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and the Interpretation of Baryon Resonances**

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APPLICATIONS OF THE $1/N_c$ EXPANSION TO BARYONS

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Outline

- Spin-flavor symmetry in baryons
- $1/N_c$ expansion
- Excited baryons: $SU(6) \times O(3)$ scheme
- Some general results
- Partial widths of the $[70, 1^-]$
- Configuration mixings
- Summary

SU(6)xO(3)

Multiplet	Baryon	Name, status	Exp.(MeV)
[56, 0 ⁺]	$N_{1/2}$	N * * * *	939 ± 1
	$\Lambda_{1/2}$	Λ * * * *	1116 ± 1
	${}^8\Sigma_{1/2}$	Σ * * * *	1192 ± 4
	${}^8\Xi_{1/2}$	Ξ * * * *	1318 ± 3
	$\Delta_{1/2}$	Δ * * * *	1232 ± 1
	${}^{10}\Sigma_{3/2}$	Σ^* * * * *	1383 ± 3
	${}^{10}\Xi_{3/2}$	Ξ^* * * * *	1532 ± 1
	$\Omega_{3/2}$	Ω^- * * * *	1672 ± 2
[56, 0 ⁺]	$N_{1/2}$	$N(1440)$ * * * *	1440 ± 20
	$\Delta_{3/2}$	$\Delta(1600)$ * **	1600 ± 75
	$\Lambda_{1/2}$	$\Lambda(1600)$ * **	1600 ± 75
	$\Sigma_{1/2}$	$\Sigma(1660)$ * **	1660 ± 30
[56, 2 ⁺]	$N_{3/2}$	$N(1720)$ * * * *	1700 ± 50
	$\Lambda_{3/2}$	$\Lambda(1890)$ * * * *	1880 ± 30
	$N_{5/2}$	$N(1680)$ * * * *	1683 ± 8
	$\Lambda_{5/2}$	$\Lambda(1820)$ * * * *	1820 ± 5
	${}^8\Sigma_{5/2}$	$\Sigma(1915)$ * * * *	1918 ± 18
	$\Delta_{1/2}$	$\Delta(1910)$ * * * *	1895 ± 25
	$\Delta_{3/2}$	$\Delta(1920)$ * **	1935 ± 35
	$\Delta_{5/2}$	$\Delta(1905)$ * * * *	1895 ± 25
	$\Delta_{7/2}$	$\Delta(1950)$ * * * *	1950 ± 10
	${}^{10}\Sigma_{7/2}$	$\Sigma(2030)$ * * * *	2033 ± 8
[56, 4 ⁺]	$N_{9/2}$	$N(2220)$ * * * *	2245 ± 65
	$\Lambda_{9/2}$	$\Lambda(2350)$ * **	2355 ± 15
	$\Delta_{7/2}$	$\Delta(2390)$ *	2387 ± 88
	$\Delta_{9/2}$	$\Delta(2300)$ *	2318 ± 132
	$\Delta_{11/2}$	$\Delta(2420)$ *	2400 ± 100
[56, 6 ⁺]	$N_{13/2}$	$N(2700)$ * *	2806 ± 207
	$\Delta_{15/2}$	$\Delta(2950)$ * *	2920 ± 122

Multiplet	Baryon	Name, status	Exp. (MeV)
[70, 1 ⁻]	$N_{1/2}$	$N(1535)$ ****	1538 ± 18
	${}^8\Lambda_{1/2}$	$\Lambda(1670)$ ****	1670 ± 10
	$N_{3/2}$	$N(1520)$ ****	1523 ± 8
	${}^8\Lambda_{3/2}$	$\Lambda(1690)$ ****	1690 ± 5
	${}^8\Sigma_{3/2}$	$\Sigma(1670)$ ****	1675 ± 10
	${}^8\Xi_{3/2}$	$\Xi(1820)$ ***	1823 ± 5
	$N'_{1/2}$	$N(1650)$ ****	1660 ± 20
	${}^8\Lambda'_{1/2}$	$\Lambda(1800)$ ***	1785 ± 65
	${}^8\Sigma'_{1/2}$	$\Sigma(1750)$ ***	1765 ± 35
	$N'_{3/2}$	$N(1700)$ ***	1700 ± 50
	$N'_{5/2}$	$N(1675)$ ****	1678 ± 8
	${}^8\Lambda'_{5/2}$	$\Lambda(1830)$ ****	1820 ± 10
	${}^8\Sigma'_{5/2}$	$\Sigma(1775)$ ****	1775 ± 5
	$\Delta_{1/2}$	$\Delta(1620)$ ****	1645 ± 30
	$\Delta_{3/2}$	$\Delta(1700)$ ****	1720 ± 50
	${}^1\Lambda_{1/2}$	$\Lambda(1405)$ ****	1407 ± 4
	${}^1\Lambda_{3/2}$	$\Lambda(1520)$ ****	1520 ± 1
[70, 2 ⁺]	$N'_{1/2}$	$N(2100)$ *	1926 ± 26
	$N'_{5/2}$	$N(2000)$ **	1981 ± 200
	$\Lambda'_{5/2}$	$\Lambda(2110)$ ***	2112 ± 40
	$N'_{7/2}$	$N(1990)$ **	2016 ± 104
	$\Lambda'_{7/2}$	$\Lambda(2020)$ *	2094 ± 78
	$\Delta_{5/2}$	$\Delta(2000)$ **	1976 ± 237
[70, 3 ⁻]	$N_{5/2}$	$N(2200)$ **	2057 ± 180
	$N_{7/2}$	$N(2190)$ ****	2160 ± 49
	$N'_{9/2}$	$N(2250)$ ****	2239 ± 76
	$\Delta_{7/2}$	$\Delta(2200)$ *	2232 ± 87
	${}^1\Lambda_{7/2}$	$\Lambda(2100)$ ****	2100 ± 20
[70, 5 ⁻]	$N_{11/2}$	$N(2600)$ ***	2638 ± 97

Global symmetries of QCD

$$U_B(1) \times SU_L(3) \times SU_L(3) \times \text{Poincare}$$

SU(6) - dynamical symmetry only for baryons

Must be non-relativistic at baryonic level

What can explain it in QCD?

QCD expansion parameters

$$m_{u,d,s} \quad 1/N_c$$

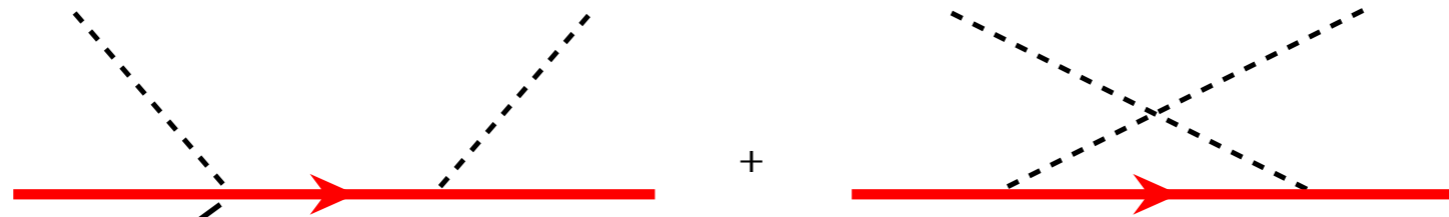
$$1/N_c \text{ expansion } SU_c(3) \longrightarrow SU_c(N_c)$$

Provides additional way of ordering non-perturbative
QCD

Scalings in N_c

	Mesons	Baryons
Mass	N_c^0	N_c
Γ	$1/N_c$	N_c^0
coupling to pions	$N_c^{-n_\pi/2}$	$N_c^{(1-n_\pi)/2}$

SU(6) from consistency: Gervais & Sakita; Dashen & Manohar



$$g_A \frac{N_c}{F_\pi} \partial_i \pi^a X^{ia} = \mathcal{O}(\sqrt{N_c})$$

$$\propto \frac{N_c^2}{F_\pi^2} \left\{ \langle B' | X^{jb} X^{ia} | B \rangle \frac{i}{k_0} - \langle B' | X^{ia} X^{jb} | B \rangle \frac{i}{k'_0} \right\}$$

(since $k_0 = k'_0 + \mathcal{O}(1/N_c)$:)

$$= \frac{N_c^2}{F_\pi^2} \frac{i}{k_0} (\langle B' | [X^{jb}, X^{ia}] | B \rangle + \mathcal{O}(1/N_c^2))$$

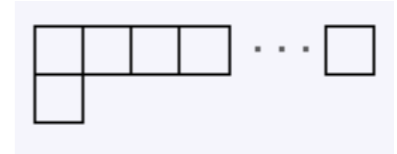
$$\{T^a, S^i, G^{ia} = N_c X^{ia}\}$$

SU(6)

$$\langle B' | [X^{jb}, X^{ia}] | B \rangle = \mathcal{O}(1/N_c)$$

Implementing the $1/N_c$ expansion

Baryons in multiplets of $SU(6)$

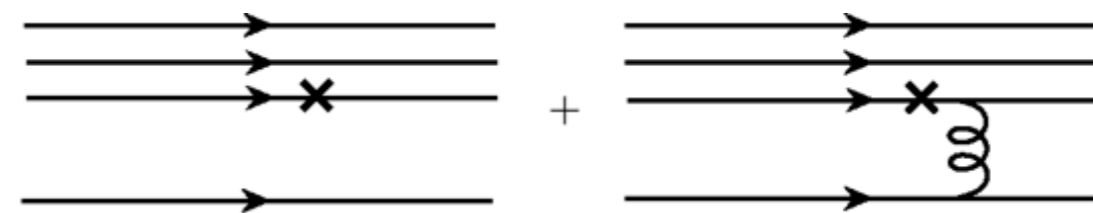


Effective operators:

$$\hat{Q} = \bar{q} \Gamma q$$

$$\hat{Q} = \sum_{n,i} C_{ni} Q_{ni}$$

$1/N_c$ power counting



Q_{ni} n-body operators

$$\left(\frac{1}{N_c}\right)^{n-1-\kappa}$$

1-body

	1	S^i	T^a	G^{ia}
$\kappa :$	1	0	0	1

2-body

	$\frac{1}{N_c} S^i S^j$	$\frac{1}{N_c} S^i G^{ja}$	$\frac{1}{N_c} T^a T^b$	$\frac{1}{N_c} G^{ia} T^b$	$\frac{1}{N_c} G^{ia} G^{jb}$
$\kappa :$	0	1	0	1	2

Effective operators built with tensor products of generators

Effective coefficients

$C_{n,i}(\bar{m}_q, q)$ contain the unknown QCD dynamics

$$C_{n,i} = \sum_{\nu=0} C_{n,i}(\nu) \left(\frac{1}{N_c} \right)^\nu$$

Masses of GS baryons

$$M_{GS} = c_1 N_c + \frac{c_{HF}}{N_c} (S^2 - \frac{3}{4} N_c) - c_S \frac{m_s - m_{u,d}}{\Lambda} S + \mathcal{O}(1/N_c^2; m_s/N_c)$$

Gursey-Radicati

Mass relation among 8 and 10

	$\Sigma - \Lambda = \mathcal{O}(m_s/N_c)$	74 MeV
GMO	$\Xi_8 - \Sigma_8 = \frac{1}{2}(3\Lambda - \Sigma_8) - N$	128 vs 141 MeV
ES	$\Sigma_{10} - \Delta = \Xi_{10} - \Sigma_{10}$	153 vs 145
"	$\Omega^- - \Xi_{10} = \Xi_{10} - \Sigma_{10}$	142 vs 145
8-10	$\Sigma_{10} - \Sigma_8 = \Xi_{10} - \Xi_8$	212 vs 195

Excited baryons

Need to extend to $SU(6) \times O(3)$

Not a consequence of large N_c but of phenomenology

$$[56, 0^+], [56, 2^+], [70, 1^-], \textit{etc}$$

Masses

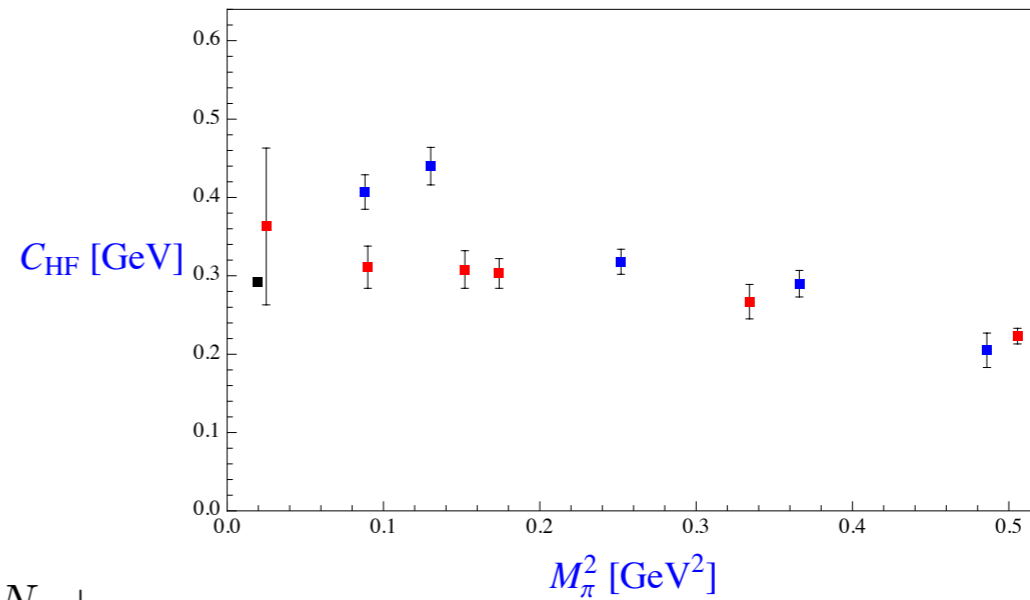
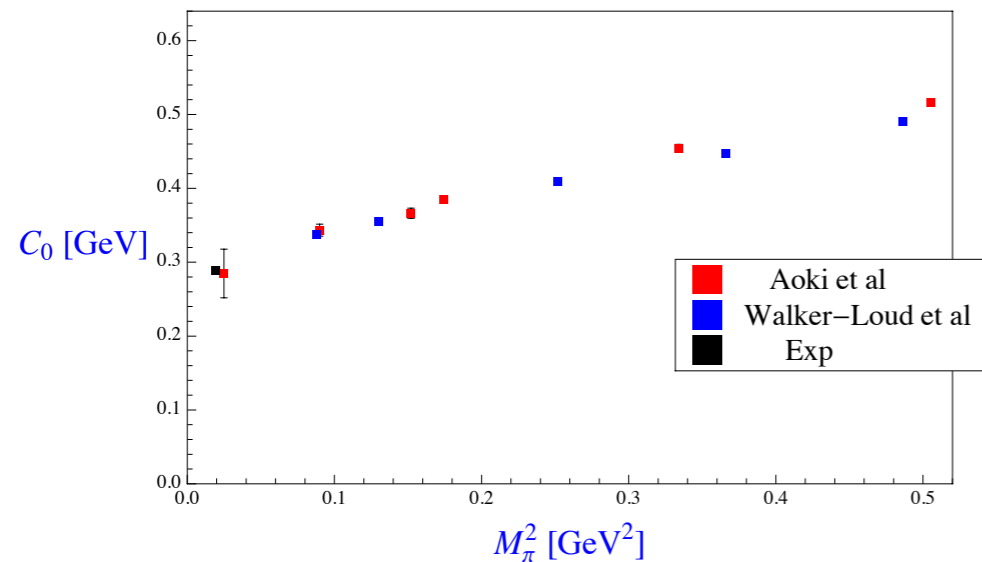
$$M = N_c M_0 + M_1 + \frac{1}{N_c} M_2 + \frac{m_s - m_{u,d}}{\Lambda} M_{SB} + \dots$$

From mass analyses: BW masses from PDG

- Subleading order of natural size (or smaller)
- Hyperfine effects most important for $SU(6)$ breaking
- Numerous mass relations well satisfied in $[56, 2^+]$

Observations on masses

Quark mass dependencies of GS mass coefficients



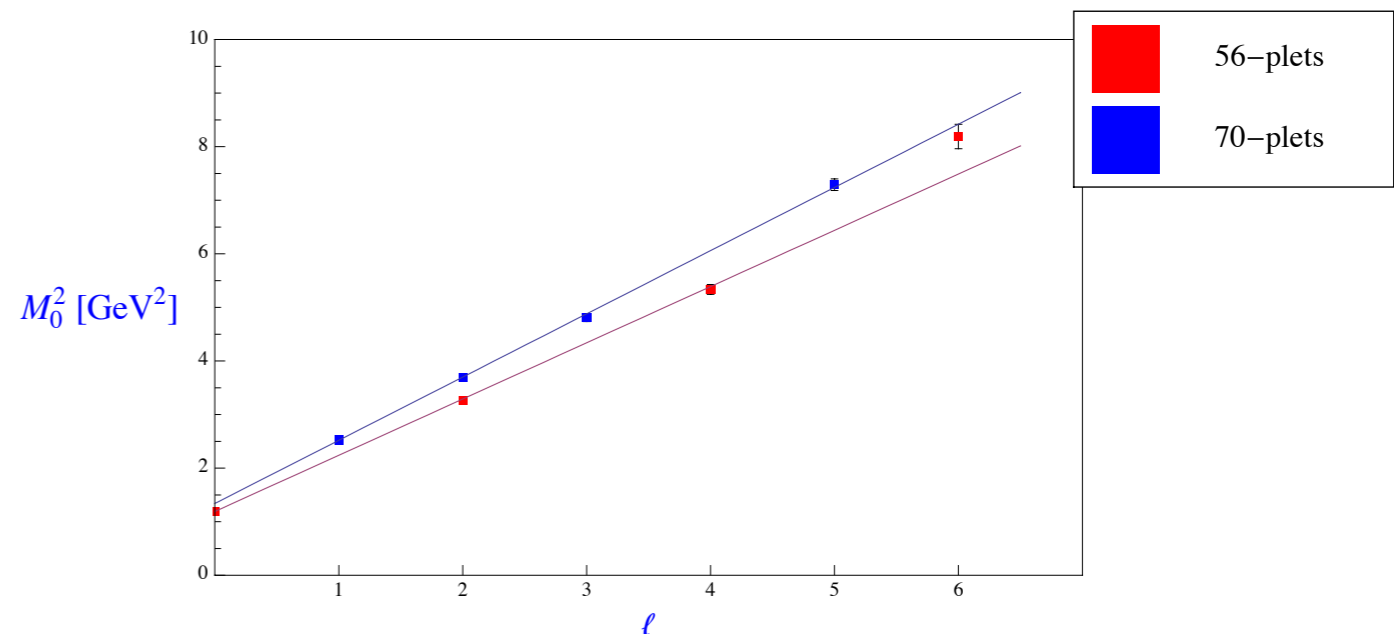
$$\sigma = \frac{\partial m_N}{\partial m_q} = \sigma_0 + \sigma_1/N_c + \dots$$

$$\sigma_0 = 43 \pm 0.2 \text{ MeV} \quad (\text{Aoki et al})$$

$$55 \pm 0.25 \text{ MeV} \quad (\text{Walker - Loud et al})$$

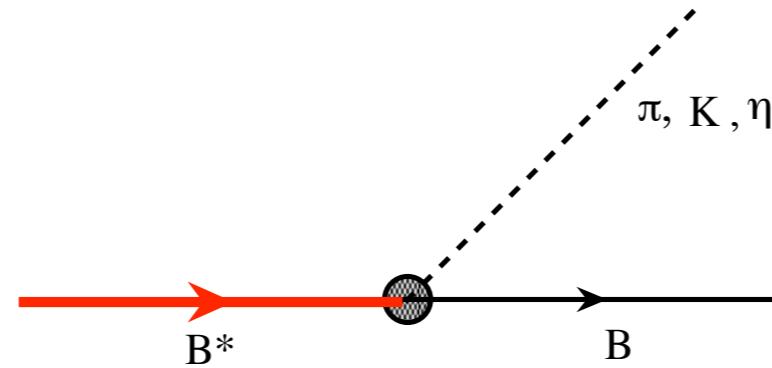
Linear trajectories for LO mass term

JLG & Matagne



Strong decays of 70-plet

JLG, Schat, Scoccola; JLG, Jayalath, Gonzalez, Scoccola



$$\mathcal{M}(\ell_\pi(Y_\pi, I_\pi), B, B^*) = (-1)^{\ell_\pi} \sqrt{2M_{B^*}} \frac{\sqrt{N_c}}{F_\pi} \langle B_{GS} | \mathbf{B}^{\ell_\pi(Y_\pi, I_\pi)} | B^* \rangle$$

$$\mathbf{B}^{\ell_\pi, (Y_\pi, I_\pi)} = \left(\frac{k_\pi}{\Lambda} \right)^{\ell_\pi} \sum_n C_n O_n^{\ell_\pi(Y_\pi, I_\pi)}$$

$$O_n^{\ell_\pi(Y_\pi, I_\pi)} = [\xi^\ell \mathcal{G}_n^{j_n(Y_\pi, I_\pi)}]^{(\ell_\pi(Y_\pi, I_\pi))}$$

PDG Name	State	Mass [MeV]	Γ_T [MeV]	BR %	
				S – wave	D – wave
$N(1535)$	$N_{1/2}$	1535(10)	150(25)	$N\pi : 45(10)$ $N\eta : 52.5(7.5)$	$\Delta\pi < 1$
$N(1520)$	$N_{3/2}$	1520(5)	113(12.5)	$\Delta\pi : 8.5(3.5)$	$N\pi : 60(5)$ $\Delta\pi : 12(2)$
$N(1650)$	$N'_{1/2}$	1657(13)	165(20)	$N\pi : 77.5(17.5)$ $N\eta : 6.5(3.5)$ $\Lambda K : 7(4)$	$\Delta\pi : 4(3)$
$N(1700)$	$N'_{3/2}$	1700(50)	100(50)		$N\pi : 10(5)$ $\Lambda K < 3$
$N(1675)$	$N_{5/2}$	1675(5)	148(18)		$N\pi : 40(5)$ $\Lambda K < 1$
$\Lambda(1670)$	$\Lambda_{1/2}$	1670(10)	37.5(12.5)	$NK : 25(5)$ $\Lambda\eta : 17.5(7.5)$ $\Sigma\pi : 40(15)$	
$\Lambda(1690)$	$\Lambda_{3/2}$	1690(5)	60(10)		$NK : 25(5)$ $\Sigma\pi : 30(10)$
$\Lambda(1800)$	$\Lambda'_{1/2}$	1785(65)	300(100)	$NK : 32.5(7.5)$	
$\Lambda(1830)$	$\Lambda_{5/2}$	1820(10)	85(25)		$NK : 6.5(3.5)$ $\Sigma\pi : 55(20)$ $\Sigma^*\pi > 15$
$\Lambda(1405)$	$\Lambda''_{1/2}$	1406(4)	50(2)	$\Sigma\pi : 100$	
$\Lambda(1520)$	$\Lambda''_{3/2}$	1519(1)	15.6(1)		$NK : 45(1)$ $\Sigma\pi : 42(1)$

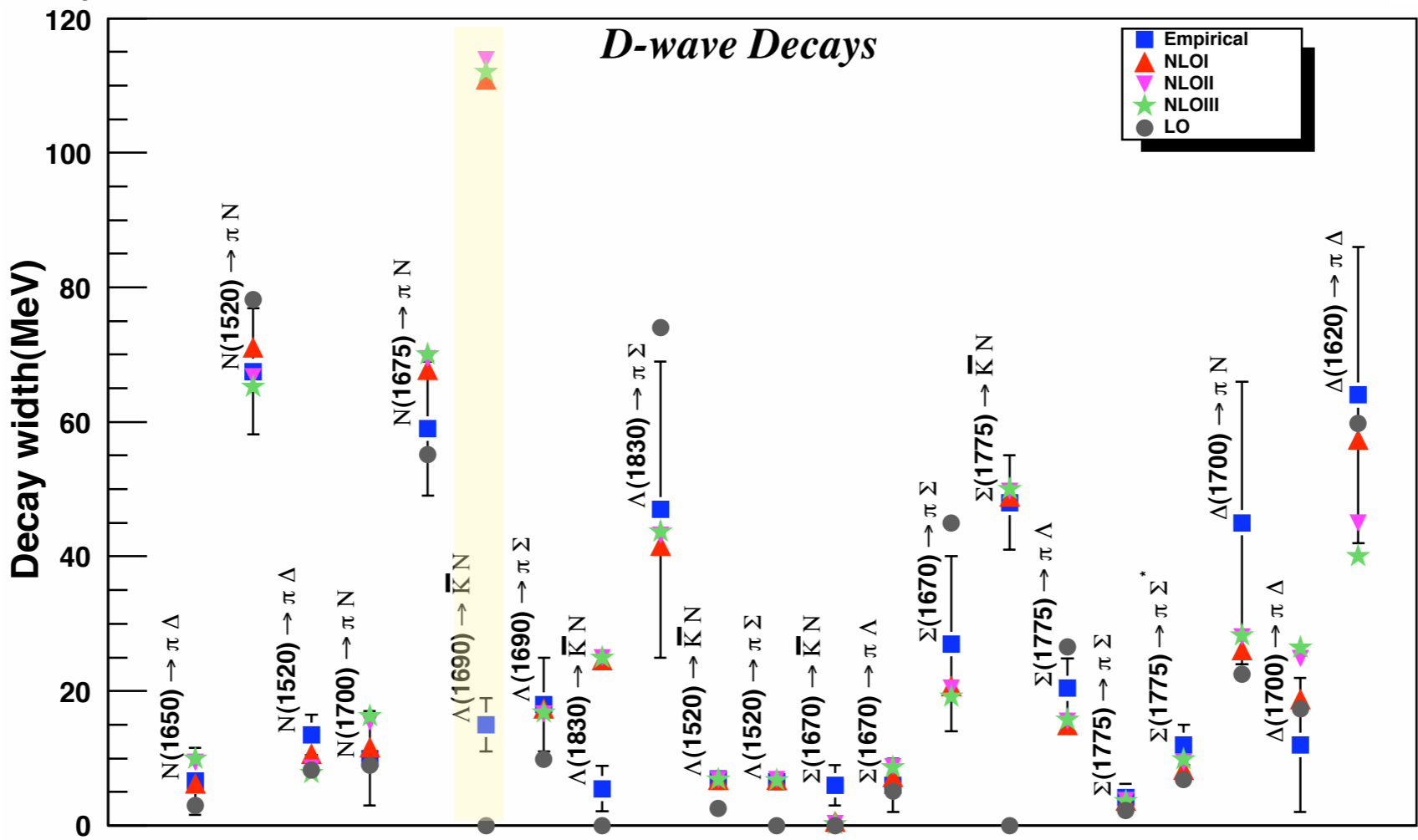
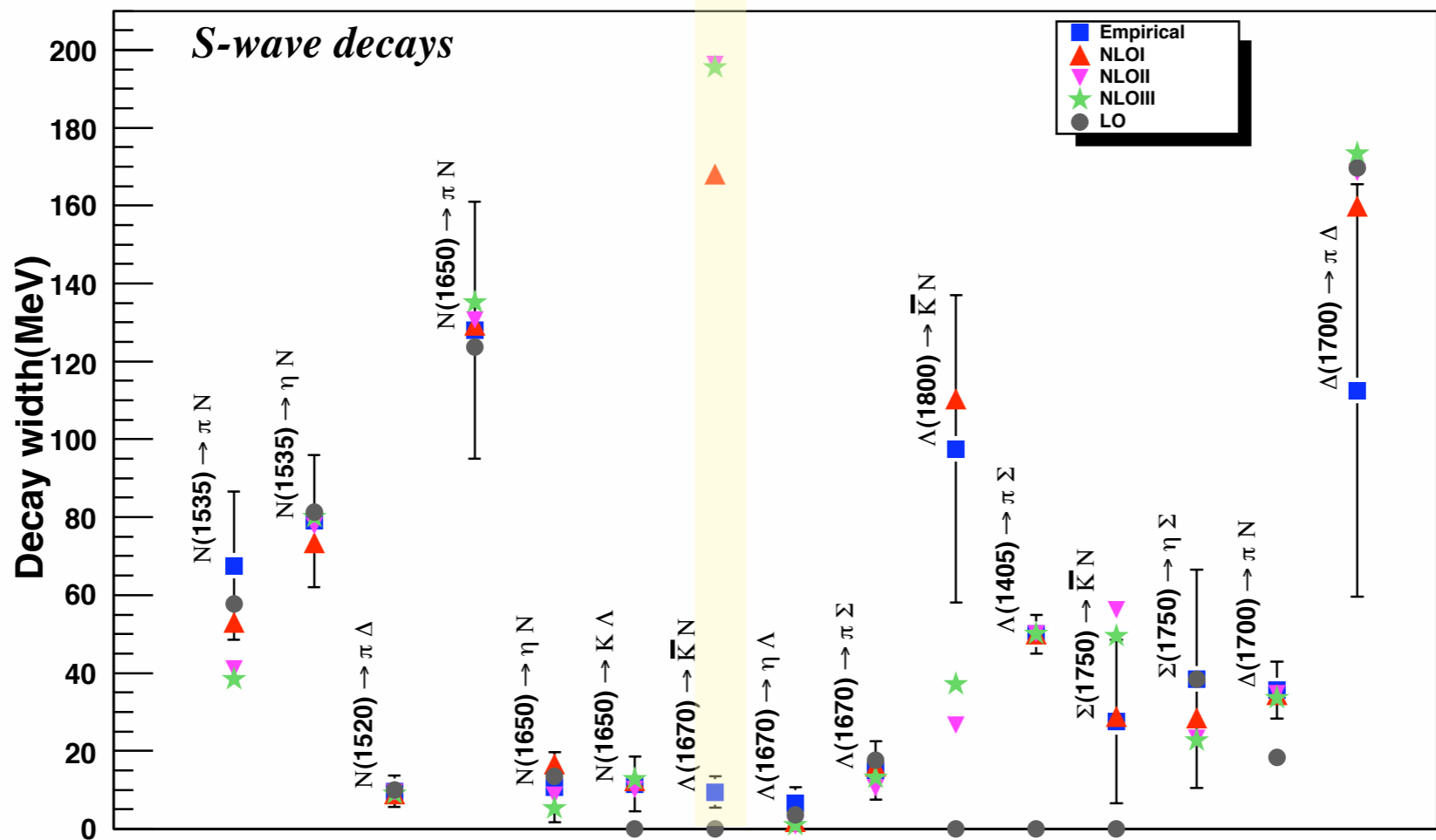
PDG Name	State	Mass [MeV]	Γ_T [MeV]	BR %	
				S – wave	D – wave
$\Sigma(1670)$	$\Sigma_{3/2}$	1675(10)	60(20)		$NK : 10(3)$ $\Lambda\pi : 10(5)$ $\Sigma\pi : 45(15)$
$\Sigma(1750)$	$\Sigma'_{1/2}$	1765(35)	110(50)	$NK : 25(15)$ $\Sigma\pi < 8$ $\Sigma\eta : 35(20)$	
$\Sigma(1775)$	$\Sigma_{5/2}$	1775(5)	120(15)		$NK : 40(3)$ $\Lambda\pi : 17(3)$ $\Sigma\pi : 3.5(1.5)$ $\Sigma^*\pi : 10(2)$
$\Delta(1620)$	$\Delta_{1/2}$	1630(30)	143(7.5)	$N\pi : 25(5)$	$\Delta\pi : 45(15)$
$\Delta(1700)$	$\Delta_{3/2}$	1710(40)	300(100)	$\Delta\pi : 37.5(12.5)$	$N\pi : 15(5)$ $\Delta\pi : 4(3)$

Analysis to $1/N_c$ and 1-body
SU(3) breaking

Operator bases

S-wave: 3 LO, 2 NLO, 2 SU(3) breakers

D-wave: 4 LO, 3 NLO, 1 SU(3) breaker



Mixing angle	θ_{N_1}	θ_{N_3}	θ_{Λ_1}	θ_{Λ_3}	θ_{Σ_1}	θ_{Σ_3}
Decays	0.39	2.75	0.21	2.51	1.14	2.19
Decays + Masses	0.48	2.81	0.81	2.57	0.95	3.0
Decays + Masses + Photo – couplings	0.40	2.81	Gonzalez & Scoccola			

S-wave relations at LO

$$\frac{\tilde{\Gamma}(N(1535) \rightarrow N\pi) - \tilde{\Gamma}(N(1650) \rightarrow N\pi)}{\tilde{\Gamma}(N(1535) \rightarrow N\pi) + \tilde{\Gamma}(N(1650) \rightarrow N\pi)} = \frac{1}{5} (3 \cos 2\theta_{N_1} - 4 \sin 2\theta_{N_1}) \rightarrow \theta_{N_1} = 0.46(10) \text{ or } 1.76(10)$$

$$\frac{\tilde{\Gamma}(N(1535) \rightarrow N\eta) - \tilde{\Gamma}(N(1650) \rightarrow N\eta)}{\tilde{\Gamma}(N(1535) \rightarrow N\eta) + \tilde{\Gamma}(N(1650) \rightarrow N\eta)} = \sin 2\theta_{N_1} \rightarrow \theta_{N_1} = 0.51(27)$$

$$\tilde{\Gamma}(N(1535) \rightarrow N\pi) + \tilde{\Gamma}(N(1650) \rightarrow N\pi) = \tilde{\Gamma}(\Delta(1535) \rightarrow \Delta\pi) \quad 51(10) \text{ (th) vs } 31(15) \text{ (exp)}$$

$$\frac{\tilde{\Gamma}(\Delta(1620) \rightarrow N\pi)}{\tilde{\Gamma}(\Delta(1700) \rightarrow \Delta\pi)} = 0.1 \text{ (th) vs } 0.29(15) \text{ (exp)}$$

D-wave relations at LO

$$2\tilde{\Gamma}(\Delta(1620) \rightarrow \Delta\pi) + \tilde{\Gamma}(\Delta(1700) \rightarrow \Delta\pi) = 15\tilde{\Gamma}(\Delta(1620) \rightarrow N\pi) + 32\tilde{\Gamma}(\Delta(1700) \rightarrow N\pi)$$

$$5.9(1.9) \text{ vs } 8.3(2.3)$$

$$\tilde{\Gamma}(N(1535) \rightarrow \Delta\pi) + \tilde{\Gamma}(N(1650) \rightarrow \Delta\pi) + 11\tilde{\Gamma}(\Delta(1620) \rightarrow \Delta\pi) = 132\tilde{\Gamma}(\Delta(1700) \rightarrow N\pi) + 90\tilde{\Gamma}(N(1675) \rightarrow N\pi)$$

$$32(11) \text{ vs } 41(10)$$

<i>S – wave Relation</i>	<i>Exp Test</i>
$\frac{N(1650) \rightarrow \pi N}{N(1535) \rightarrow \eta N} = \frac{N(1535) \rightarrow \pi N}{N(1650) \rightarrow \eta N}$	$0.6 \pm 0.2 \text{ vs } 4.4 \pm 4.0$
$\frac{N(1650) \rightarrow \eta N}{\Sigma(1750) \rightarrow \eta \Sigma} = 1$	0.12 ± 0.14
$\frac{N(1535) \rightarrow \eta N}{\Lambda(1670) \rightarrow \eta \Lambda} = 1$	5.4 ± 3.2
$\frac{\Delta(1620) \rightarrow \pi N}{\Delta(1700) \rightarrow \pi \Delta} = 2/5$	0.29 ± 0.15
$\frac{N(1535) \rightarrow \pi N}{\Lambda(1670) \rightarrow \pi \Sigma} = 1$	4.4 ± 2.5
$\frac{N(1650) \rightarrow \pi N}{\Lambda(1670) \rightarrow \eta \Lambda} = \frac{\Lambda(1670) \rightarrow \pi \Sigma}{N(1650) \rightarrow \eta N}$	$2.1 \pm 0.9 \text{ vs } 0.8 \pm 0.6$
$\frac{N(1650) \rightarrow \pi N}{N(1535) \rightarrow \eta N} = \frac{N(1535) \rightarrow \pi N}{N(1650) \rightarrow \eta N}$	$0.6 \pm 0.2 \text{ vs } 4.4 \pm 4.0$
$\frac{N(1535) \rightarrow \pi N}{N(1650) \rightarrow \pi N}$	$\theta_1 = 0.30 \pm 0.08 \quad 1.6 \pm 0.08$
$\frac{N(1535) \rightarrow \pi N}{\Delta(1620) \rightarrow \pi N}$	$\theta_1 = 0.33 \pm 0.08 \quad 1.57 \pm 0.08$
$\frac{N(1535) \rightarrow \eta N}{N(1650) \rightarrow \eta N}$	$\theta_1 = 0.68 \pm 0.14 \quad 1.22 \pm 0.14$
$\frac{N(1520) \rightarrow \pi \Delta}{\Delta(1700) \rightarrow \pi \Delta}$	$\theta_3 = 2.48 \pm 0.08 \quad 2.96 \pm 0.09$

<i>D – wave Relation</i>	<i>Exp Test</i>
$\frac{N(1675) \rightarrow \pi N}{\Lambda(1830) \rightarrow \pi \Sigma} = 1$	0.92 ± 0.46
$\frac{\Sigma(1670) \rightarrow \pi \Lambda}{\Sigma(1670) \rightarrow \pi \Sigma} = 1/2$	0.12 ± 0.10
$\frac{\Sigma(1775) \rightarrow \pi \Lambda}{\Sigma(1775) \rightarrow \pi \Sigma} = 1/2$	3.1 ± 1.6
$\frac{\Sigma(1775) \rightarrow \pi \Sigma}{\Sigma(1775) \rightarrow \pi \Sigma_{10}} = 8/7$	1.3 ± 0.6
$\frac{2\Delta(1620) \rightarrow \pi \Delta + \Delta(1700) \rightarrow \pi \Delta}{8\Delta(1700) \rightarrow \pi N + N(1675) \rightarrow \pi N} = 1$	2.9 ± 1.2
$\frac{\frac{2}{9} N(1535) \rightarrow \pi \Delta + \frac{2}{9} N(1650) \rightarrow \pi \Delta + \frac{20}{3} \Delta(1620) \rightarrow \pi \Delta}{16\Delta(1700) \rightarrow \pi N + 15N(1675) \rightarrow \pi N} = 1$	2.6 ± 1.2
$\frac{\frac{1}{36} N(1520) \rightarrow \pi N + \frac{1}{36} N(1700) \rightarrow \pi N + \frac{5}{12} \Delta(1620) \rightarrow \pi \Delta}{\Delta(1700) \rightarrow \pi N + N(1675) \rightarrow \pi N} = 1$	2.5 ± 1.2

SU(3) breaking very important

Two observations on decays

56-plets

$$\Gamma(N^*, \Delta^* \rightarrow N\eta, \Delta\eta) = \mathcal{O}(1/N_c^2)$$

70-plets

Strict large N_c

$$\theta_1 = \cos^{-1}(1/\sqrt{3}) = 54.7^\circ \quad \textit{Phen} : 23^\circ$$

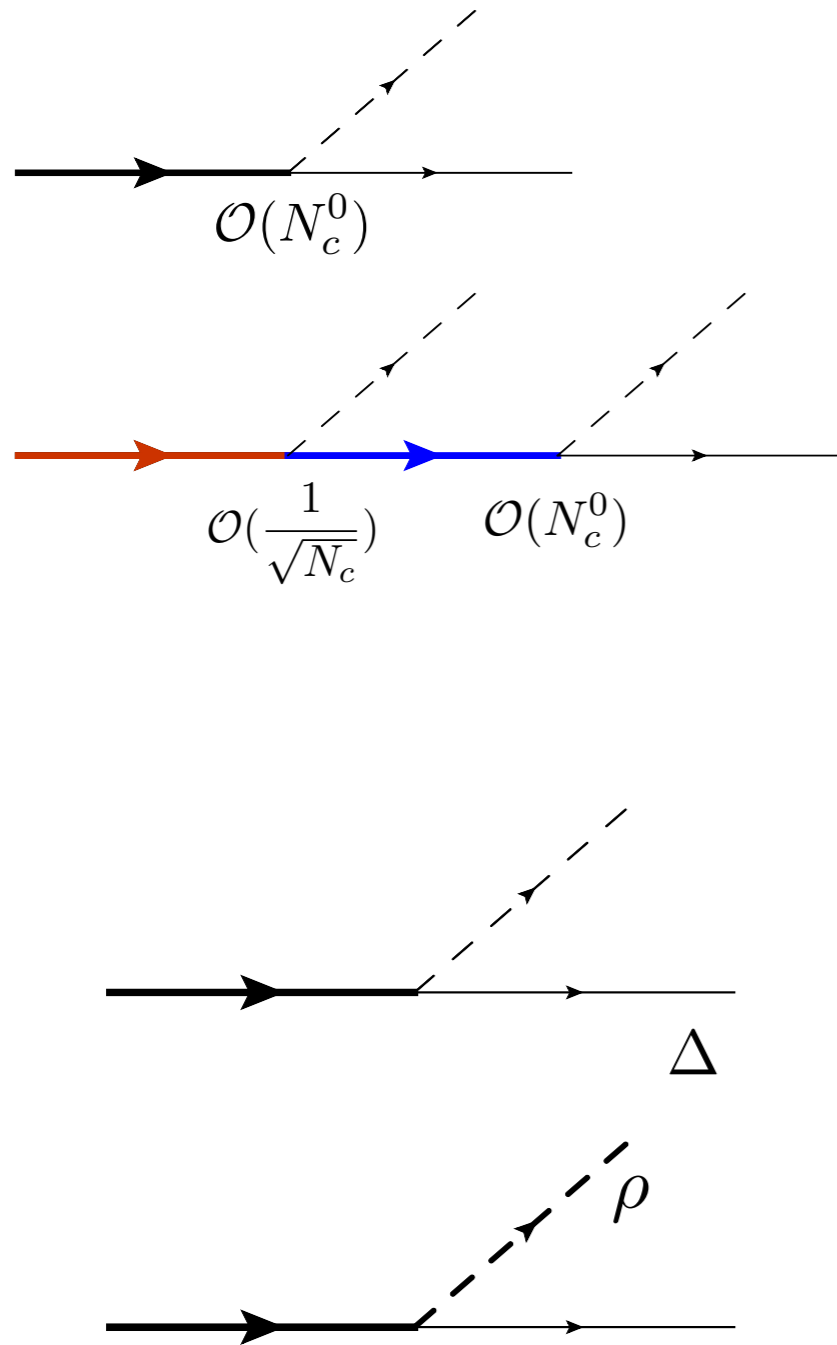
$$N(1535) \not\rightarrow N\pi$$

$$N(1535) \rightarrow N\eta$$

$$N(1650) \rightarrow N\pi$$

$$N(1650) \not\rightarrow N\eta$$

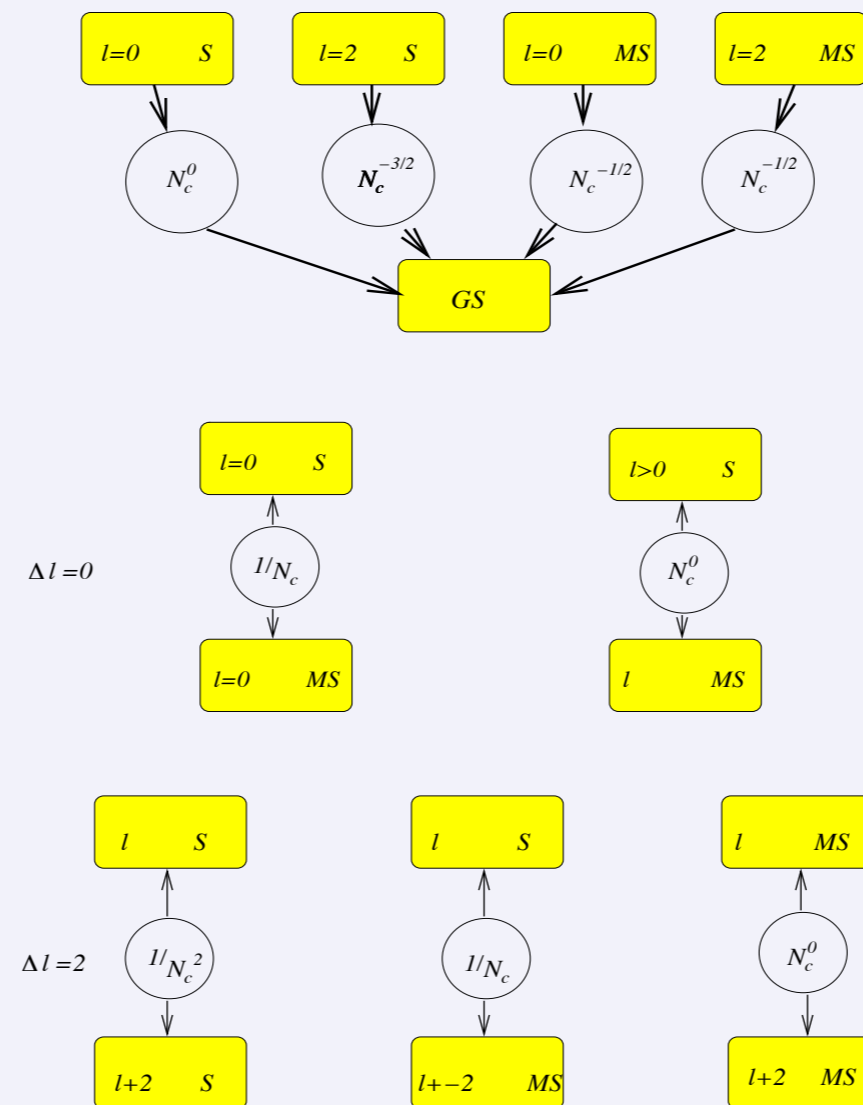
2 pion channels



<i>State</i>	$N_{\pi\pi}(\%)$	
$D_{13}(1520)$	40 – 50	$\Delta\pi \sim 15 - 20$ $N_\rho \sim 15 - 25$
$S_{11}(1650)$	12 – 20	$\Delta\pi \sim 1 - 7$ $N_\rho \sim 4 - 12$ $N(1440)\pi < 5$
$D_{13}(1700)$	85 – 95	$\Delta\pi > 30$ $N_\rho \sim 7$
$D_{15}(1675)$	50 – 60	$\Delta\pi \sim 50 - 60$ $N_\rho < 1 - 3$
$S_{31}(1620)$	40 – 70	$\Delta\pi \sim 30 - 60$ $N(1440)\pi \sim 11$
$D_{33}(1700)$	80 – 90	$\Delta\pi \sim 30 - 60$ $N_\rho \sim 30 - 55$ $N(1535)\pi \sim 4$
...
$P_{31}(1910)$	80 – 95	$\Delta\pi \sim 6$ $N_\rho \sim 30$ $N(1440)\pi \sim 56$
$P_{33}(1920)$	~ 100	$\Delta\pi \sim 41$ $N(1535)\pi \sim 6$ $N(1440)\pi \sim 53$

Configuration mixings

Mixing summary



Summary on $1/N_c$ in baryons

- In principle, $1/N_c$ gives rigorous connection to QCD
- An expansion for sorting out effects by magnitude
- In baryons it basically organizes how $SU(6)$ symmetry is broken
- Gives relationships which are beyond dynamics at a given order in $1/N_c$ - parameter independent relations, which serve as tests
- Important dynamical insight associated with small coefficients: small spin-orbit; small 3-body forces; dominance of 1-body operators in strong decays and photo-couplings
- Relevant dynamics is hidden in effective coefficients: both long and short distance effects
- It can be applied to lattice results (mass spectrum) which is very promising
- It should be no problem to implement $1/N_c$ in phenomenological models used in

PWA