mu B^\mu + \bar{\psi}\,(i\not{\partial} - m)\,\psi + g\,\bar{\psi}\,\gamma_\mu \psi\, B^\mu, \\
\mathcal{L}_{\text{ct}} &= \frac{\delta z}{2}\, B_\mu B^\mu - \delta m\, \bar{\psi}\,\psi\,.
\end{aligned}
$$

Counter term Lagrangian $\mathcal{L}_{\text{ct}}$ is treated perturbatively and the propagators read

$$
\begin{aligned}
i\, S_F(p) &= \frac{i}{\not{p} - m + i\,\epsilon}\,, \\
i\, S_{\mu\nu}(p) &= -i\, \frac{g_{\mu\nu} - p_\mu p_\nu / z}{p^2 - z}\,. \qquad (10)
\end{aligned}
$$

Perturbative unitarity of the S-matrix

Diagrams contributing in the $\bar{f}f \to \bar{f}f$ amplitude:

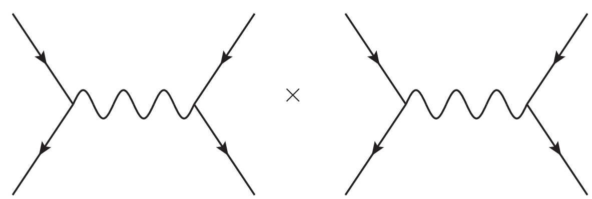


Imaginary part of the tree-order amplitude shown in Fig. a)

$$\text{Im}\,[T_a] \;=\; \text{Im}\left[V^{\mu}\,\frac{s-z^*}{(s-z)(s-z^*)}\,V_{\mu}\right] = -V^{\mu}\,\frac{M\,\Gamma}{(s-z)(s-z^*)}\,V_{\mu}\,. \quad (11)$$

Imaginary parts of one-loop diagrams c) and i)

$$\text{Im}\,[T_i+T_c] = V^{\mu}\,\frac{\text{Im}\,[\Pi(s)+\delta z_1]}{(s-z)(s-z^*)}\,V_{\mu} + \mathcal{O}\left(\Gamma^3\right)\,. \quad (12)$$



The "square" of the tree-order amplitude above reads

$$TT^{\dagger} \;=\; 2\,V^{\mu}\,\frac{\text{Im}\,[\Pi(s)]}{(s-z)(s-z^*)}\,V_{\mu}\,. \quad (13)$$

From Eqs. (11), (12) and (13), using $\text{Im}[\delta z_1] = M\,\Gamma + \mathcal{O}(\hbar^2)$, follows that unitarity is satisfied up to $\mathcal{O}\left(\Gamma^3\right)$.

Summary

- Magnetic moment of the Roper resonance in the framework of the low-energy EFT of QCD.

- Vector form factor of the pion.

- Perturbative unitarity of the scattering amplitude within CMS for one-loop diagrams.

- Result obtained under assumption that the expansion parameter remains real.

- Unstable particles do not appear as asymptotic states.

- Generalization of cutting rules to multi-loop diagrams is straightforward, analysis of perturbative unitarity - more involved.