The helicity amplitudes in the hypercentral Constituent Quark Model

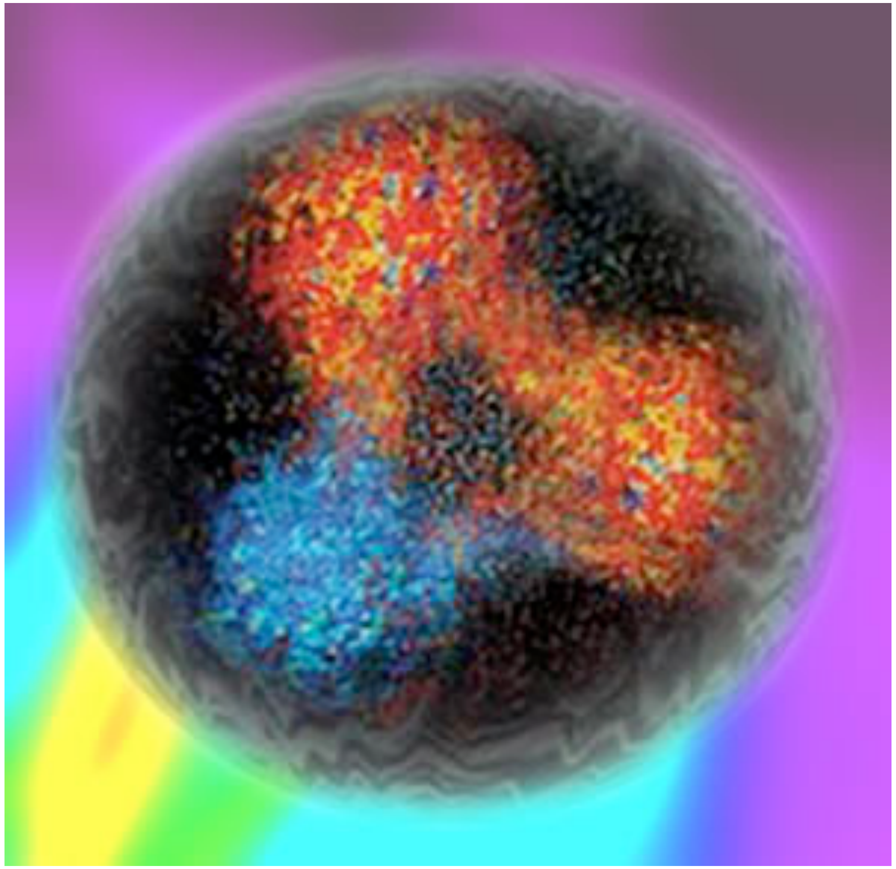
Mauro Giannini
University of Genova - INFN

PWA6 - George Washington University, May 27, 2011

Outline of the talk

- The spectrum in the hCQM
- The helicity amplitudes
- Relativity
- q-antiquark pair effects - meson cloud

Basic idea of Constituent Quark Models (CQM)



Constituent Quarks

At variance with QCD quarks

CQ acquire mass & size
carrier of the proton spin

Various CQM for bayons

GROUP	Kin. Energy	SU(6) inv	SU(6) viol	date
Isgur-Karl	non rel	h.o. + shift	OGE	1978-9
Capstick-Isgur	rel	string + coul-like	OGE	1986
Iachello et al.	non rel	U(7) Casimir	group chain	1994
Genoa	non rel/rel	hypercentral	OGE/isospin	1995
Glozman-Riska	rel	linear	GBE	1996
Bonn	rel	linear 3-body	instanton	2001

Hypercentral Constituent Quark Model
hCQM

free parameters fixed from the spectrum

Predictions for:

photocouplings

transition form factors

elastic from factors

……..

describe data (if possible)

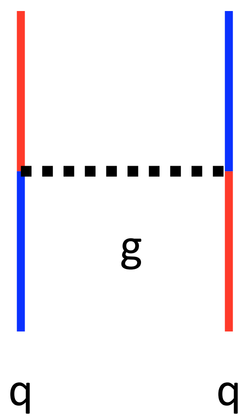
understand what is missing

Introducing dynamics

LQCD (De Rújula, Georgi, Glashow, 1975)

the quark interaction contains

- a long range spin-independent confinement
  $\longrightarrow$ SU(6) configurations
- a short range spin dependent term



One Gluon Exchange

V_{OGE} = -a/r + Hyperfine interaction

THREE-QUARK WAVE FUNCTION

$$\Psi_{3q} = \theta_{colour} \times \chi_{spin} \times \phi_{iso} \times \psi_{space}$$

SU(3)$_c$ SU(2) SU(3)$_f$ O(3)

SU(6) limit $$\Psi_{3q} = \theta_{colour} \times \Phi \times \psi_{space}$$

SU(3)$_c$ SU(6)$_{sf}$ O(3)

A the rest must be symmetric

SU(6) x O(3) wf have the same symmetry (A, MS, MA, S)

SU(6) configurations for three quark states

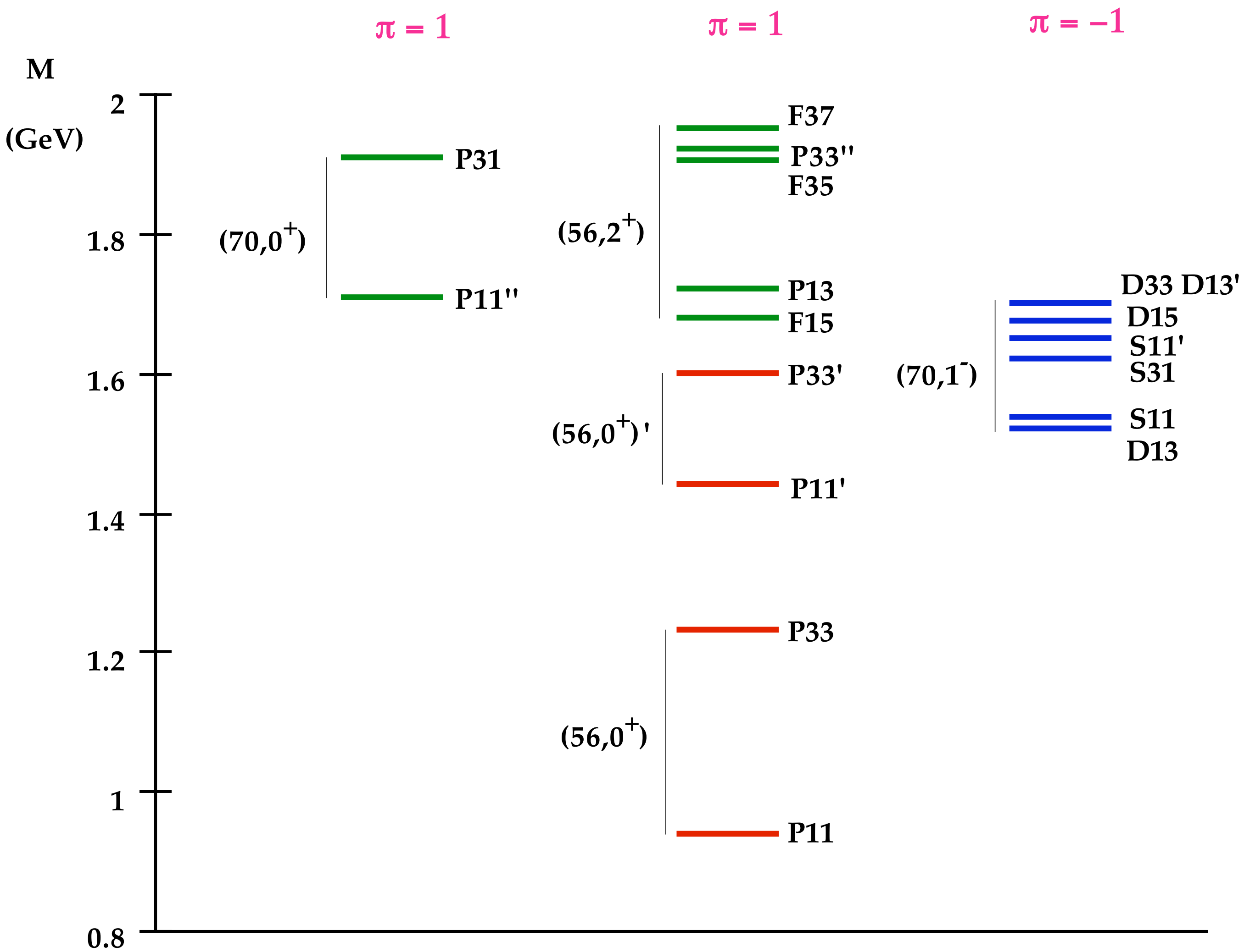
$$6 \times 6 \times 6 = \underset{A}{20} + \underset{M}{70} + \underset{M}{70} + \underset{S}{56}$$

Notation

(d, L^{π})

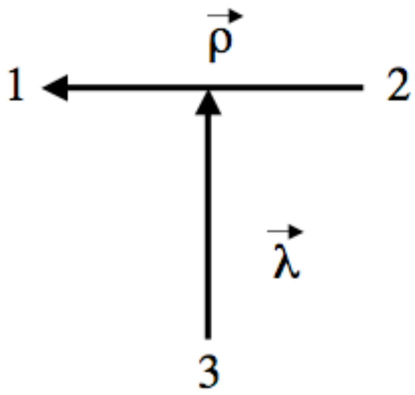
d = dim of SU(6) irrep
L = total orbital angular momentum
π = parity

PDG 4* & 3*



π = 1
π = 1
π = −1
M
(GeV)
2
1.8
1.6
1.4
1.2
1
0.8
P31
(70,0+)
P11''
F37
P33''
F35
(56,2+)
P13
F15
P33'
(56,0+)'
P11'
P33
(56,0+)
P11
D33 D13'
D15
S11'
S31
(70,1-)
S11
D13

Jacobi coordinates



Hyperspherical Coordinates

$$(\rho, \Omega_\rho, \lambda, \Omega_\lambda) \Rightarrow (x, t, \Omega_\rho, \Omega_\lambda)$$

$$x = \sqrt{\rho^2 + \lambda^2}$$

$$t = arctg\frac{\rho}{\lambda}$$

$$L^2(\Omega)Y_{[\gamma]}(\Omega) = -\gamma(\gamma+4)Y_{[\gamma]}(\Omega)$$

$$L^2(\Omega) \Leftrightarrow C_2(O(6))$$

γ grand angular quantum number

$Y_{[\gamma]}(\Omega)$ Hyperspherical harmonics

$$\sum_{i<j} V(\mathbf{r}_{ij}) \approx V(x) + \ldots$$

$\gamma = 2n + l_\rho + l_\lambda$

Hasenfratz et al. 1980:

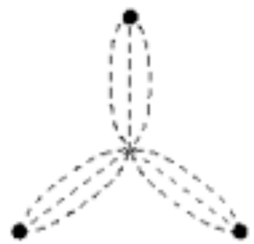
$\Sigma\, V(r_i,r_j)$ is approximately hypercentral

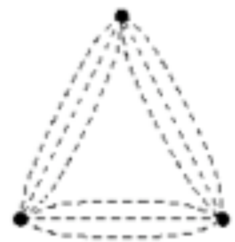
- QCD fundamental mechanism

3-body forces

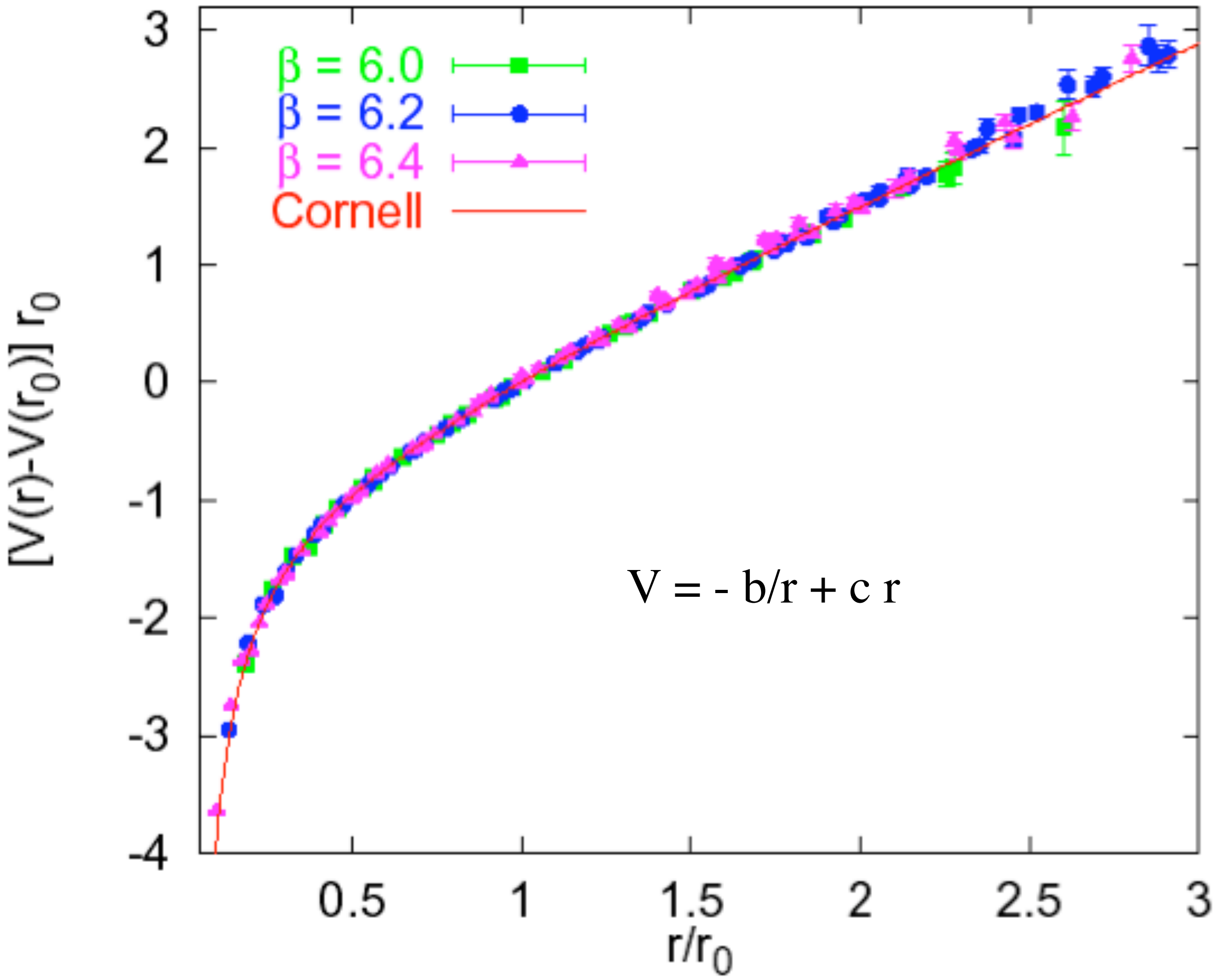
Carlson et al, 1983
Capstick-Isgur 1986
hCQM 1995

- Flux tube model



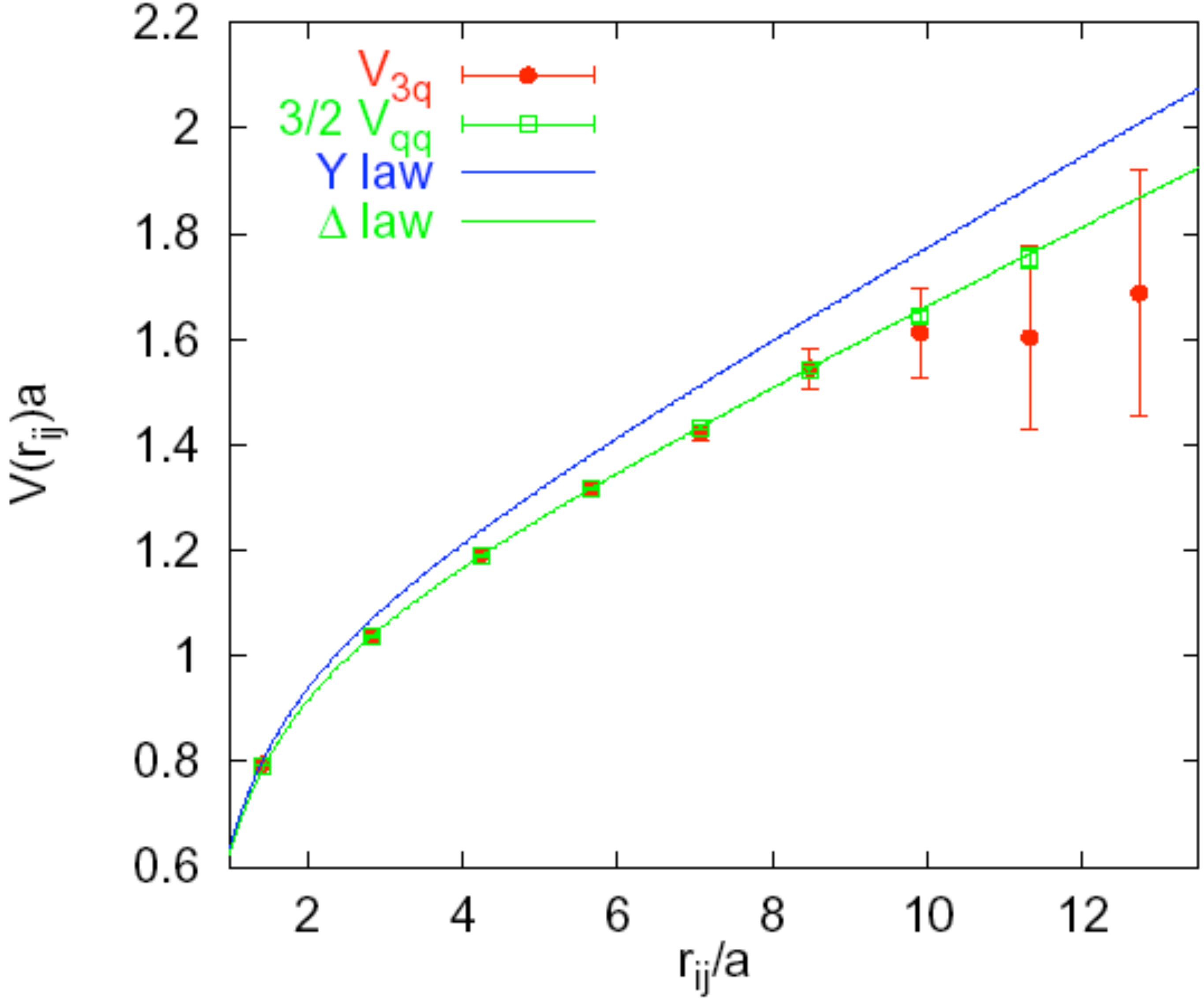


Quark-antiquark lattice potential G.S. Bali Phys. Rep. 343, 1 (2001)



3-quark lattice potential

G.S. Bali Phys. Rep. 343, 1 (2001)



Hypercentral Hypothesis

$$V = V(x)$$

Factorization

$$\psi(x, t, \Omega_\rho, \Omega_\lambda) = \quad \psi_{\nu\gamma}(x) \qquad Y_{[\gamma, l_\rho, l_\lambda]}$$

("dynamics") ("geometry")

Only one differential equation in x (hyperradial equation)

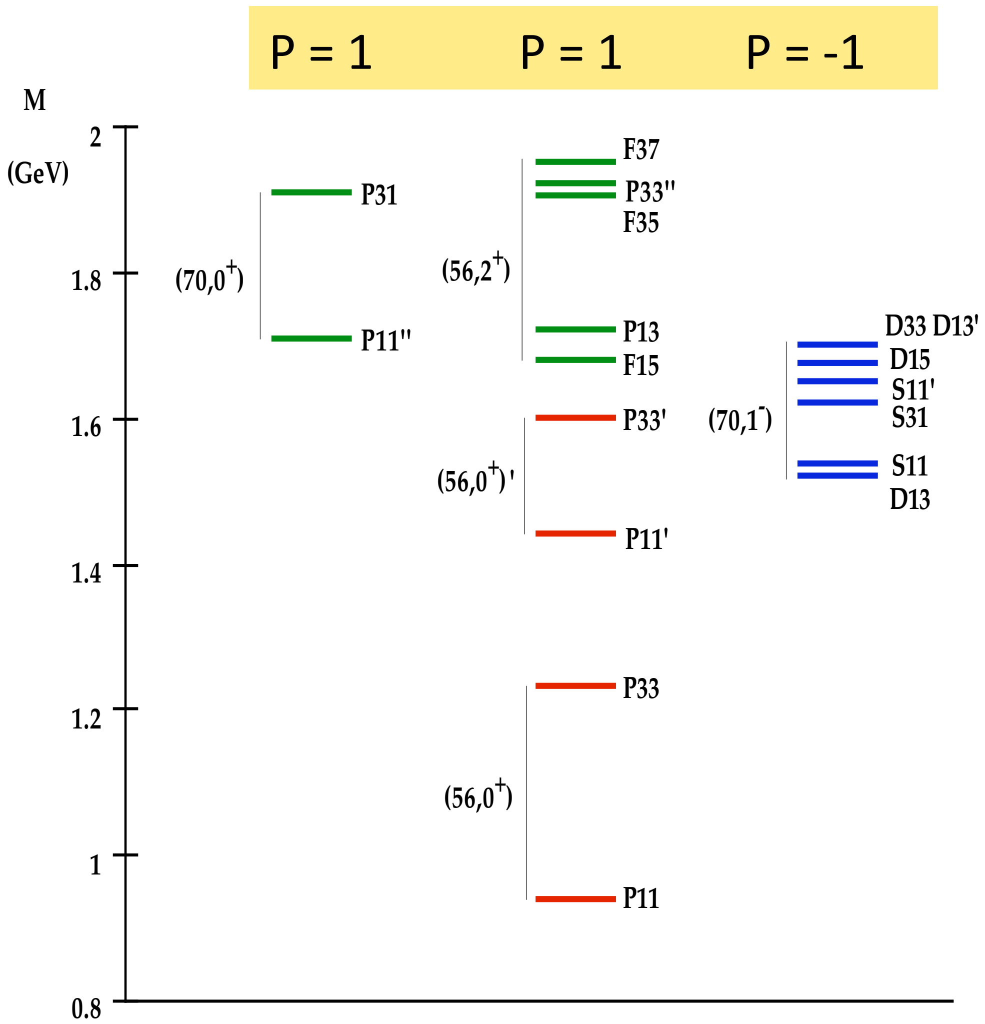
Hypercentral Model

Genoa group, 1995

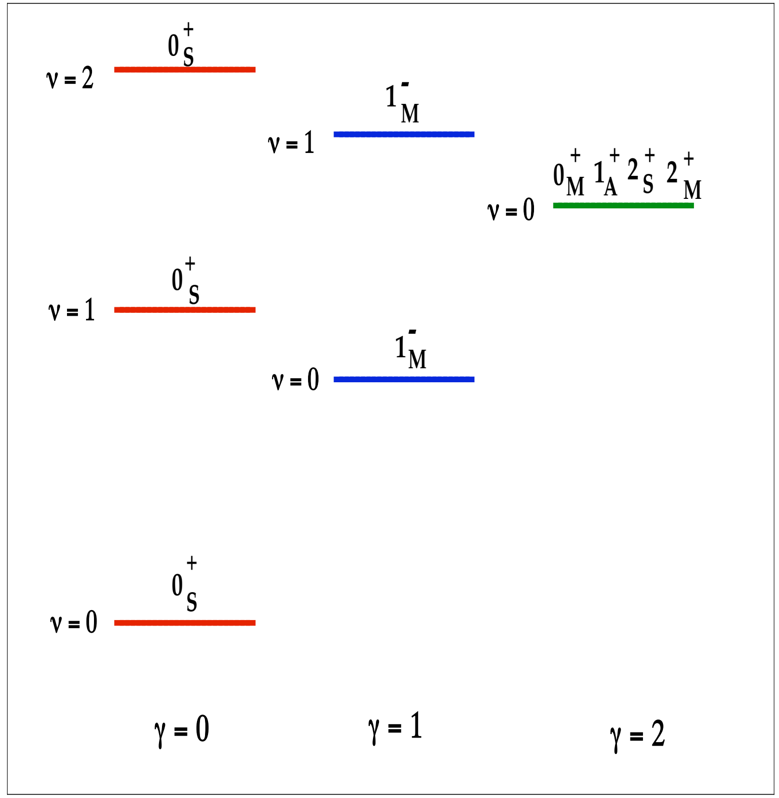
$V(x) = -\tau/x + \alpha\, x$

Hypercentral approximation of $V = - b/r + c\, r$

PDG 4* & 3*

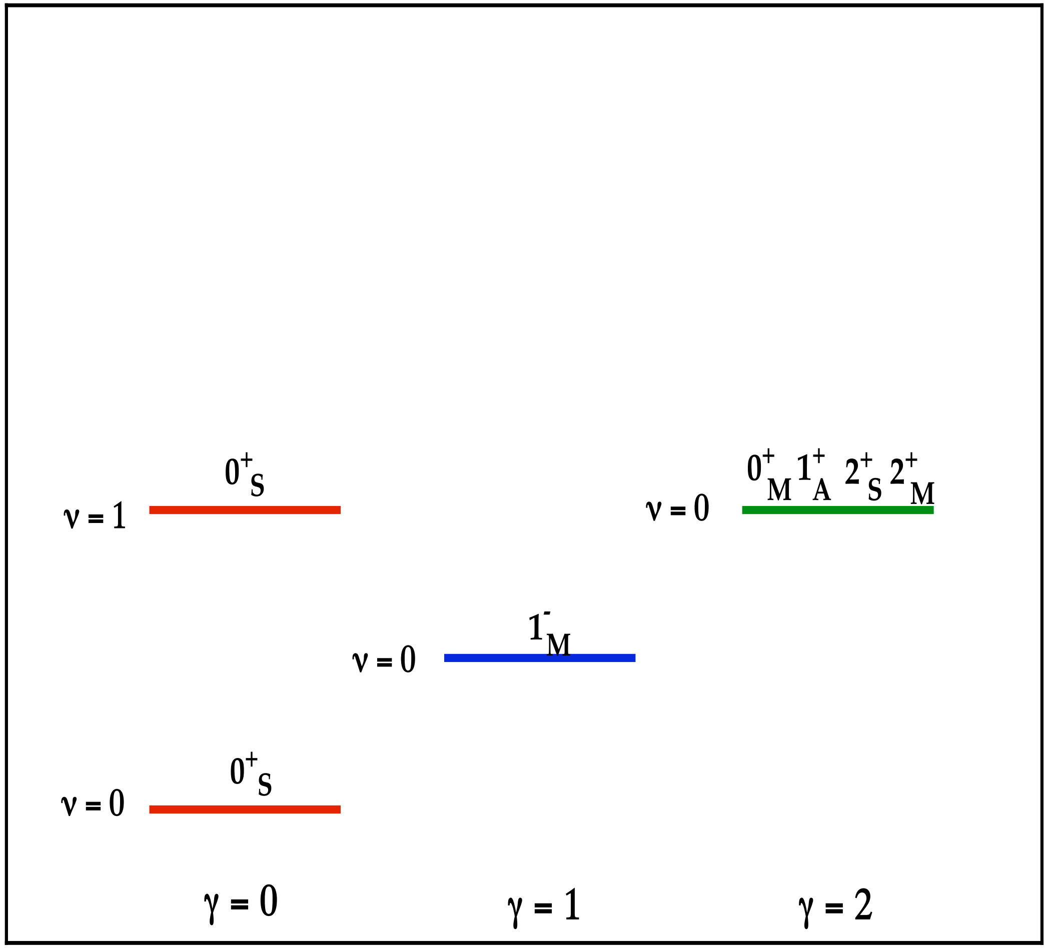


$$V(x) = -\tau/x + \alpha x$$



$$\Sigma_{i<j}\, 1/2\; k\, (r_i - r_j)^2 = 3/2\; k\, x^2$$

b) **H. O.**

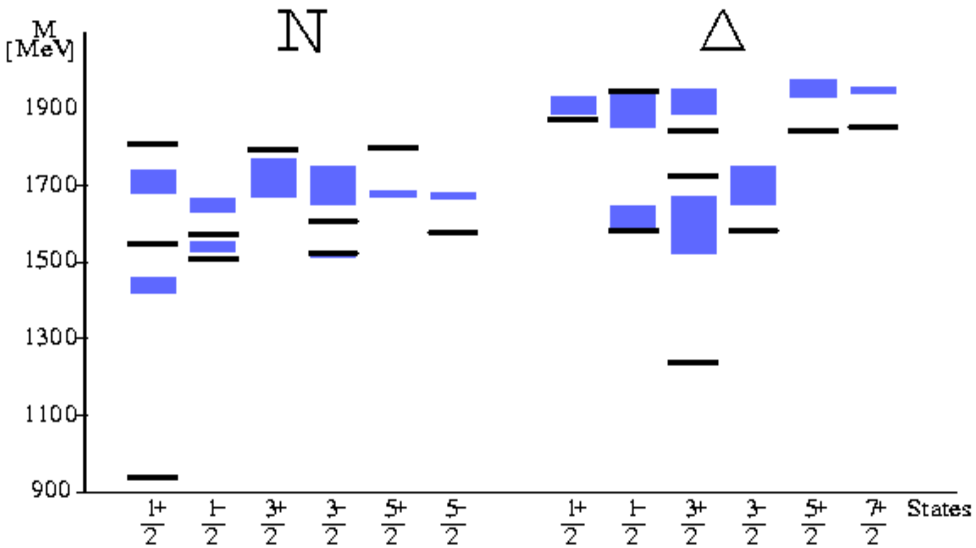


Hypercentral Model (1)

$$H_{3q} = 3m + \sum_{i=1}^{3} \frac{\mathbf{p}_i^2}{2m} + V(x) + H_{hyp}$$

M. Ferraris, M. M. Giannini, M. Pizzo, E. Santopinto, L. Tiator, Phys. Lett. B364 (1995), 231

- $V(x) = -\frac{\tau}{x} + \alpha x; \qquad H_{hyp} = A \left[\sum_{i<j} V^S(\mathbf{r}_i, \mathbf{r}_j) \ \ \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + \text{tensor} \right]$
- 3 parameters τ α A $\leftarrow$ fixed to the spectrum, $m = \frac{M}{3}$



$\tau = 4.59$

$\alpha = 1.61 \text{ fm}^{-1}$

$A \Leftarrow (N - \Delta)$

$x = \sqrt{\rho^2 + \lambda^2}$

hyperradius

The helicity amplitudes

HELICITY AMPLITUDES

Definition

$A_{1/2} = < N^* \; J_z = 1/2 \mid H^T_{em} \mid N \; J_z = -1/2 > * \zeta$ §

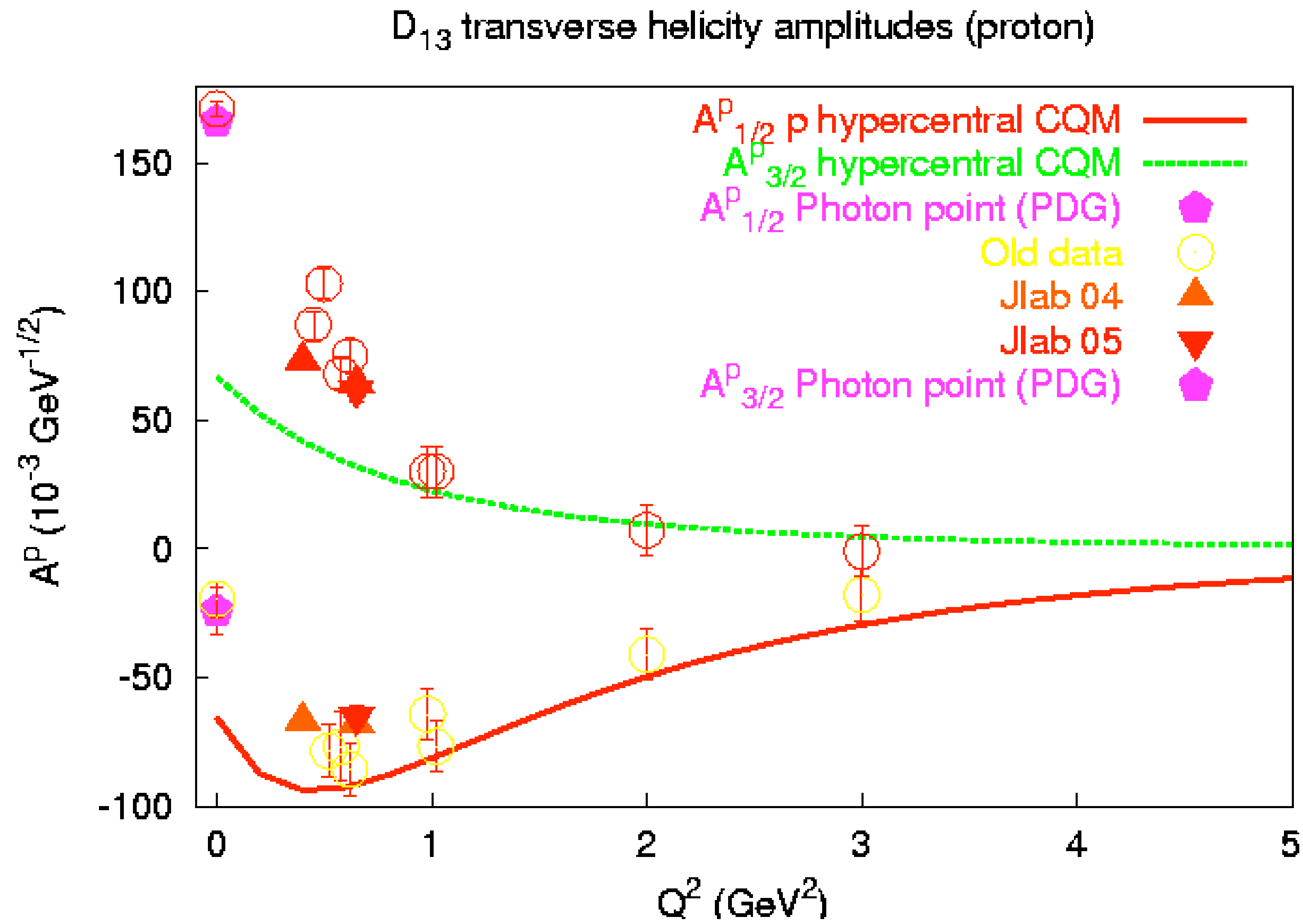
$A_{3/2} = < N^* \; J_z = 3/2 \mid H^T_{em} \mid N \; J_z = 1/2 > * \zeta$ §

$S_{1/2} = < N^* \; J_z = 1/2 \mid H^L_{em} \mid N \; J_z = 1/2 > * \zeta$

N, N* nucleon and resonance as 3q states
mixed by OGE interaction

H^T_{em} H^l_{em} model transition operator

§ results for the negative parity resonances: M. Aiello et al. J. Phys. G24, 753 (1998)

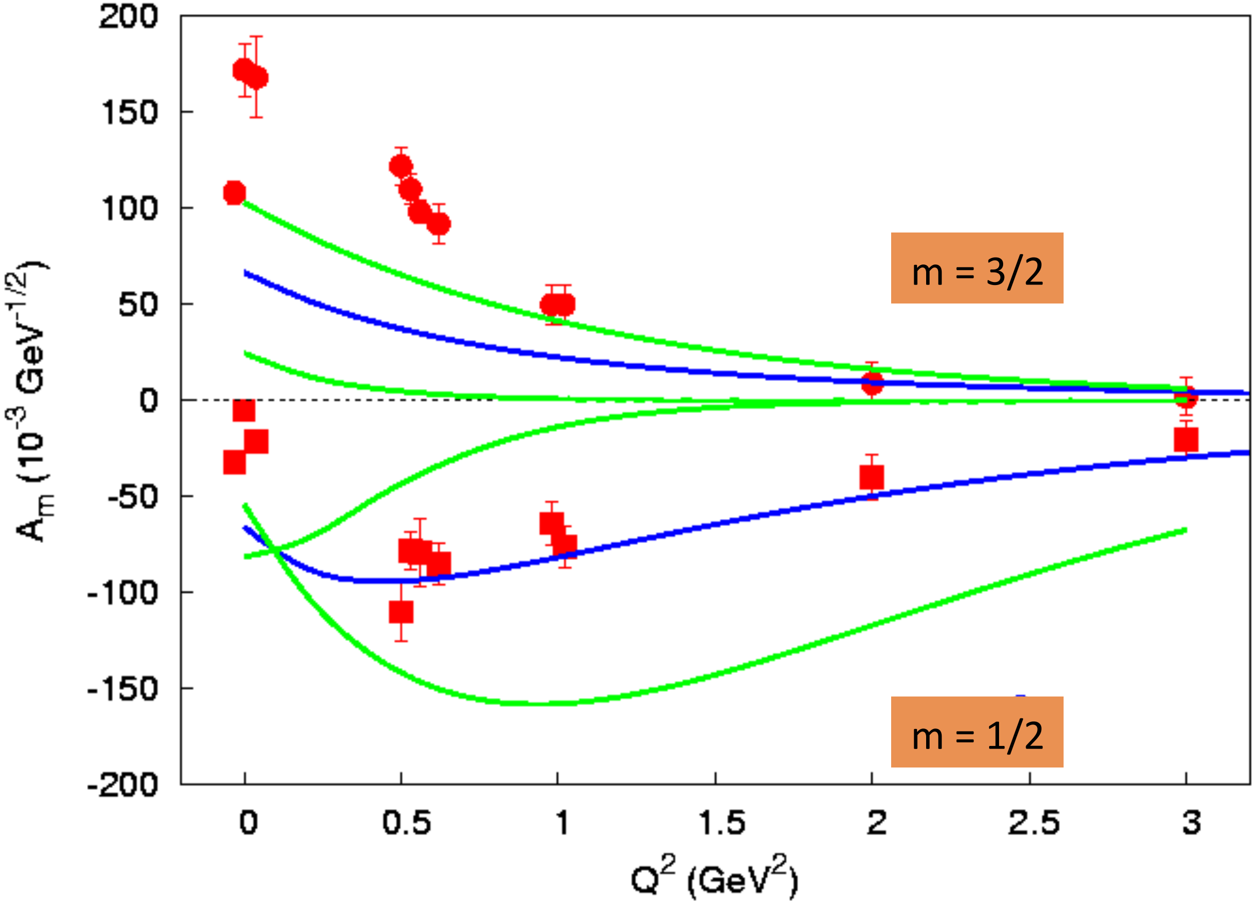


D13 transverse helicity amplitudes (proton)
A^p_1/2 p hypercentral CQM
A^p_3/2 hypercentral CQM
A^p_1/2 Photon point (PDG)
Old data
Jlab 04
Jlab 05
A^p_3/2 Photon point (PDG)
150
100
50
0
-50
-100
0
1
2
3
4
5
A^p (10^-3 GeV^-1/2)
Q^2 (GeV^2)

A^p_m $N(1520)D13$

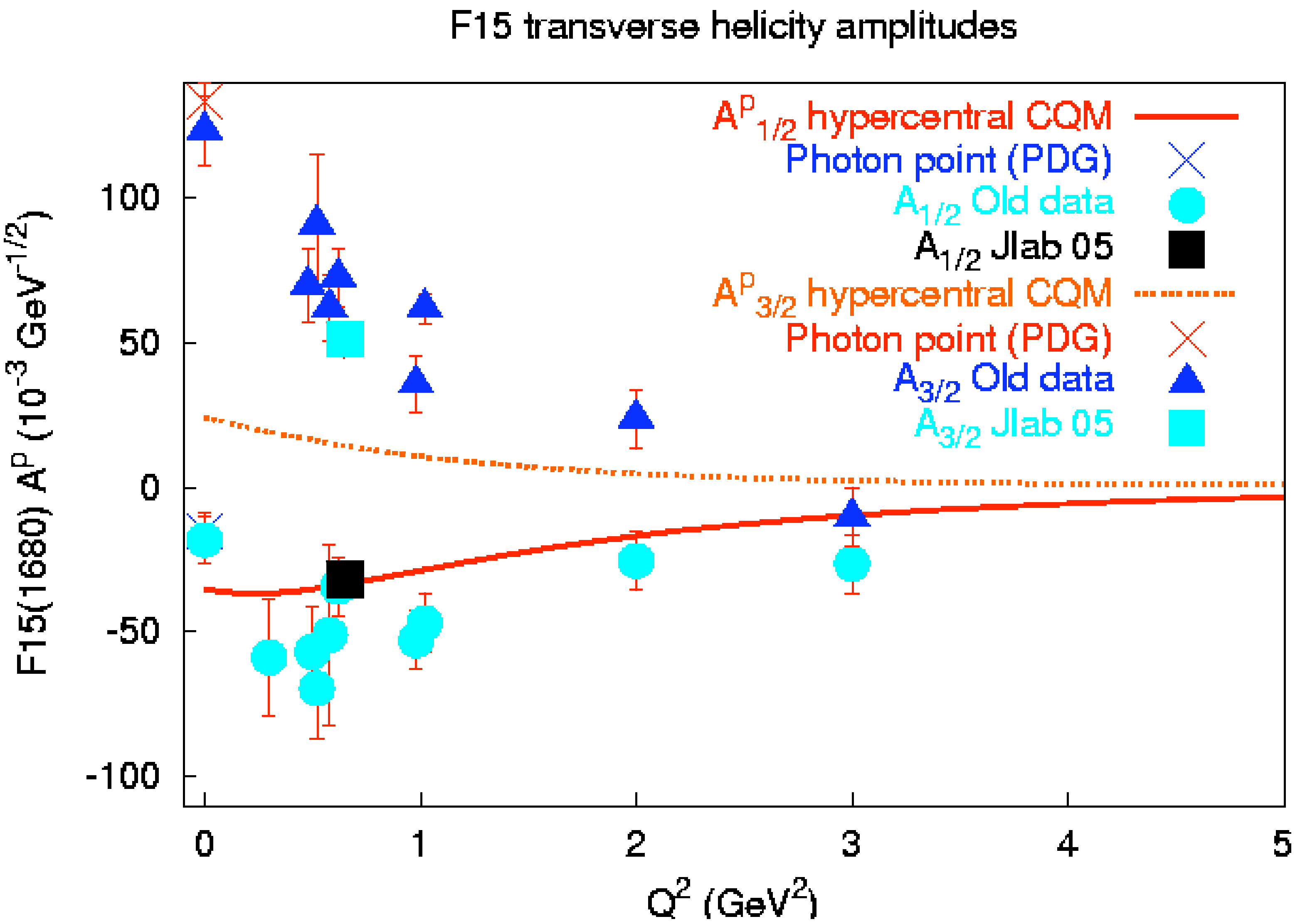
r_p 0.5 fm

r_p 0.86 fm

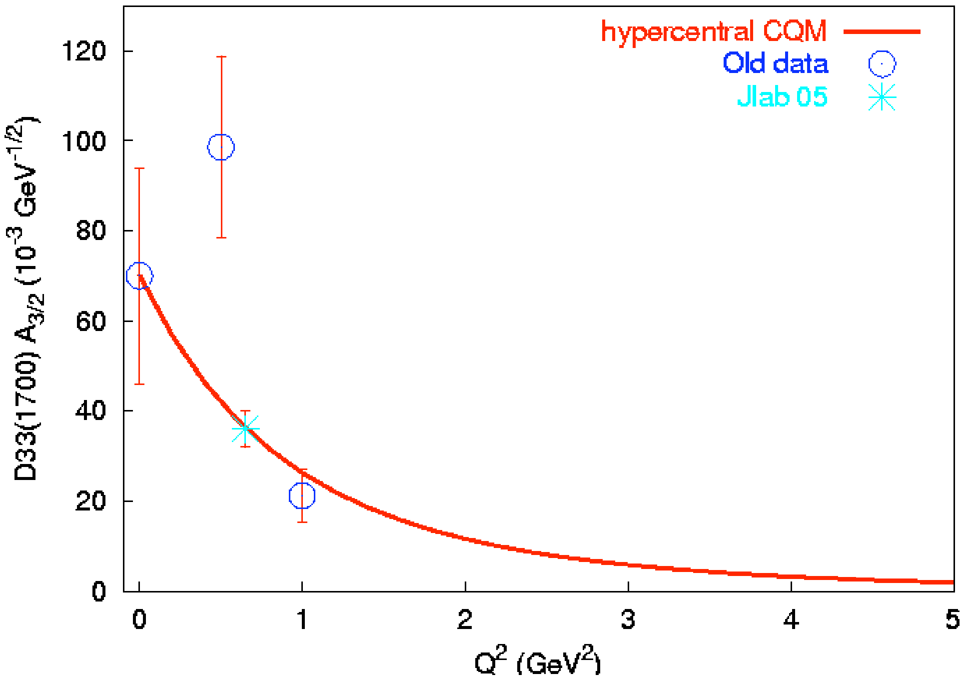


Green curves H.O.

Blue curves hCQM



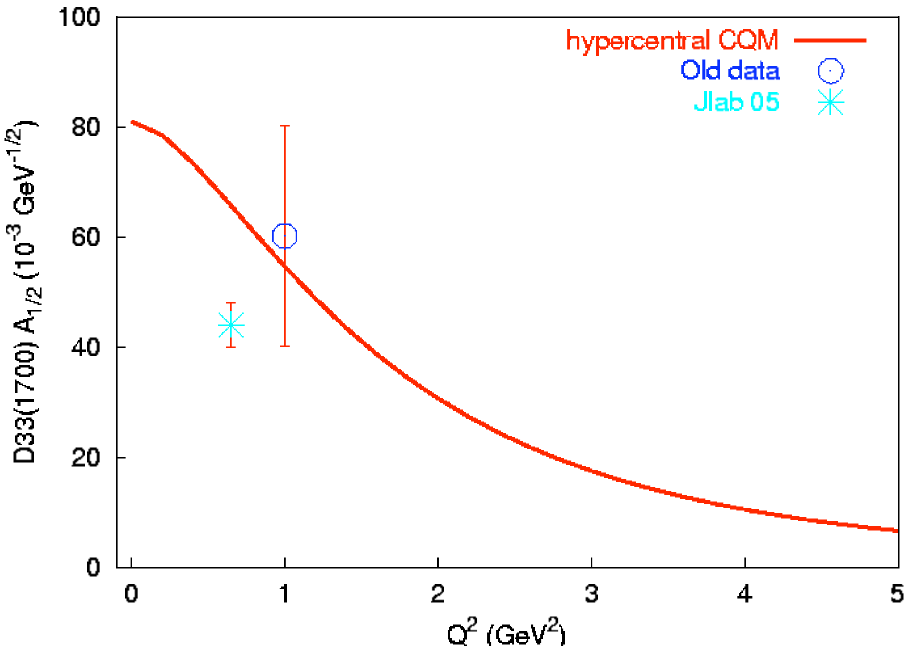
F15 transverse helicity amplitudes
$A^p_{1/2}$ hypercentral CQM
Photon point (PDG)
$A_{1/2}$ Old data
$A_{1/2}$ Jlab 05
$A^p_{3/2}$ hypercentral CQM
Photon point (PDG)
$A_{3/2}$ Old data
$A_{3/2}$ Jlab 05
F15(1680) A^p (10^{-3} GeV$^{-1/2}$)
Q^2 (GeV^2)
100
50
0
-50
-100
0
1
2
3
4
5

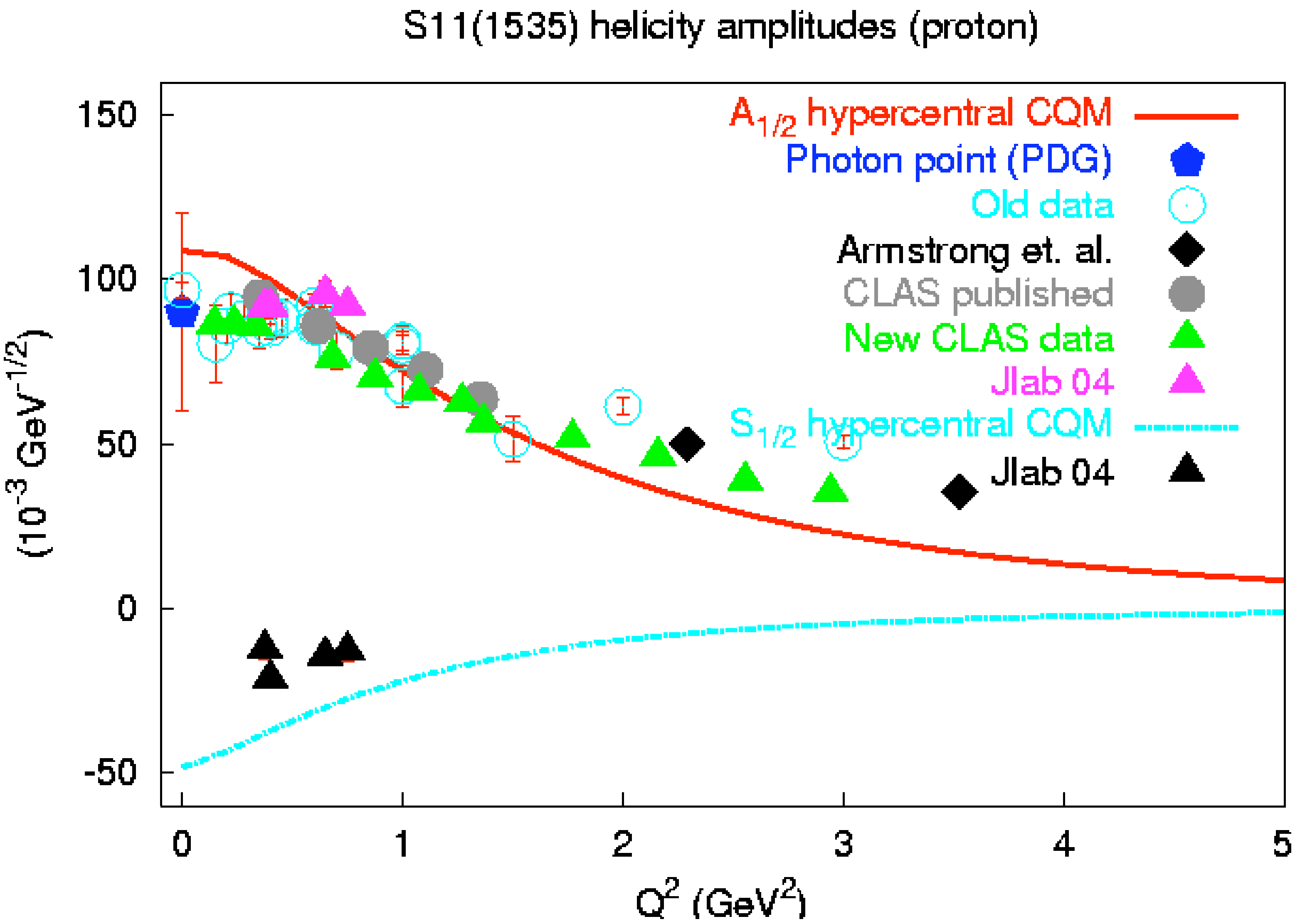


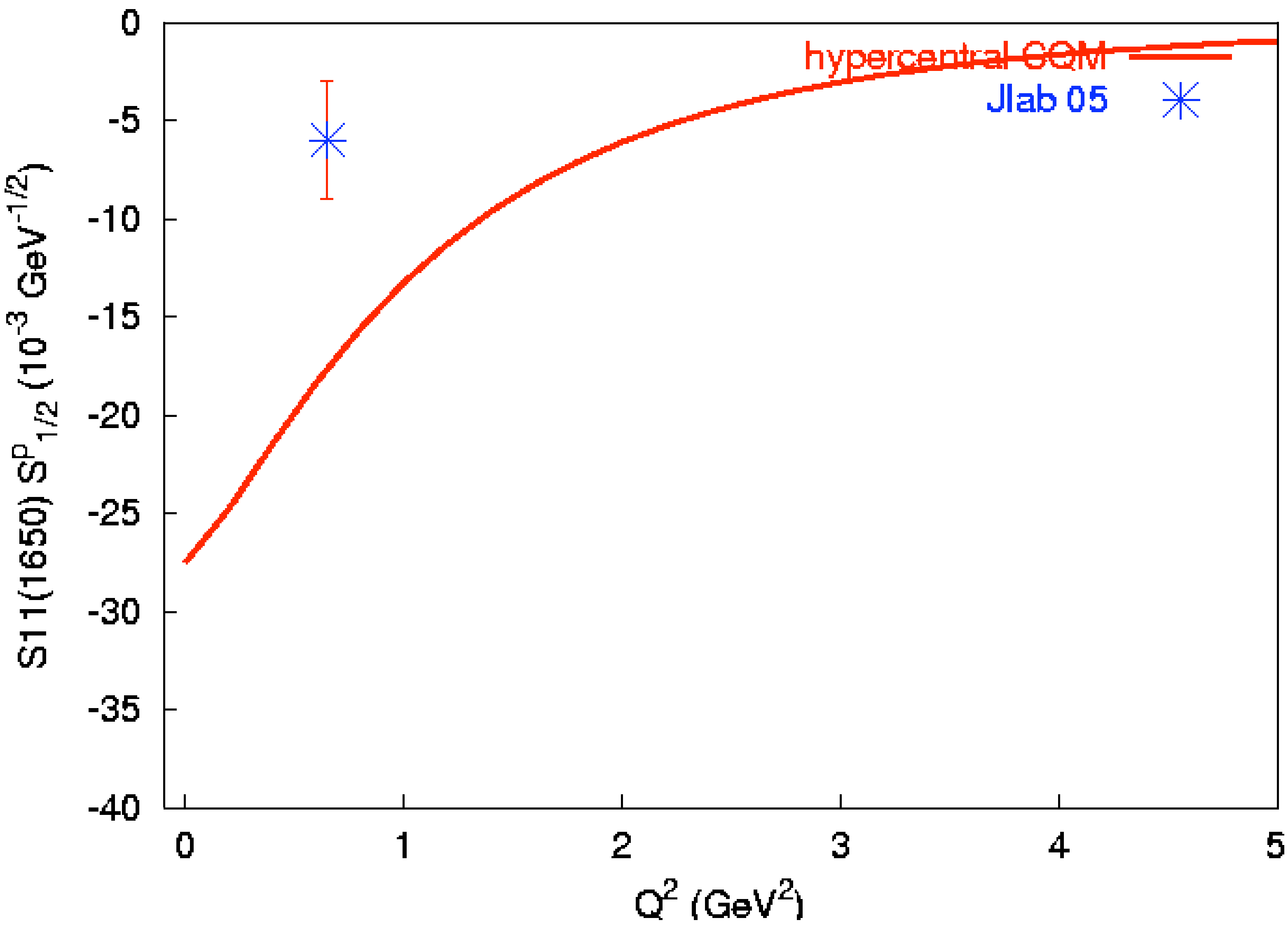
D33(1700)

A 3/2

A 1/2




S11(1535) helicity amplitudes (proton)
A1/2 hypercentral CQM
Photon point (PDG)
Old data
Armstrong et. al.
CLAS published
New CLAS data
Jlab 04
S1/2 hypercentral CQM
Jlab 04
150
100
50
0
-50
(10-3 GeV-1/2)
0
1
2
3
4
5
Q2 (GeV2)



hypercentral CQM
Jlab 05
S11(1650) S^P_1/2 (10^-3 GeV^-1/2)
Q^2 (GeV^2)
0
-5
-10
-15
-20
-25
-30
-35
-40
0
1
2
3
4
5

observations

- the calculated proton radius is about 0.5 fm

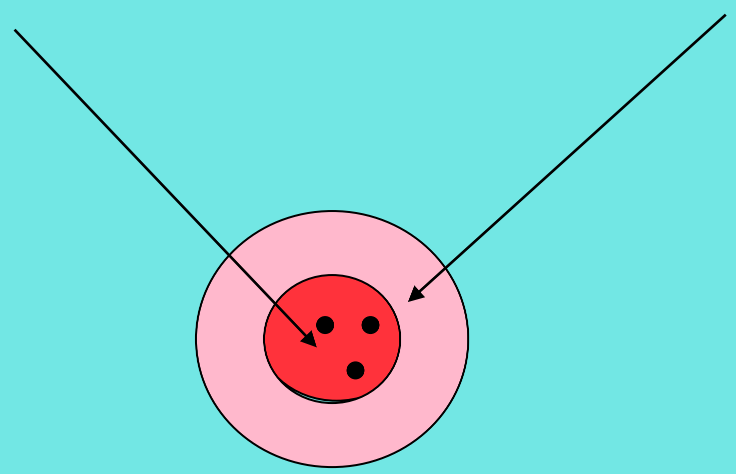
  (value previously obtained by fitting the helicity amplitudes)

- the medium Q^2 behaviour is fairly well reproduced (1/x potential)
- there is lack of strength at low Q^2 (outer region) in the e.m. transitions

  specially for the A 3/2 amplitudes

- emerging picture: quark core (0.5 fm) plus (meson or sea-quark) cloud

Quark-antiquark pairs effects are important for the low Q^2 behavior



What is
missing?

Relativity
Quark-antiquark effects

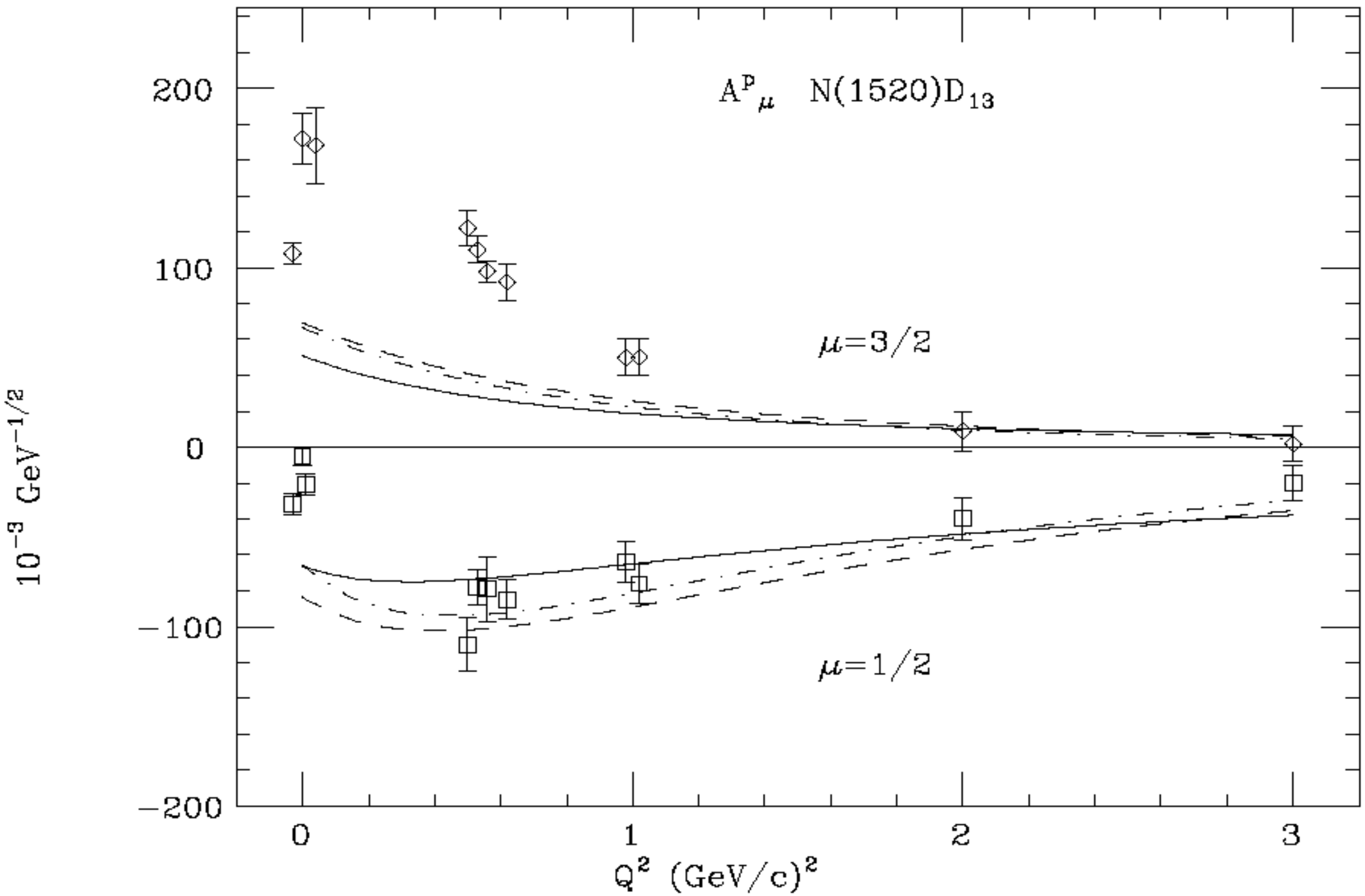
Relativistic corrections to form factors

- Breit frame
- Lorentz boosts applied to the initial and final state
- Expansion of current matrix elements up to first order in quark momentum
- Results

$$A_{rel}(Q^2) = F\ A_{n.rel}(Q^2_{eff})$$

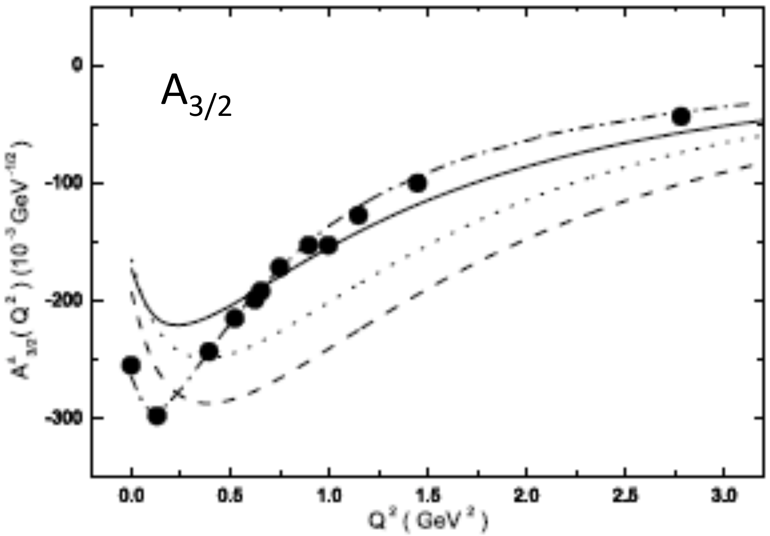
F = kin factor $\qquad Q^2_{eff} = Q^2\,(M_N/E_N)^2$

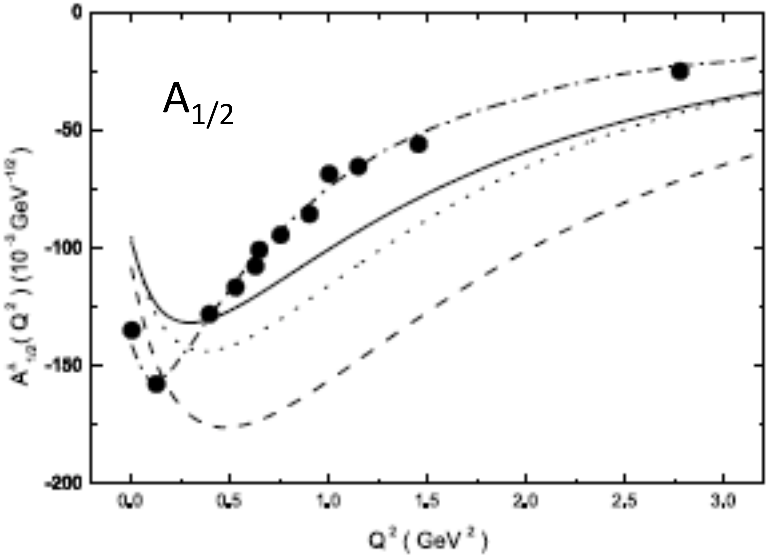
De Sanctis et al. EPJ 1998



Full curves: hCQM with relativistic corrections
Dashed curves: hCQM in different frames

Chen, Dong, M.G., Santopinto, Trieste 2006





dot bare
dash dressed
full rel. corr (prelimnary calculation)
dash-dot MAID

Construction of a fully relativistic theory

Relativistic Hamiltonian Dynamics
for a fixed number of particles (Dirac)

Construction of a representation of the Poincaré generators
P_μ (tetramomentum), J_k (angular momenta), K_i (boosts)

obeying the Poincaré group commutation relations
in particular

$$[P_k , K_i] = i\, \delta_{kj}\, H$$

Three forms: instant, front, point

Point form: P_μ interaction dependent
J_k and K_i free

Quark spins undergo the same Wigner rotation

Composition of angular momentum states as in the non relativistic case

Bakamjian-Thomas construction

$M = M_0 + M_I$

$M_0 = \Sigma_i \sqrt{\mathbf{p}_i^2 + m^2}$

$\Sigma_i \, \mathbf{p}_i = 0$

Free mass operator

M_I introduced sucht that:
commutes with J_k and K_i (free)
V_μ four velocity (free)

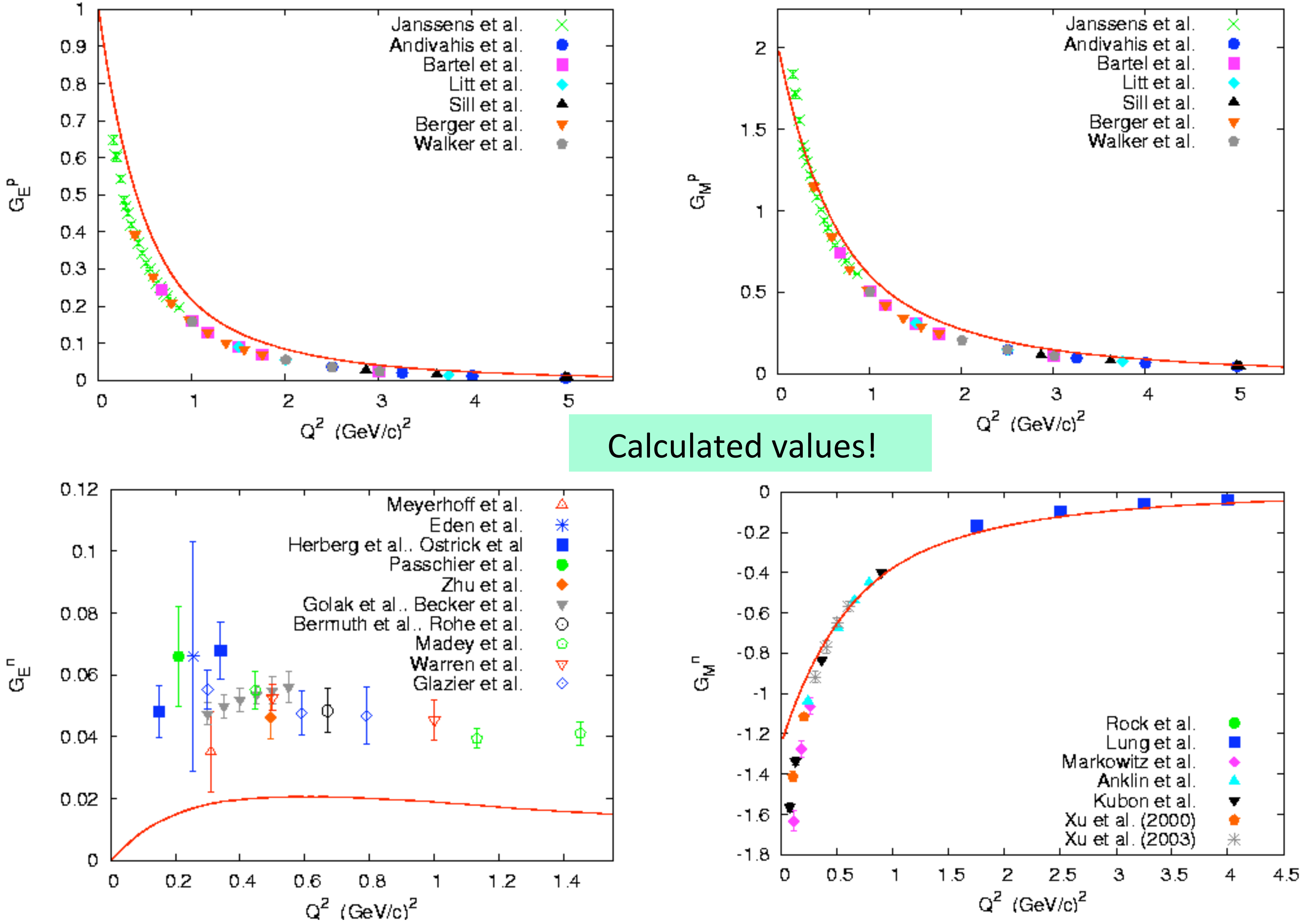
The interaction is contained in $P_\mu = M \, V_\mu$

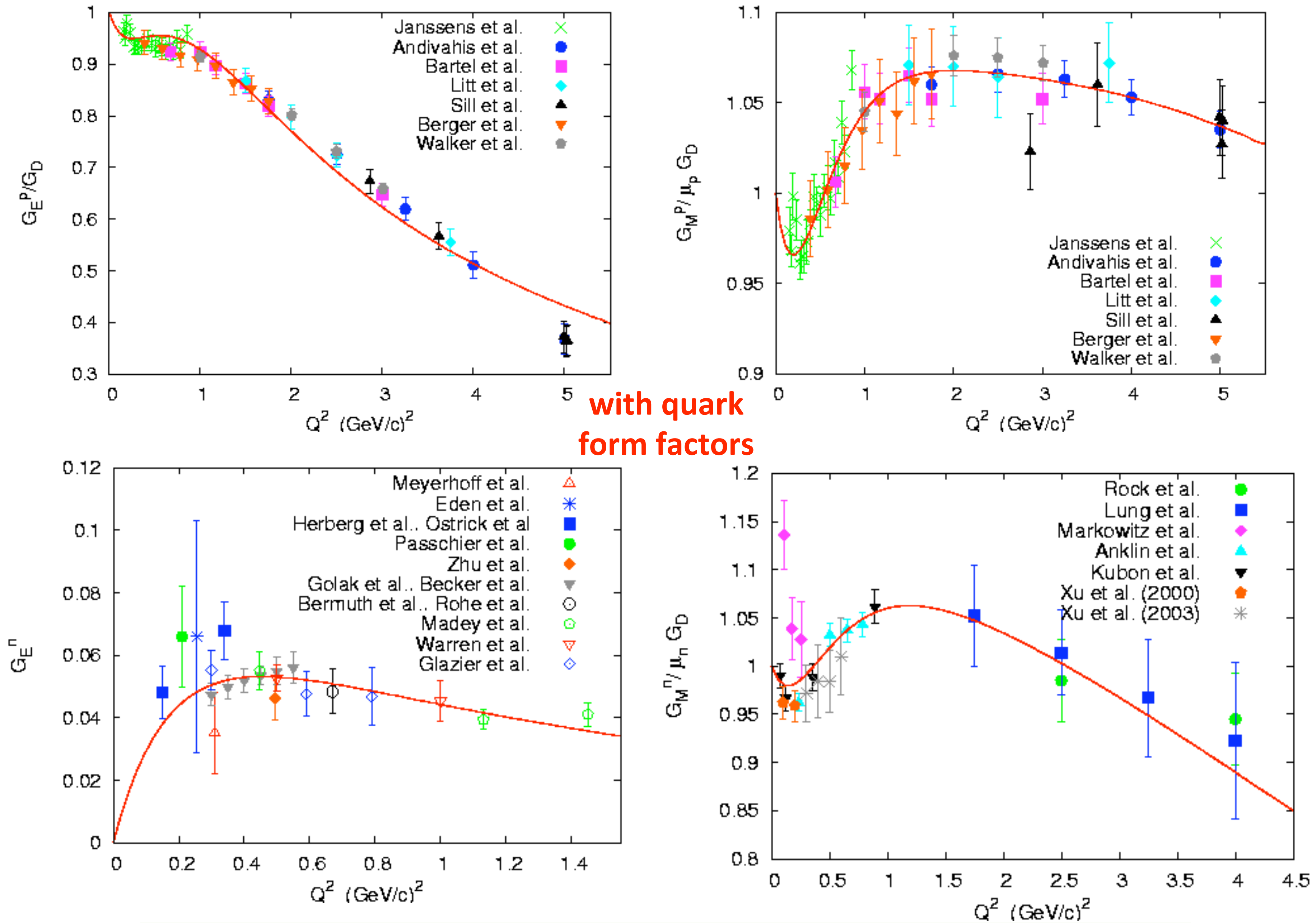
The eigenstates of the relativistic hCQM are interpreted as
eigenstates of the mass operator M

Moving three-quark states are obtained through
(interaction free) Lorentz boosts (velocity states)

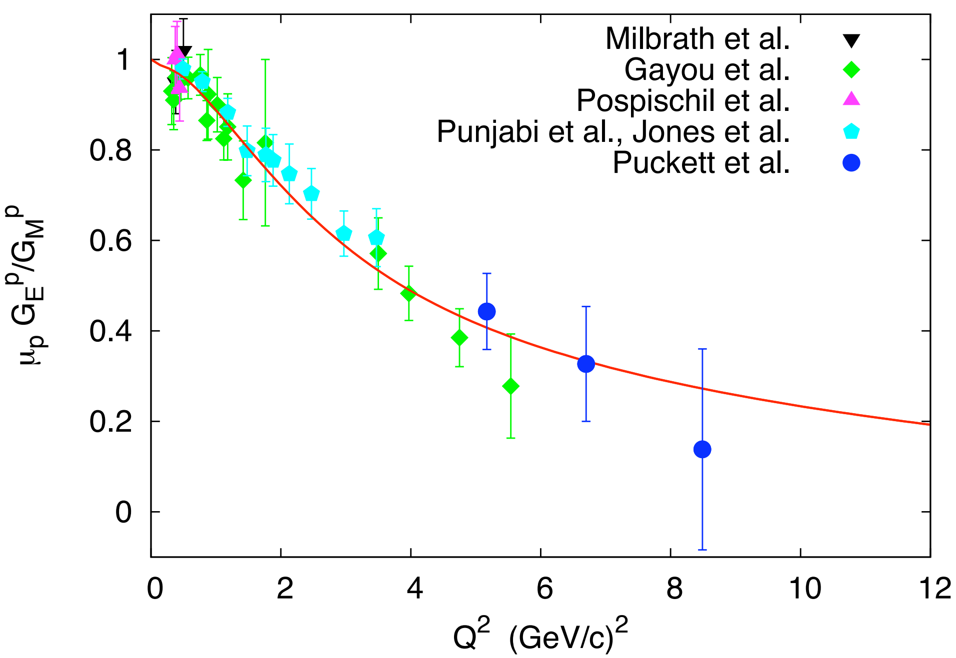
Covariant e.m. quark current

$$\bar{u}_i(p_i) j_{i\mu} u_i(p_i') = \bar{u}_i(p_i) e_i \gamma_\mu(i) u_i(p_i'),$$

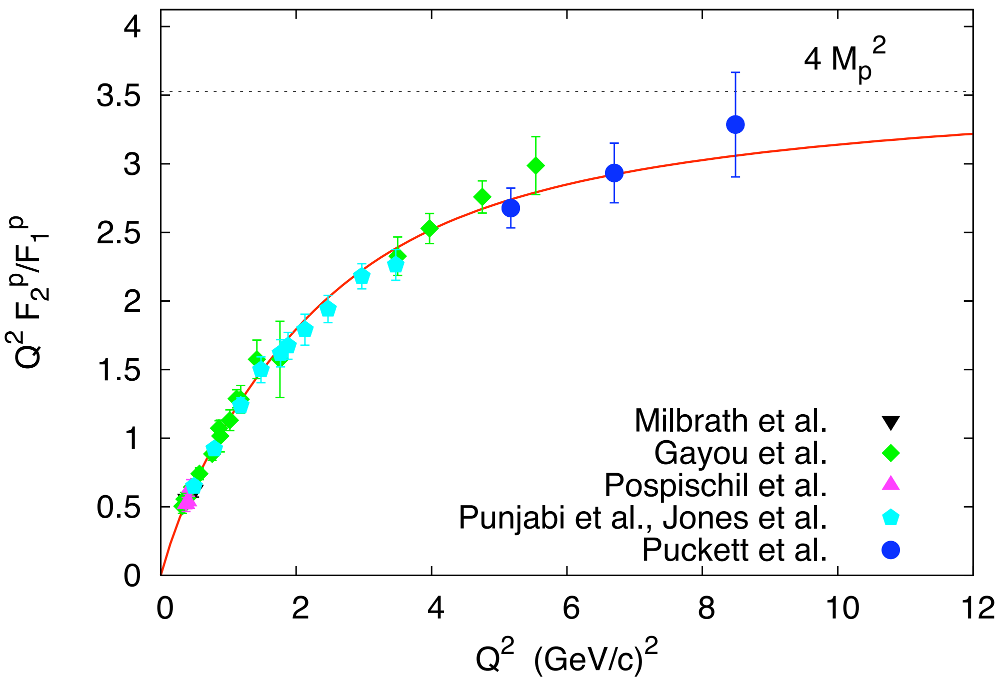

Janssens et al.
Andivahis et al.
Bartel et al.
Litt et al.
Sill et al.
Berger et al.
Walker et al.
G_E^p
Q^2 (GeV/c)^2
G_M^p
Q^2 (GeV/c)^2
Calculated values!
Meyerhoff et al.
Eden et al.
Herberg et al., Ostrick et al
Passchier et al.
Zhu et al.
Golak et al., Becker et al.
Bermuth et al., Rohe et al.
Madey et al.
Warren et al.
Glazier et al.
G_E^n
Q^2 (GeV/c)^2
Rock et al.
Lung et al.
Markowitz et al.
Anklin et al.
Kubon et al.
Xu et al. (2000)
Xu et al. (2003)
G_M^n
Q^2 (GeV/c)^2



M. De Sanctis, M. G., E. Santopinto, A. Vassallo, Phys. Rev. C76, 062201 (2007)



With quark form factors



Santopinto et al.
PR C **82**, 065204 (2010)

Relativistic treatment

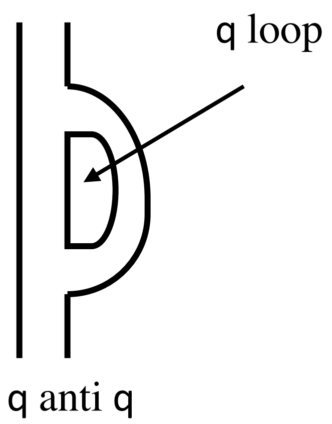
- elastic form factors: necessary
- helicity amplitudes: probably necessary
  exciting higher resonances the recoil is smaller
- Delta excitation: g.s. in the SU(6) limit
  probably more important

Relativity is an important issue for the description of elastic and inelastic form factors

but it is not the only important issue

Unquenching the quark model

Mesons P. Geiger, N. Isgur, Phys. Rev. D41, 1595 (1990)
D44, 799 (1991)

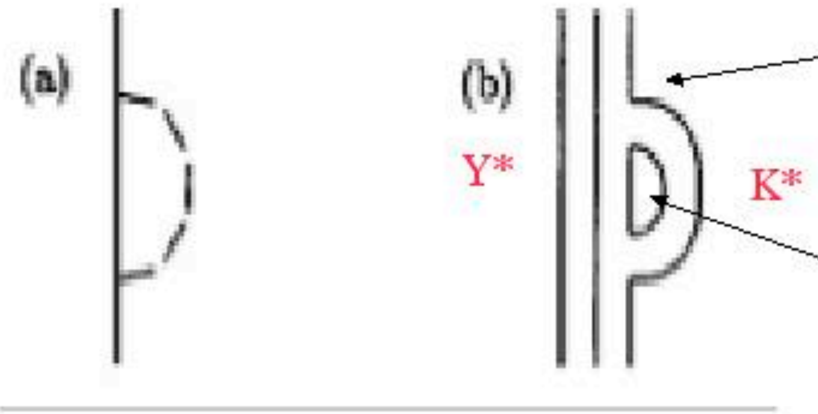


Note:

- sum over all intermediate states
  necessary for OZI rule
- linear interaction is preserved
  renormalization of the string constant

baryons

The $q\bar{q}$-pair creation mechanism is introduced at the microscopical level
→ string-like $q\bar{q}$ pair creation mechanism



Pair creation vertex

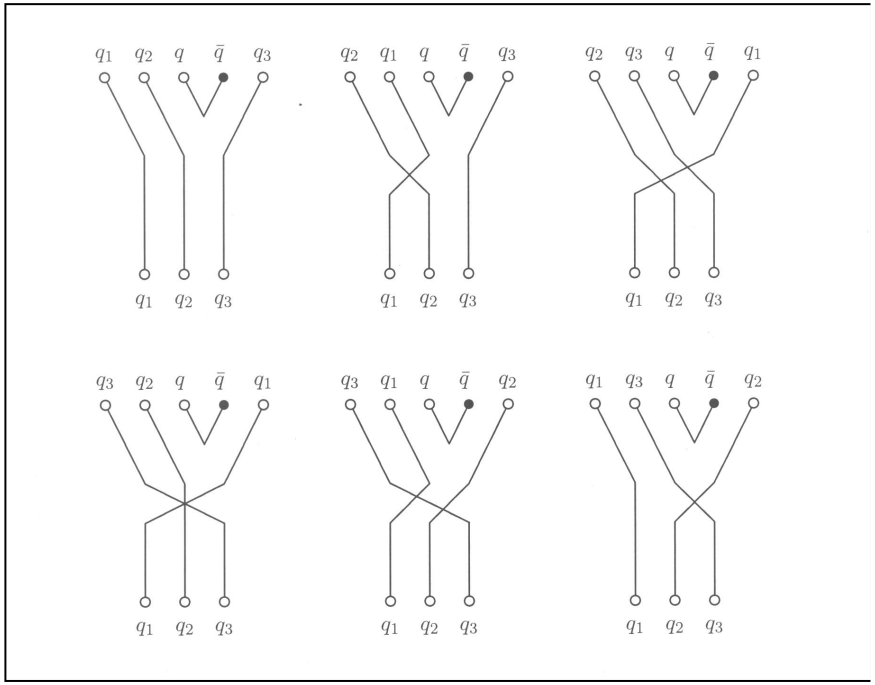
Σ over all the big towers of intermediate states

R. Bijker, E. Santopinto, Phys.Rev.C80:065210,2009

Problems that have been solved for baryons:

- sum over the big tower of intermediate states
- permutational symmetry

(both with group theoretical methods)



- find a quark QCD inspired pair creation mechanism $^3P_0$

- implementation of the mechanism in such a way to do not destroy the good CQMs results

The good magnetic moment results of the CQM are preserved by the UCQM

Bijker, Santopinto,Phys.Rev.C80:065210,2009.

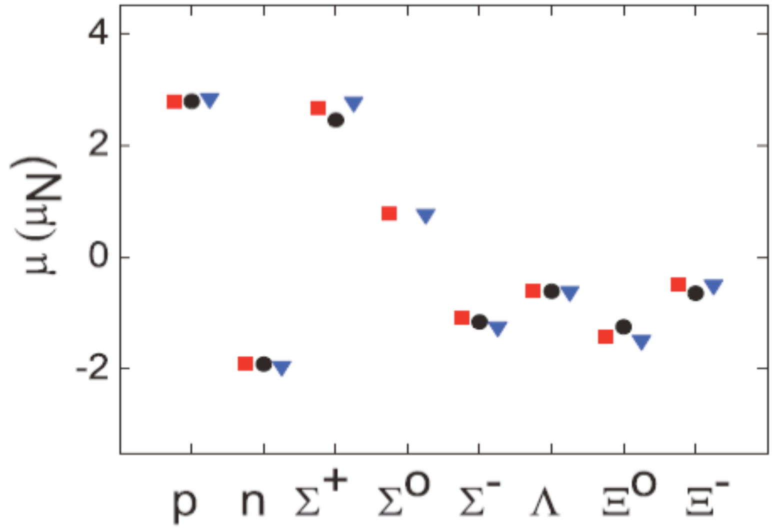
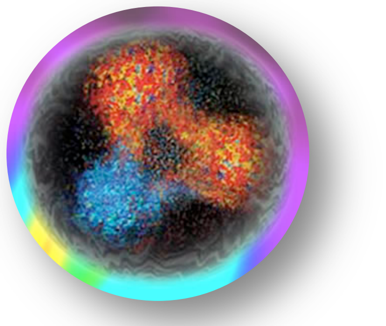


FIG. 3. (Color online) Magnetic moments of octet baryons: experimental values from the Particle Data Group [34] (circles), CQM (squares), and unquenched quark model (triangles).

Possible structure of the nucleon



3-quark core (about 0.5 fm)
+
quark-antiquark pairs
outside and inside the core

Unquenching the CQM:

effects on spectrum
e.m. excitation
consistent evaluation of electroproduction

Conclusions

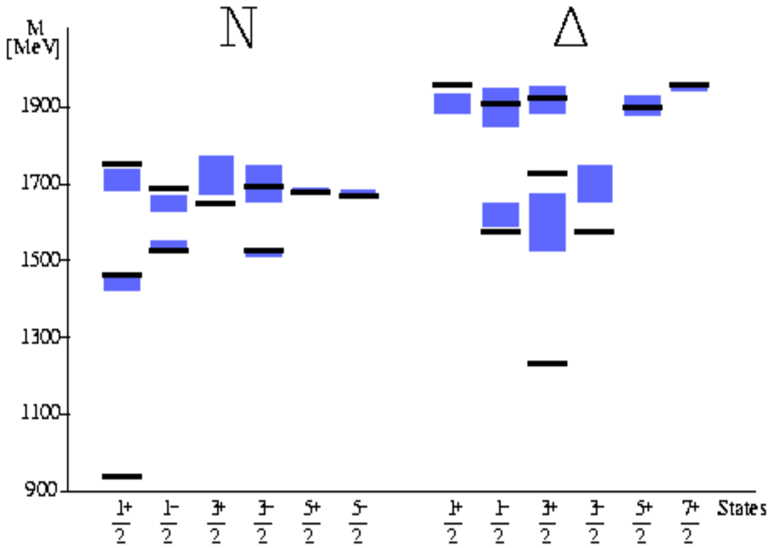
- CQM provide a good systematic frame for baryon studies
- fair description of e.m. properties (specially n-N* transitions)
- possibility of understanding missing mechanisms
- quark antiquark pairs effects
- unquenching: important break through

Hypercentral Model (2)

$$H_{3q} = 3m + \sum_{i=1}^{3} \frac{\mathbf{p}_i^2}{2m} + V(x) + H_{hyp} + H_I + H_{SI}$$

M. M. Giannini, E. Santopinto and A. Vassallo, Nucl. Phys. A 699 (2002) 308.

- $H_I = A_I \sum_{i<j} V^I(\mathbf{r}_i, \mathbf{r}_j, \sigma_I) \ \ \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$
- $H_{SI} = A_{SI} \sum_{i<j} V^{SI}(\mathbf{r}_i, \mathbf{r}_j, \sigma_{SI}) \ \ (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)$



Roper & S11 $\Leftarrow H_{SI}$

$N - \Delta \Leftarrow$ 50% H_{hyp}, 15% H_I, 50% H_{SI}

Photocouplings $(Q^2 = 0)$

	Ap 1/2 PDG	±	Ap 1/2 hCQM	Ap 3/2 PDG	±	Ap 3/2 hCQM	An 1/2 PDG	±	An 1/2 hCQM	An 3/2 PDG	±	An 3/2 hCQM
D13(1520)	-24	9	-65,7	166	5	66,8	-59	9	-1,4	-139	11	-61,1
D13(1700)	-18	13	8	-2	24	-10,9	0	50	12	-3	44	70,1
D15(1675)	19	8	1,4	15	9	1,9	-43	12	-36,6	-58	13	-51,1
D33(1700)	104	15	80,9	85	22	70,2						
F15(1680)	-15	6	-35,4	133	12	24,1	29	10	37,7	-33	9	14,8
F35(1905)	26	11	-16,6	-45	20	-50,5						
F37(1950)	-76	12	-28	-97	10	-36,2						
P11(1440)	-65	4	-87,7				40	10	57,9			
P11(1710)	9	22	42,5				-2	14	-21,7			
P13(1720)	18	30	94,1	-19	20	-17,2	1	15	-47,6	-29	61	3
P33(1232)	-135	6	-96,9	-250	8	-169						
S11(1535)	90	30	108				-46	27	-81,7			
S11(1650)	53	16	68,8				-15	21	-21			
S31(1620)	27	11	29,7									