Chiral perturbation theory with explicit spin-3/2 DOF

Outline

- Introduction
- Heavy Baryon ChEFT with explicit $\Delta(1232)$
  - $\pi N$ scattering, nuclear forces
- Beyond the Heavy Baryon approach
  - $V^2$CS & spin-dependent polarizabilities of the nucleon
- Summary and outlook
1. QCD  \[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \bar{q}(i\slashed{D} - \mathcal{M})q \]

2. Effective Lagrangian for hadronic DOF (\(\pi, N, \ldots\))

Most general form (infinitely many terms), restricted only by symmetries, approximate spontaneously-broken chiral symmetry
Chiral perturbation theory

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Perturb. expansion in powers of soft scales over the \( \chi \) symmetry breaking scale \( Q \in (p_i/\Lambda_\chi, M_\pi/\Lambda_\chi) \)

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4. Heavy baryon expansion
Special care needed to ensure that the nucleon mass does not spoil the power counting
\[ \delta m_N = -\frac{3g_A^2m_N^3}{(4\pi F_\pi)^2} \left( 16\pi^2 L(\mu) + \frac{1}{2} \ln \frac{m_N^2}{\mu^2} \right) + \mathcal{O}(d-4) \]
HB approach: (covariant) nonrelativistic expansion of the Lagrangian
The imaginary part of the triangle diagram is proportional to

\[ \arctan x \quad \text{with} \quad x = \frac{\sqrt{(t - 4M_\pi^2)(4m_N^2 - t)}}{t - 2M_\pi^2} \]

Near threshold, formally: \( x = O\left(\frac{m_N}{M_\pi}\right) \) \( \rightarrow \) the HB approach corresponds to the expansion:

\[ \arctan x = \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^2} + \ldots \]

which converges for \( |x| > 1 \). This condition is violated for \( |t - 4M_\pi^2| \leq \frac{M_\pi^4}{m_N^2} \) ...
ChPT: beyond heavy baryon

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**Solutions (extraction of the soft part of the amplitude):**

- **Infrared regularization:** expand the integrand, evaluate the integrals using DR and resum...
  Ellis & Tang; Becher & Leutwyler

- **Extended on-mass-shell renormalization:** covariant approach + DR + properly chosen subtraction to get rid of hard pieces.
  Fuchs, Gegelia, Japaridze, Scherer

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Inclusion of the spin-3/2 DOF

Why to include $\Delta(1232)$ as an explicit DOF?

- Low excitation energy: $\Delta \equiv m_\Delta - m_N = 293$ MeV $\sim 2M_\pi$
- Strong coupling to the pion-nucleon system
- In standard ChPT, effects of the $\Delta$ are included implicitly (through LECs)
  - Large values of the ($\Delta$-saturated) LECs may spoil convergence
  - Explicit treatment in SSE: $\Delta \sim O(M_\pi)$ Hemmert, Holstein, Kambor

Expansion parameter: $\epsilon \in \left( \frac{M_\pi}{\Lambda_\chi}, \frac{p_i}{\Lambda_\chi}, \frac{\Delta}{\Lambda_\chi} \right)$

Price to pay: more LECs, calculations considerably more involved...
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Inclusion of spin-3/2 fields (Rarita-Schwinger formalism) in chiral EFT is non-trivial...

- Maintaining the proper number of DOF in the interacting theory, chiral & gauge invariance
  - Pascalutsa; EE, Krebs, Meißner; Wies, Gegelia, Scherer; Shklyar, Lenske

- Off-shell parameters in the effective Lagrangian
  - Ellis, Tang; Pascalutsa; EE, Krebs, Meißner
Heavy baryon Chiral EFT with explicit $\Delta$ (1232)
πN scattering: \( \Delta \)-less vs \( \Delta \)-full

Tree level: πN scattering at NLO in SSE:

\[ Q^2, \text{no } \Delta \quad | \quad Q^2 \text{ with } \Delta \quad | \quad Q^3 \text{ no } \Delta \quad | \quad \text{EM98} \]

\[
\begin{array}{|c|c|c|c|}
\hline
&a^+_0 & b^+_0 & a^-_{0+} & b^-_{0+} & a^-_{1-} & a^+_{1-} & a^-_{1+} & a^+_{1+} \\
\hline
Q^2, \text{no } \Delta & 0.41 & -4.46 & 7.74 & 3.34 & -0.05 & -2.81 & -6.22 & 9.68 \\
Q^2 \text{ with } \Delta & 0.41 & -4.46 & 7.74 & 3.34 & -1.32 & -5.30 & -8.45 & 12.92 \\
Q^3 \text{ no } \Delta & 0.49 & -5.23 & 7.72 & 1.62 & -1.19 & -5.38 & -8.16 & 13.66 \\
EM98 & 0.41 \pm 0.09 & -4.46 & 7.73 \pm 0.06 & 1.56 & -1.19 \pm 0.08 & -5.46 \pm 0.10 & -8.22 \pm 0.07 & 13.13 \pm 0.13 \\
\hline
\end{array}
\]

The LECs \( c_1, c_2, c_3, c_4 \) are determined from a fit to S- and P-wave threshold parameters.

One finds:

- \( Q^2 \) with \( \Delta \) is performing better/worth than \( Q^2/Q^3 \) in the \( \Delta \)-less theory

- Numerical values of \( c_2, c_3 \) and \( c_4 \) strongly reduced once \( \Delta \) is included explicitly

\[
c_2 = -2.84 \rightarrow -0.25 , \quad c_3 = -3.87 \rightarrow -0.79 , \quad c_4 = 2.89 \rightarrow 1.33
\]

(all values in units of GeV\(^{-1}\))
πN scattering: Δ-less vs Δ-full

Similar conclusions from the leading loop analysis: $\epsilon^3$ more accurate than $Q^3$
Fettes, Meißner ’98

+ many more diagrams...

from: Fettes, Meißner, Nucl. Phys. A679 (01) 629

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Extensions to the $\Delta$-region

The generic assignment $\omega \sim M_\pi \sim \Delta$ in the SSE does not account for enhancement of the one-delta-reducible graphs in the delta region (and thus converges only for $\omega$ well below $\Delta$). Extension to the Delta-region requires resummation of $1\Delta R$ graphs.

Ellis, Tang '98; Pascalutsa, Phillips '03; Pascalutsa, Vanderhaeghen '05-'08; Long, van Kolck '10

$\gamma p$ differential cross section in the $\delta$-expansion

$P_{33}$ $\pi N$ phase shift

$\delta$-expansion: $\delta \in \left( \frac{\Delta}{\Lambda_X}, \frac{M_\pi}{\Delta} \right)$

from: Pascalutsa, Phillips, PRC67 (03) 055202

from: Long, van Kolck, NPA840 (10) 39
Implications for nuclear forces

Given the importance of the $\Delta$ in the $\pi N$ system, one expects implications for nuclear forces as well...
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Two-nucleon force in EFT with and without $\Delta$

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Ordonez, Ray & van Kolck ’96, Kaiser, Gerstendorfer & Weise ’98


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Implications for nuclear forces

$2\pi$-exchange up to N$^2$LO

with $\Delta$

without $\Delta$

- a much better convergence for the potential when $\Delta$ is included explicitly
- clearly visible in NN peripheral waves

$^3F_3$ partial wave up to N$^2$LO

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V\textsuperscript{2}CS in EFT with explicit \Delta: Lorentz-invariant approach

in collaboration with Veronique Bernard, Hermann Krebs & Ulf-G. Mei\ss{}ner

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V²CS and polarizabilities

Spin-dependent forward tensor for V²CS:

\[
i \int d^4 x \, e^{i q \cdot x} \langle PS | T J^\mu(x) J^\nu(0) | PS \rangle = -\frac{i}{2} \epsilon^{\mu \nu \alpha \beta} q_\alpha \left[ S_\beta S_1(\nu, Q^2) + \frac{1}{m_N^2} (P \cdot q S_\beta - S \cdot q P_\beta) S_2(\nu, Q^2) \right]
\]

The structure functions \( S_{1,2} \) are related via dispersion integrals to the ones \( G_{1,2} \) measured in polarized spin-dependent inclusive lepton-nucleon scattering.
Spin-dependent forward tensor for $V^2$CS:

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For small photon energy, $\bar{S}_{1,2}$ (elastic contrib. subtracted) can be expanded powers of $\nu^2$:

\[ \bar{S}_1(\nu, Q^2) = \sum_{i=0}^{\infty} \bar{S}_1^{(2i)}(0, Q^2) \nu^{2i}, \quad \bar{S}_2(\nu, Q^2) = \sum_{i=0}^{\infty} \bar{S}_2^{(2i+1)}(0, Q^2) \nu^{2i+1} \]
Spin-dependent forward tensor for $V^2$CS:

$$i \int d^4x \ e^{iqx} \langle PS|T J^\mu(x) J^\nu(0)|PS\rangle = -\frac{i}{2} \epsilon^{\mu\nu\alpha\beta} q_\alpha \left[ S_\beta \ S_1(\nu, Q^2) + \frac{1}{m_N^2} (P \cdot q S_\beta - S \cdot q P_\beta) S_2(\nu, Q^2) \right]$$

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moments of structure functions
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$$i \int d^4x e^{iq\cdot x} \langle PS|TJ^\mu(x)J^\nu(0)|PS\rangle = -\frac{i}{2}e^{\mu\nu\alpha\beta} q_\alpha \left[ S_\beta S_1(\nu, Q^2) + \frac{1}{m_N^2} (P \cdot q S_\beta - S \cdot q P_\beta) S_2(\nu, Q^2) \right]$$

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Generalized polarizabilities:

$$\gamma_0(Q^2) = \frac{1}{8\pi} \left( \tilde{S}^{(2)}_1(0, Q^2) - \frac{Q^2}{m_N} \tilde{S}^{(3)}_2(0, Q^2) \right), \quad \delta_0(Q^2) = \frac{1}{8\pi} \left( \tilde{S}^{(2)}_1(0, Q^2) + \frac{1}{m_N} \tilde{S}^{(1)}_2(0, Q^2) \right)$$

can be measured (using dispersion integrals) and computed in ChPT.
V²CS and polarizabilities

Previous calculations within chiral EFT

- HB ChPT up to order $Q^4$
  Ji, Kao, Osborne, Spitzenberg, Vanderhaeghen, Birse, McGovern, Kumar
  - Only well-known LECs from $\mathcal{L}^{(2)}_{\pi N}$ contribute
  - Large discrepancy for $\gamma_0$ even at the photon point

- HB ChPT with explicit $\Delta$ up to order $\varepsilon^3$ (leading loop)
  Kao, Spitzenberg, Vanderhaeghen '02
  - Sizable, negative contribution to $\gamma_0$ ; $\delta_0$ less sensitive...

- IR ChPT up to order $Q^4$
  Bernard, Hemmert, Meißner '03
  - Test of the HB expansion: poor/good convergence for $\gamma_0/\delta_0$:
    
    $\gamma^P_0 = 4.45 - 8.31 + 6.03 + 3.22 + O(\mu^2) = 4.64$
    $\delta^P_0 = 2.23 - 0.75 + 0.53 + 0.12 + O(\mu^2) = 2.04$

    (values at the photon point, all numbers in units of $10^{-4} \text{fm}^4$)

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FIGURE 6. 

The left panel compares HBChPT at $O(p^4)$ with data from E94010 [43]. Solid lines: MAID [14], data from E94010 [43]. The right panel shows a comparison of the generalized polarizabilities with other models.

- IR: $Q^4 + \Delta$ (tree)
- HB: $\varepsilon^3$
- HB: $Q^4$
- IR: $Q^4 + \Delta/VM$ (tree)
- MAID

The validity of the BC sum rule is demonstrated by the upper panel of Fig. 7. The data points at the highest values of $Q^2$ show agreement with the theoretical predictions.$^p_4$
We worked out the contributions of the $\Delta$ up to $\varepsilon^3$ within the covariant framework (both IR and DR).

The LECs $h_A/b_1$ determined from the strong/em width of the $\Delta$:

$h_A = 1.45 \pm 0.02$

$b_1 = (4.84 \pm 0.21) m_N^{-1}$
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V\textsuperscript{2}CS and polarizabilities

- Strong improvement for $\gamma_0$; insensitivity of $\delta_0$ to the $\Delta$-contributions confirmed
- HB expansion in the presence of $\Delta$ seems not to converge even at the photon point
- Consequently, some scheme dependence observed (IR vs DR), further study needed...
- Order-$\varepsilon^4$ calculation needed to draw final conclusions
Summary and outlook

- Explicit treatment of the $\Delta(1232)$ in chiral EFT within the SSE improves the description of the pion-nucleon system and nuclear forces.

- Generalized forward spin polarizabilities are calculated up to order $\varepsilon^3$ in the Lorentz-invariant formulation. $\Delta$ loop contributions to $\gamma_0$ are large and strongly improve the description of the data; $\delta_0$ appears to be less sensitive.

Still to be done (work in progress):

- Nuclear forces to order $\varepsilon^4$; $V^2$CS at order $\varepsilon^4$ in Lorentz-invariant ChEFT; global analysis of $\pi N$ data, form factors, $\pi$ photo-/electroproduction in Baryon ChEFT with explicit $\Delta$
## Implications for nuclear forces

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EE, Krebs, Meißner, NPA 806 (08) 65
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- Δ contributions at N^3LO are large!
- Long-range part is parameter free
- Much richer spin/isospin structure compared to the Illinois model
- Complete analysis still to be done

Krebs, E.E., in progress

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