EXTRACTION OF RESONANCE PROPERTIES
CAN WE MEASURE S-MATRIX POLES?

Sasa Ceci
Rudjer Boskovic Institute

Sixth International Workshop on πN PWA and the Interpretation of Baryon Resonances, 23-27 May, 2011 ● George Washington University, Washington DC, U.S.A.
We want to match theory and experiment

The matching point has to be uniquely defined, physical, and measurable

In excited nucleon physics, we match resonance parameters
\[
\sigma_{\text{tot}}(s) \sim |\tau(s)|^2 \\
\tau(s) \sim \frac{1}{D(s)} \\
D(s) = s - m_0^2 + \Sigma(s)
\]

- **Bare poles**: \( m_B = m_0 \)
- **Breit – Wigner**:
  \( \text{Re} \, D(x_0) = 0 \)
  \( m_{BW} = \sqrt{x_0} \)
  \( \Gamma_{BW} = \text{Im} \, \Sigma(x_0)/\sqrt{x_0} \)
- **S – matrix poles**:
  \( D(s_0) = 0 \)
  \( m_P = \text{Re} \, \sqrt{s_0} \)
  \( \Gamma_P = -2 \, \text{Im} \, \sqrt{s_0} \)
Resonance Parameters
Where Did The Curve Come From?

PDG \( N^* \) convention (\( W = \sqrt{s} \))

\[
\tau(W) = \frac{|r_P| e^{i\theta_P}}{m_P - W - i\Gamma_P/2} + \tau_b(W)
\]

PDG

- \( m_P = 1210 \text{ MeV} \)
- \( \Gamma_P = 100 \text{ MeV} \)
- \( |r_P| = 50 \text{ MeV} \)
- \( \theta_P = -47^\circ \)

Unitarity

\[
\text{Im} \tau(W) = \tau(W)^\dagger \tau(W)
\]

\[
|r_P| = \Gamma_P/2
\]

\[
\tau_b(W) = e^{i\theta_P/2} \sin \theta_P/2
\]
RESONANCE PARAMETERS
S-MATRIX POLES ARE (UN)MEASURABLE?

Figure 41.6: World data on the total cross section of $e^+e^-\rightarrow$ hadrons and the ratio $R(s) = \sigma(e^+e^-\rightarrow$ hadrons, $s)/\sigma(e^+e^-\rightarrow\mu^+\mu^-,s)$. $\sigma$ is experimentally corrected for initial state radiation and electron-positron vertex loops, $\sigma(e^+e^-\rightarrow\mu^+\mu-,s) = 4\pi\alpha^2(s)/3s$. Data errors are total below 2 GeV and statistic above 2 GeV. The curves are used to guide the eye: the broken one (green) is a naive quark-parton model prediction, and the solid one (red) is 3-loop pQCD prediction (see "Quantum Chromodynamics" section of this Review, Eq. (9.7) or, for more details, K. G. Chetyrkin et al., Nucl. Phys. B586, 56 (2000) (Erratum ibid. B634, 413 (2002)). Breit-Wigner parameterizations of $J/\psi$, $\psi(2S)$, and $\Upsilon(nS)$ are also shown. The full list of references to the original data and the details of the $R$ ratio extraction from them can be found in [arXiv:hep-ph/0312114]. Corresponding computer-readable data are available at http://pdg.lbl.gov/current/xsect/ (Courtsey of the COMPAS (Protvino) and HEPDATA (Durham) Groups, May 2010). See full-color version on color pages at end of book.
RESONANCE PARAMETERS
S-MATRIX POLES ARE (UN)MEASURABLE?

Statistics of data interval fits

\[ \tau(W) \approx \frac{|r_P| e^{i \theta_P}}{m_P - W - i \Gamma_P / 2} + \tau_b \]

\[ \sigma(W) \approx \text{const.} \times |\tau(W)|^2 \]

\[ \sigma(W) \approx \sigma_\infty \frac{(m_0 - W)^2 + \Gamma_0^2 / 4}{(m_P - W)^2 + \Gamma_P^2 / 4} \]

TABLE I: Pole parameters of Z obtained in this work. PDG values of pole and BW parameters are given for comparison.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>91159 ± 8</td>
<td>91162 ± 2</td>
<td>91188 ± 2</td>
</tr>
<tr>
<td>Γ</td>
<td>2484 ± 10</td>
<td>2494 ± 2</td>
<td>2495 ± 2</td>
</tr>
</tbody>
</table>
RESONANCE PARAMETERS
S-MATRIX POLES ARE (UN)MEASURABLE?

Figure 41.6: World data on the total cross section of $e^+e^-$ collisions and the ratio $R(s) = \sigma(e^+e^- \rightarrow \text{hadrons}, s)/\sigma(e^+e^- \rightarrow \mu^+\mu^-, s)$. The data errors are below 2 GeV and statistical above 2 GeV. The curves are guides: the broken one (green) is a naive quark-parton model prediction, and the solid one (red) is 3-loop pQCD prediction (see "Quantum Chromodynamics" section of this Review, Eq. (9.7) or, for more details, K. G. Chetyrkin et al., Nucl. Phys. B586, 56 (2000) (Erratum ibid. B634, 413 (2002)). Breit-Wigner parameterizations of $J/\psi$, $\psi(2S)$, and $\Upsilon(nS)$ are also shown. The full list of references to the original data and the details of the $R$ ratio extraction from them can be found in [arXiv:hep-ph/0312114]. Corresponding computer-readable data files are available at http://pdg.lbl.gov/current/xsect/ (courtesy of the COMPAS (Protvino) and HEpdata (Durham) Groups, May 2010). See full-color version on color pages at end of book.
TABLE II: Parameters of Υ(11020) meson. Pole parameters are results of this work.

<table>
<thead>
<tr>
<th>Pole</th>
<th>BABAR [1, 8]</th>
<th>PDG [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>M/MeV</td>
<td>10996 ± 2</td>
<td>11019 ± 8</td>
</tr>
<tr>
<td>Γ/MeV</td>
<td>79 ± 16</td>
<td></td>
</tr>
</tbody>
</table>


TABLE IV: The connection between S-matrix pole and Breit-Wigner parameters using only the PDG values.

<table>
<thead>
<tr>
<th>θ/°</th>
<th>M_p/MeV PDG[1]</th>
<th>M_p/MeV Eq. (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ(1232)</td>
<td>-23.0</td>
<td>1210 ± 1</td>
</tr>
<tr>
<td>N(1440)</td>
<td>-37.3</td>
<td>1365 ± 15</td>
</tr>
<tr>
<td>Υ(11020)</td>
<td>-46.8</td>
<td>10996a ± 2</td>
</tr>
<tr>
<td>Z</td>
<td>-1.26</td>
<td>91162 ± 2</td>
</tr>
</tbody>
</table>

*aBABAR value.

\[ \tau(11020) \] 

\[ jG(jPC) = 0^-(1-\cdot) \]

\[ \Gamma(11020) \] 

\[ \text{TABLE IV: The connection between S-matrix pole and Breit-Wigner parameters.} \]

\[ \Delta \approx 10 \text{ GeV} \]

\[ \Theta = \pi/2 \]

\[ \Gamma_p = \cos^2 \theta \Gamma_b \]

\[ M_p = M_b + \sin \theta \cos \theta \Gamma_b /2 \]

\[ \text{TABLE II: Parameters of } \Upsilon(11020) \text{ meson. Pole parameters are results of this work.} \]

\[ \text{TABLE III: } \Upsilon(11020) \text{ resonance parameters.} \]

\[ \text{TABLE IV: The connection between S-matrix pole and Breit-Wigner parameters using only the PDG values.} \]

\[ \text{TABLE IV: The connection between S-matrix pole and Breit-Wigner parameters using only the PDG values.} \]

\[ \text{TABLE IV: The connection between S-matrix pole and Breit-Wigner parameters using only the PDG values.} \]

\[ \text{TABLE IV: The connection between S-matrix pole and Breit-Wigner parameters using only the PDG values.} \]
In order to pinpoint the statistical strategy to be used, we did a substantial number of simulations with the data sets that had known poles and zeros. It turned out that the most successful strategy was to make an ordered list of all fit results, from best to worst, and then to drop the worst three quarters using the following goodness-of-fit measures:

- Akaike information criterion [11],
- Schwartz (Bayesian information) criterion [12],
- P-values of the extracted fit parameters (in particular, $M_p$ and $\Gamma_p$).

Eventually, we kept the intersection of the fits that satisfied all criteria.

Results closest to the original poles were produced by averaging the obtained pole positions of all good fits. The standard deviation turned out to be a good estimate for errors of obtained parameters.

All other approaches we tested, such as keeping only a handful of the best fits, or keeping just those whose values of reduced $\chi^2$ were close to one, failed to accurately reproduce the original pole parameters.

**TABLE III: N(1440) resonance parameters.**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$/MeV</td>
<td>1370 ± 6</td>
<td>1365 ± 15</td>
<td>1440 ± 3.0</td>
</tr>
<tr>
<td>$\Gamma$/MeV</td>
<td>197 ± 6</td>
<td>190 ± 30</td>
<td>300 ± 1.5</td>
</tr>
</tbody>
</table>

$W = M - i \Gamma / 2 = M_p - i \Gamma_p$.

When we introduce the BW mass and $\Gamma_p$, the $\chi^2$ becomes

$$
\chi^2 = \sum \left( \frac{W_{\text{data}} - W_{\text{fit}}}{\sigma_{\text{data}}} \right)^2,
$$

where the $\sigma_{\text{data}}$ is the error of $W_{\text{data}}$.

The field-theory reason for the resonance pole shift is $W / MeV$.

The plot represents the PDG range for the resonance pole. The $\Gamma_p$ in the plot shows the width of the resonance, which is obtained by our method. The plots for $1100, 1200, 1300, 1400, 1500, 1600, 1700$ MeV show the resonance pole obtained by our method.
In order to pinpoint the statistical strategy to be used, we did a substantial number of simulations with the data sets that had known poles and zeros. It turned out that the most successful strategy was to make an ordered list of all fit results, from best to worst, and then to drop the worst three quarters using the following goodness-of-fit measures:

- Akaike information criterion [11],
- Schwartz (Bayesian information) criterion [12],
- P-values of the extracted fit parameters (in particular, \( M_p \) and \( \Gamma_p \)).

Eventually, we kept the intersection of the fits that satisfied all criteria.

Results closest to the original poles were produced by averaging the obtained pole positions of all good fits. The standard deviation turned out to be a good estimate for errors of obtained parameters.

All other approaches we tested, such as keeping only a handful of the best fits, or keeping just those whose values of reduced \( \chi^2 \) were close to one, failed to accurately reproduce the original pole parameters.

**TABLE III: \( N(1440) \) resonance parameters.**

<table>
<thead>
<tr>
<th>( N(1440) )</th>
<th>Pole</th>
<th>Pole PDG [1]</th>
<th>BW PDG [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M/\text{MeV} )</td>
<td>1370 ± 6</td>
<td>1365 ± 15</td>
<td>1440 ± 30/20</td>
</tr>
<tr>
<td>( \Gamma/\text{MeV} )</td>
<td>197 ± 6</td>
<td>190 ± 30</td>
<td>300 ± 15/100</td>
</tr>
</tbody>
</table>
We could not find a simple (nor unique) parameterization of the amplitude that would result in the model independent Breit-Wigner parameters.

For narrow resonances with small background, BW parameters are similar to the S-matrix pole parameters.

Closest match, depending exclusively on the full amplitude in a model independent way, were K-matrix poles / PLB 659 (2008) 228
Analyticity is assumed to be necessary model/parameterization feature for the proper S-matrix pole extraction.

We just showed that the S-matrix pole mass can (sometimes) be extracted without assuming analyticity.

Is this the only such exception?
A CRAZY NON-ANALYTICITY EXAMPLE

How do we fix this?

\[ D(s) = s - m_0^2 - \Sigma(s) \]
\[ \text{Im } \Sigma(s) \sim \frac{q_{\pi\pi}(s)}{\sqrt{s}} \]
\[ \text{Im } \Sigma(s) = m_0 \Gamma_0 \frac{\sqrt{m_0^2}}{q_{\pi\pi}(m_0^2)} \frac{q_{\pi\pi}(s)}{\sqrt{s}} \]
\[ \text{Re } \Sigma(s) = \text{Disp Rel} \ldots \]

\[ m_0 = 500 \text{ MeV} \]
\[ \Gamma_0 = 500 \text{ MeV} \]

\[ m_0 \rightarrow 730 \text{ MeV} \]
Can we say now that **S-matrix mass** may be **measured** directly?

If not, what about the **Breit-Wigner mass**? Can it be measured?

In both cases we need a particular (mathematical) parameterization or some (physical) model.

All in all, by using **simple parameterization** and **local sequential** fitting excellent estimate of the **S-matrix pole mass** can be obtained.

Current parameterization works all right for the S-matrix pole widths (we are improving it!)

**The question:** should we really abandon what we have learned just because the approach was not unitary, and had no (proper!) analyticity?
Thank you for your attention!