Key issues:

2. Connecting amplitudes (real world) and resonances ("unphysical sheets")

3. What is the connection between resonances and QCD?

>> amplitude analysis << (analytic properties, dispersion relations, QCD and model input)
Outline:

* Aspects of partial wave dispersion relations

* isovector P-wave

* things to do: example forces vs particles

in collaboration with

Peng Guo, Marco Battaglieri, Raffaela De Vita, Matt Shepherd, Ryan Mitchel
Towards a connection between data and resonances

\[ \pi^+\pi^- \rightarrow \pi^0 \pi^0 \]

Partial amplitude \( A(s) = \frac{N(s)}{D(s)} \)

Potential (left hand cut)

Unitarity (right hand cut)

"forces": cross channel exchanges: production mechanisms are important (dynamically generated resonances)

Resonances: poles on unphysical sheets

CDD poles "cut in" and produce bumps
Towards a connection between data and resonances

\[ \pi\pi \rightarrow \pi\pi \]

Partial amplitude \( A(s) = \frac{N(s)}{D(s)} \)

CDD pole (of D)
(Zero of A)

Potential (left hand cut)

Unitarity (right hand cut)

CDD poles “cut in” and produce bumps

Resonances: poles on unphysical sheets

“Forces”: cross channel exchanges: production mechanisms are important (dynamically generated resonances)

CDD pole corresponds to an elementary particle (move out from inelastic cut when coupling is decreased)
"Schroedinger" equation for the scattering amplitude

\[ A(s) = \frac{1}{\pi} \int_{-\infty}^{0} ds' \frac{\Im A(s')}{s' - s} + \frac{1}{\pi} \int_{\text{st}}^{\infty} ds' \frac{\Im A(s')}{s' - s} \]

\[ \Im A(s) = R(s) \rho(s)|A(s)|^2 \]

input ("potential") : through crossing lhc is related to other physical amplitudes
“Schroedinger” equation for the scattering amplitude

\[ A(s) = \frac{1}{\pi} \int_{-\infty}^{0} ds' \frac{\text{Im}A(s')}{s' - s} + \frac{1}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\text{Im}A(s')}{s' - s} \]

\[ \text{Im}A(s) = R(s)\rho(s)|A(s)|^2 \]

input ("potential") : through crossing lhc is related to other physical amplitudes

- Dispersion relations ca 1970
- potential not known everywhere
- in principle many (\( \infty \)) channels contribute
- x-sections known over limited energy range
- solutions are not unique (CDD)
- analyticity in all channels: complex angular momentum

Thursday, May 26, 2011
“Schroedinger” equation for the scattering amplitude

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input (“potential”): through crossing lhc is related to other physical amplitudes

\[ ImA(s) = R(s)\rho(s)|A(s)|^2 \]

\* Dispersion relations ca 1970

\* QCD: interpretation of the ambiguities (CDD pols)

\* modern developments

\* potential not known everywhere

\* chiral symmetry: low energy constraints

\* in principle many (\(\infty\)) channels contribute

\* x-sections known over limited energy range

\* solutions are not unique (CDD)

\* analyticity in all channels: complex angular momentum
if single hadron states exist: lattice is the place to find them

- On finite volume multi-meson state and single hadron states are discrete.

- If there are single hadron states, use volume dependence to disentangle

- Continuum states can have any J,P,C but not single hadron states

- The choice of operators minimizes overlap with multi-meson states

- In the continuum these states should disappear through cuts onto unphysical sheets (as CDD poles)

>> there is evidence for single hadron states << (no surprising, quark model, CDD poles, etc.)

J. Dudek et al.
We will focus on the $l=1$, P-wave

PDG (before 1988) lists two resonances: rho(770) and rho(1600)

$\rho(770)$

P-wave $\pi\pi \rightarrow \pi\pi$ scattering data: phase shift and inelasticity

$\rho(1600)$?

B.Hyams et al. Nucl.Phys.B64(1973) 134
We will focus on the \( I=1, P \)-wave 

PDG (before 1988) lists two resonances: 
\( \rho(770) \) and \( \rho(1600) \)

PDG (after 1988) replaces \( \rho(1600) \) by 
\( \rho(1450) \) and \( \rho(1700) \)
analysis based on a coherent sum of 
three BW’s parametrization to explain 
both photoproduction \((2\pi, 4\pi)\) and 
pion form factor

\[ \rho(770) \]

P-wave \( \pi\pi \rightarrow \pi\pi \) scattering data: phase shift and inelasticity

\[ \rho(1600) \]?
**PDG 2010**
**rho(770) and rho(1600)**

\[ \rho(1700) \text{ DECAY MODES} \]

<table>
<thead>
<tr>
<th>Mode</th>
<th>Fraction ((\Gamma_i/\Gamma))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Gamma_1)</td>
<td>(4\pi)</td>
</tr>
<tr>
<td>(\Gamma_2)</td>
<td>(2(\pi^+\pi^-))</td>
</tr>
<tr>
<td>(\Gamma_3)</td>
<td>(\rho\pi\pi)</td>
</tr>
<tr>
<td>(\Gamma_4)</td>
<td>(\rho^0\pi^+\pi^-)</td>
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<tr>
<td>(\Gamma_5)</td>
<td>(\rho^0\pi^0\pi^0)</td>
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<tr>
<td>(\Gamma_6)</td>
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<tr>
<td>(\Gamma_7)</td>
<td>(a_1(1260)\pi)</td>
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<tr>
<td>(\Gamma_8)</td>
<td>(h_1(1170)\pi)</td>
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<tr>
<td>(\Gamma_9)</td>
<td>(\pi(1300)\pi)</td>
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<tr>
<td>(\Gamma_{10})</td>
<td>(\rho\rho)</td>
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<tr>
<td>(\Gamma_{11})</td>
<td>(\pi^+\pi^-)</td>
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<tr>
<td>(\Gamma_{12})</td>
<td>(\pi\pi)</td>
</tr>
<tr>
<td>(\Gamma_{13})</td>
<td>(K\bar{K}^*(892) + \text{c.c.})</td>
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<tr>
<td>(\Gamma_{14})</td>
<td>(\eta\rho)</td>
</tr>
<tr>
<td>(\Gamma_{15})</td>
<td>(a_2(1320)\pi)</td>
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<tr>
<td>(\Gamma_{16})</td>
<td>(K\bar{K})</td>
</tr>
<tr>
<td>(\Gamma_{17})</td>
<td>(e^+e^-)</td>
</tr>
<tr>
<td>(\Gamma_{18})</td>
<td>(\pi^0\omega)</td>
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</table>

*2 channel K-matrix parametrization*  
(Hyams et al. KK channel)
Fig. 8. (a) $m_{\pi^+\pi^-}$ from $\gamma p \rightarrow \pi^+\pi^- p$. Data points from ref. [42] corrected for a contribution from the $\rho_1$. Dashed line fits the data from ref. [43]. Full line is explained in the text. (b) $e^+ e^- \rightarrow \pi^+\pi^-$, $\sqrt{s} < 1.4$ GeV, ref. [44]; $1.4 \leq \sqrt{s} \leq 2.1$ GeV, ref. [45].
Amplitude construction I

2 channel K-matrix parametrization
K-matrix: use “many” uncontrolled CDD poles and left hand poles

\( \rho(s) \rightarrow \sqrt{s} \rho(s) \)

(Hyams et al. used an “approximation”)

and the K-matrix representation becomes

\[
[t^{-1}(s)]_{\alpha\beta} = [K^{-1}(s)]_{\alpha\beta} + \delta_{\alpha\beta}(s - s_{\alpha})\sqrt{s_{\alpha} - s}.
\]

The “standard” K-matrix approximation

\[
Im t^{-1} = -\rho \\
t^{-1}(s) = -i\rho(s)
\]

while what is should be is

\[
t^{-1}(s) = \frac{1}{\pi} \int ds' \frac{\rho(s')}{s' - s}
\]

\[
K_{\pi\pi} = \frac{\alpha_{\pi}^2}{M_{\rho}^2 - s} + \frac{\beta_{\pi}^2}{s_2 - s} + \gamma_{\pi\pi}, \\
K_{KK} = \frac{\beta_{K}^2}{s_2 - s} + \gamma_{KK} \\
K_{\pi K} = K_{K\pi} = \frac{\beta_{\pi} \beta_{K}}{s_2 - s} + \gamma_{\pi K}, \\
\]  

(32)
Analytical structure on first Riemann sheet

\[
t_{\alpha\beta}(s) = \frac{N_{\alpha\beta}(s)}{D_{\alpha\beta}(s)}
\]

\[
t_{\pi\pi}(s) = \lambda_{\pi\pi} \frac{(s - 4m_{\pi}^2)(s - z_{\pi\pi})}{(s - s_{L,1})(s - s_{L,2})} e^{\frac{s}{\pi} \int_{4m_{\pi}^2}^\infty ds' \frac{\varphi_{\pi\pi}(s')}{s'(s' - s - i0)}},
\]

\[
t_{\pi K}(s) = (q_{\pi} q_{K}) \lambda_{\pi K} \frac{(s - m_{\rho}^2)(s - z_{\pi K})}{(s - s_{L,1})(s - s_{L,2})} e^{\frac{s}{\pi} \int_{4m_{\pi}^2}^\infty ds' \frac{\varphi_{\pi K}(s')}{s'(s' - s - i0)}},
\]

\[
t_{KK}(s) = \lambda_{KK} \frac{(s - 4m_{K}^2)(s - z_{KK})}{(s - s_{L,1})(s - s_{L,2})} e^{\frac{s}{\pi} \int_{4m_{\pi}^2}^\infty ds' \frac{\varphi_{KK}(s')}{s'(s' - s - i0)}}.
\]

\[
\varphi_{\alpha\beta}(s) = \tan^{-1} \frac{Im[t_{\alpha\beta}(s)]}{Re[t_{\alpha\beta}(s)]}
\]
2 channel K-matrix fit looking good but...
2 channel K-matrix fit looking good but...

\[ \eta = 1 \]

\[ 4 \mu^2 \]

\[ 4m_k^2 \]
2 channel K-matrix fit    looking good but...

\[ s_L = -13.87 \text{ GeV}^2 \]
\[ s_L = -0.787 \text{ GeV}^2 \]

\[ \eta < 1 \]

\[ 4\mu^2 \]
\[ \eta = 1 \]
\[ 4m_k^2 \]

\[ s_{CDD} = \infty \]

\[ \text{in } t_{mm \rightarrow mm} \]

Thursday, May 26, 2011
2 channel K-matrix fit looking good but...

\[
s_L = -13.87 \text{ GeV}^2 \quad s_L = -0.787 \text{ GeV}^2
\]

\[
s_{CDD} = m_{\rho}^2
\]

\[
\eta < 1 \quad \text{in } t_{\pi\pi \to KK}
\]

\[
\eta = 1 \quad 4\mu^2 \quad \eta(\text{scDD}) = 1
\]

and another zero in KK -> KK
2 channel K-matrix fit   looking good but...

\[ s_L = -13.87 \text{ GeV}^2 \]
\[ s_L = -0.787 \text{ GeV}^2 \]

zeros

\[ 4\mu^2 \]
\[ \eta = 1 \]
\[ 4m_k^2 \]
\[ \eta(s_{CDD}) = 1 \]

\[ s_{CDD} = m_\rho^2 \]
\[ s_{CDD} = 3.884 \text{ GeV}^2 \]
\[ s_{CDD} = \infty \]

and another zero in KK -> KK

K-matrix in general unreasonable (the “reality” of the near poles and zeros can be checked)
Amplitude construction II

- use K-matrix in the data region
- extrapolate using Regge asymptotic

Regge asymptotics
**Amplitude construction II**

* use K-matrix in the data region
* extrapolate using Regge asymptotic

![Graphs showing Re t and Im t](image)

**Amplitude construction III**

recompute phase  \( \varphi_{\alpha\beta}(s) = \tan^{-1} \frac{\text{Im}[t_{\alpha\beta}(s)]}{\text{Re}[t_{\alpha\beta}(s)]} \)

and \( D(s) \)

\[
t_{\alpha\beta}(s) = \frac{N_{\alpha\beta}(s)}{D_{\alpha\beta}(s)}
\]

via Omnes-Muskhelishvili integral (right hand cut)

fit a simple \( N \) to reproduce data  \( N_{\alpha\beta}(s) = \frac{\lambda_{\alpha\beta}}{s - s_L} \)
Comparison with dispersion relation

\[ A(s) = \frac{1}{\pi} \int_{-\infty}^{0} ds' \frac{\text{Im}A(s')}{s'-s} + \frac{1}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\text{Im}A(s')}{s'-s} \]

crossing symmetry (low energy),
Regge limit (high energy)

\[ \text{Im}A(s) = \rho(s)|A(s)|^2 \]
assume elastic unitarity
Comparison with dispersion relation

\[ A(s) = \frac{1}{\pi} \int_{-\infty}^{0} ds' \frac{\text{Im} A(s')}{s'-s} + \frac{1}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\text{Im} A(s')}{s'-s} \]

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\* CDD pole required!
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and the quark model was born or as lattice suggests there are single hadron states in the spectrum

J. Dudek et al. 2011
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most resonances do not originate from meson-meson interactions but from the underlying QCD dynamics.
Resonances are not generated dynamically from interactions between other resonances!

\[ A(s) = \frac{1}{\pi} \int_{-\infty}^{0} ds' \frac{\text{Im}A(s')}{s'-s} + \frac{1}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\text{Im}A(s')}{s'-s} \]

Crossing symmetry (low energy), Regge limit (high energy).

CDD pole required!

Bootstrap failed.

And the quark model was born or as lattice suggests there are single hadron states in the spectrum.

Most resonances do not originate from meson-meson interactions but from the underlying QCD dynamics.

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\[ \text{Im}A(s) = \rho(s)|A(s)|^2 \]

Assume elastic unitarity.

J. Dudek et al. 2011
Comparison with dispersion relation

\[ A(s) = \frac{1}{\pi} \int_{-\infty}^{0} ds' \frac{ImA(s')}{s'-s} + \frac{1}{\pi} \int_{s_{th}}^{\infty} ds' \frac{ImA(s')}{s'-s} \]

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most resonances do not originate from meson-meson interactions but from the underlying QCD dynamics.

resonances are not generated dynamically from interactions between other resonances

how does it fit in with the success of dynamically generated resonance program from a unitarized chi-PT approach?
It does in U chi-PT resonances also come form short distance (QCD) physics via subtractions, cut offs, and not meson-meson interactions

J. Dudek et al. 2011
Applications $J/\Psi \rightarrow \pi^+\pi^0\pi^0, K^+K^-\pi^0$

Isobar model interactions (diagonal and channel mixing) and re-scattering (beyond isobar)
middle of Dalitz plot is quite empty and the plot of about scattering amplitudes especially from the Dalitz seems three pions rescattering reduce the amplitude from integral equation is shown in Figowo. As we can then the solutions of all let's try to use the following input of integral equation effect can change the channel we want to see how three body rescattering can be completely explained by single channel rescattering parametrization however obvious.

\[ \Psi_i = \frac{1}{2} \text{Re}[T(s,s_m)] \\]

\[ \zeta_1 \]

FIG. 5: Real and Imaginary part of \( J/\Psi(\Psi') \)...

re-scattering corrections are small

Khuri-Treiman (1960)
Pasquier-Pasquier (1968-1970)
Aitchison, Brehm (late 70’s)
Application to $J/\psi \to \pi^+\pi^-\pi^0$

**P-wave $\pi\pi\pi$**

$$\langle \pi^0\pi^+\pi^-, out|J/\psi(\lambda), \text{in}\rangle = (2\pi)^4\delta^4\left( \sum_{i=0,\pm} p_i - P \right) T_\lambda, $$

$$ T_\lambda = \sum_{i=0,\pm} \sum_{\mu=\pm,0} D_{\lambda,\mu}^1 (r_i) d_{\mu,0}^1(\theta_i) F_\mu(s) (s) $$

$$ F_{+1} = F_{-1} = F, F_0 = 0 $$

$$ Im\hat{F}_\pi(s+i\epsilon) = i\hat{t}_{\pi\pi}(s)\hat{t}_{\pi\pi}(s)\theta(s-4m_{\pi}^2) $$

$$ + i\hat{t}_{\pi\pi}(s)\hat{t}_{\pi\pi}(s)\theta(s-4m_{\pi}^2). $$

$$ Im\hat{F}_K(s+i\epsilon) = i\hat{t}_{K\pi}(s)\hat{t}_{K\pi}(s)\theta(s-4m_{\pi}^2) $$

$$ + i\hat{t}_{K\pi}(s)\hat{t}_{K\pi}(s)\theta(s-4m_{K}^2). $$

This unitary relations have simple (algebraic) solution provided $N_{\alpha\beta}(s) = N(s)$

$$ \sim \frac{N_{\alpha\beta}(s)}{D_{\alpha\beta}(s)} $$

$$ \sim \frac{P_{\alpha\beta}(s)}{D_{\alpha\beta}(s)} $$

---

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$J/\psi \rightarrow 3\pi$

\[ F(s) \sim \frac{1}{D_{\pi\pi\rightarrow\pi\pi}(s)} \]

\[ F(s) \sim \frac{1 + c_1 s}{D_{\pi\pi\rightarrow\pi\pi}(s)} + \frac{c_0}{D_{K\bar{K}\rightarrow\pi\pi}(s)} \]

BES Collaboration
Phys.Rev.D70:012005,2004
Pion formfactor: $|F_{\pi\pi}(s)|^2$

$F(s) \sim \frac{1 + c_1 s}{D_{\pi\pi\rightarrow\pi\pi}(s)} + \frac{c_0}{D_{K\bar{K}\rightarrow\pi\pi}(s)}$

Novel interpretation of asymptotic behavior
(M.Gorshteyn,P.Guos,AS (2011))
$J/\psi \rightarrow K\bar{K}\pi^0$

Broad bump in low mass KK region is difficult to be explained by a single BW.

---

**PRL 97, 142002 (2006)**

**CLEO**

BES collaboration(b)

PRELIMINARY

...
\[ e^{i\delta(E)} \cos \delta(E) \]

force=Regge exchange

Extended source

\[ e^{i\delta(E)} \frac{\sin \delta(E)}{k} \]

Compact source

\[ \pi \pi \pi \quad \pi \pi \pi \quad \pi \pi \pi \]

\[ \text{Mass of } \pi \pi \pi \text{ System (GeV/c)} \]

Thursday, May 26, 2011
Figure 11: Fit to the $1^+$ \( \rho \pi \) intensity from \( \pi^- p \rightarrow \pi^- \pi^- \pi^+ p \) at \( E_\pi = 25 \) and \( E_\pi = 40 \) GeV, CERN data [70], with (left) both long-range production from one pion exchange and short-range direct production and (right) short-range direct production only [63].
$e^{i\delta(E)} \cos \delta(E)$

$e^{i\delta(E)} \sin \delta(E) \frac{k}{k}$

Figure 11: Fit to the $1^+ \rho \pi$ intensity from $\pi^- p \rightarrow \pi^- \pi^+ \pi^+ p$ at $E_\pi = 25$ and $E_\pi = 40$ GeV, CERN data [70], with (left) both long-range production from one pion exchange and short-range direct production and (right) short-range direct production only [63].
*** PWA work-day ****
*** Saturday June 25th, JLab ****

9:00-10:00 PWA of existing photo-production data (20" each)

   9:00 - 9:20 PWA analysis of (old) CLAS data (g6c, 3pi, BNL amplitudes)
         Dennis Weygand
   9:20 - 9:40  PWA analysis of (new) CLAS data (g12, 3pi or summary of ongoing
               analyses, BNL amplitudes,
               Paul Eugenio
   9:40 - 10:00 -PWA analysis of (new) CLAS data (g11, 2pi , moments approach)
      Marco Battaglieri/Raffaella Devita

10:00-10:45 Discussion: Amplitude construction Mike Pennington

10:45 - 11:00 Coffee break

11:00-12:00 PWA of future photo-production data (20" each)

   11:00 - 11:20 - IU tools Matt Shepherd
   11:20 - 11:40 - New tools applied to CLAS/CLAS12 data (g11, 2k , moments
                    approach Derek Glazier
   11:40 - 12:00 New tools applied to GLUEX Curtis Meyer
   12:00 - 12:20 PWA analysis issues in charmonium Ryan Mitchell

12:20-13:30 Pizza Lunch

13:30-15:00 Discussion: Interfacing theory and experiment Adam Szczepaniak
Summary:

- Dispersion relations constrain partial waves

- CDD ambiguities: use lattice as guidance

- Resonances are generated from short distance physics and not from meson-meson rescattering

- Explore full analyticity and unitarity constraints from crossed channels (L-plane singularities)