Excited state meson and baryon spectroscopy from Lattice QCD

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Auspices of the Hadron Spectrum Collaboration
Lattice QCD

Goal: resolve highly excited states

$N_f = 2 + 1 \ (u,d + s)$

Anisotropic lattices:

$(a_s)^{-1} \sim 1.6 \text{ GeV}, \ (a_t)^{-1} \sim 5.6 \text{ GeV}$
Spectrum from variational method

Two-point correlator

\[ C(t) = \langle 0 | \Phi'(t) \Phi(0) | 0 \rangle \]

\[ C(t) = \sum_n e^{-E_n t} \langle 0 | \Phi'(0) | n \rangle \langle n | \Phi(0) | 0 \rangle \]

Matrix of correlators

\[
C(t) = \begin{bmatrix}
\langle 0 | \Phi_1(t) \Phi_1(0) | 0 \rangle & \langle 0 | \Phi_1(t) \Phi_2(0) | 0 \rangle & \cdots \\
\langle 0 | \Phi_2(t) \Phi_1(0) | 0 \rangle & \langle 0 | \Phi_2(t) \Phi_2(0) | 0 \rangle & \cdots \\
& & \ddots
\end{bmatrix}
\]

Diagonalize:
- eigenvalues \( \rightarrow \) spectrum
- eigenvectors \( \rightarrow \) spectral “overlaps”

Each state optimal combination of \( \Phi_i \)

\[ \Omega_n = v_1^n \Phi_1 + v_2^n \Phi_2 + \ldots \]

Benefit: orthogonality for near degenerate states
Operator construction

Baryons: permutations of 3 objects

Permutation group $S_3$: 3 representations

- **Symmetric**: 1-dimensional
  - e.g., uud+udu+duu
- **Antisymmetric**: 1-dimensional
  - e.g., uud-udu+duu-...
- **Mixed**: 2-dimensional
  - e.g., udu - duu & 2duu - udu - uud

Color antisymmetric $\rightarrow$ Require Space [Flavor Spin] symmetric

Classify operators by these permutation symmetries:
- Leads to rich structure
Couple derivatives onto single-site spinors: Enough D’s – build any J,M

\[ \mathcal{O}^{JM} \leftarrow (CGC')_{i,j,k} \left( \vec{D} \right)_{i} \left( \vec{D} \right)_{j} [\Psi]_{k} \]

Only using symmetries of continuum QCD

Operators $\leftarrow$ Derivatives

Use all possible operators up to 2 derivatives (transforms like 2 units orbital angular momentum)

1104.5152
Baryon operator basis

3-quark operators with up to two covariant derivatives – projected into definite isospin and continuum \( J^P \)

Operators \( \left( \begin{array}{cc} \text{Flavor} & \text{Dirac} \\ \text{Space} & \text{symmetry} \end{array} \right)^{J^P} \)

Spatial symmetry classification:

Nucleons: \( N^{2S+1L_{\pi}} J^P \)

Symmetry crucial for spectroscopy

By far the largest operator basis ever used for such calculations

<table>
<thead>
<tr>
<th>( J^P )</th>
<th>#ops</th>
<th>E.g., spatial symmetries</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J=1/2^- )</td>
<td>24</td>
<td>( N^2P_M \frac{1}{2}^- ), ( N^4P_M \frac{1}{2}^- )</td>
</tr>
<tr>
<td>( J=3/2^- )</td>
<td>28</td>
<td>( N^2P_M 3/2^- ), ( N^4P_M 3/2^- )</td>
</tr>
<tr>
<td>( J=5/2^- )</td>
<td>16</td>
<td>( N^4P_M 5/2^- )</td>
</tr>
<tr>
<td>( J=1/2^+ )</td>
<td>24</td>
<td>( N^2S_{S \frac{1}{2}^+} ), ( N^2S_{M \frac{1}{2}^+} ), ( N^4D_{M \frac{1}{2}^+} ), ( N^2P_{A \frac{1}{2}^+} )</td>
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<td>( J=3/2^+ )</td>
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</tr>
<tr>
<td>( J=7/2^+ )</td>
<td>4</td>
<td>( N^4D_{M7/2^+} )</td>
</tr>
</tbody>
</table>
Operators are not states

Two-point correlator

\[ C(t) = \langle 0 | \Phi'(t) \Phi(0) | 0 \rangle \]

\[ C(t) = \sum_n e^{-E_n t} \langle 0 | \Phi'(0) | n \rangle \langle n | \Phi(0) | 0 \rangle \]

Full basis of operators: many operators can create same state

Spectral “overlaps”

\[ \langle n; J^P | \Phi_i | 0 \rangle = Z_i^n \]

States may have subset of allowed symmetries
Spin identified Nucleon & Delta spectrum

Statistical errors < 2%

$m_\pi \sim 520\text{MeV}$
arXiv:1104.5152
Spin identified Nucleon & Delta spectrum

SU(6) x O(3) counting
No parity doubling

$\rho \approx 520\text{MeV}$
Spin identified Nucleon & Delta spectrum

Discern structure: spectral overlaps

\[ m_\pi \sim 520\text{MeV} \]

\[ \begin{align*}
N^* & \quad \begin{cases}
\mathcal{P}_{1/2}^4, \mathcal{P}_{3/2}^4, \mathcal{P}_{5/2}^4, \\
\mathcal{P}_{1/2}^2, \mathcal{P}_{3/2}^2, \mathcal{P}_{5/2}^2,
\end{cases} \\
\mathcal{S}_{1/2}^2 & \quad \begin{cases}
\mathcal{P}_{1/2}^4, \mathcal{P}_{3/2}^4, \mathcal{P}_{5/2}^4, \\
\mathcal{P}_{1/2}^2, \mathcal{P}_{3/2}^2, \mathcal{P}_{5/2}^2,
\end{cases}
\end{align*} \]

\[ \begin{align*}
\Delta^* & \quad \begin{cases}
\mathcal{P}_{1/2}^2, \mathcal{P}_{3/2}^2, \mathcal{P}_{5/2}^3, \\
\mathcal{P}_{1/2}^5, \mathcal{P}_{3/2}^5, \mathcal{P}_{5/2}^5,
\end{cases} \\
\mathcal{S}_{3/2}^4 & \quad \begin{cases}
\mathcal{P}_{1/2}^2, \mathcal{P}_{3/2}^2, \mathcal{P}_{5/2}^3, \\
\mathcal{P}_{1/2}^5, \mathcal{P}_{3/2}^5, \mathcal{P}_{5/2}^5,
\end{cases}
\end{align*} \]

[56,0\text{^{-}}] S-wave

[70,1\text{^{-}}] P-wave

[56,0\text{^{+}}] S-wave

[70,1\text{^{-}}] P-wave

arXiv:1104.5152
**Nucleon $J^-$**

Overlaps

$Z_i^n = \langle J^- | \Phi_i | 0 \rangle$

Little mixing in each $J^-$

Nearly "pure" [S= 1/2 & 3/2] $1^-$

Thomas Jefferson National Accelerator Facility
Discern structure: spectral overlaps

Significant mixing in $J^+$

13 levels/ops

8 levels/ops

$N^*$

$\Delta^*$

$N=2$  $J^+$  Nucleon & Delta spectrum
Near degeneracy in $\frac{1}{2}^+$ consistent with SU(6) O(3) but heavily mixed

Discrepancies??
Operator basis – spatial structure

What else? Multi-particle operators
Spectrum of finite volume field theory

**Missing states:** “continuum” of multi-particle scattering states

- Infinite volume: continuous spectrum
  \[ E(p) = 2\sqrt{m^2_\pi + p^2} \]

- Finite volume: discrete spectrum

Deviation from (discrete) free energies depends upon interaction - contains information about scattering phase shift

\[ \Delta E(L) \leftrightarrow \delta(E) : \text{Lüscher method} \]
The idea: 1 dim quantum mechanics

Two spin-less bosons: \( \psi(x,y) = f(x-y) \)

\[
\left[ -\frac{1}{m} \frac{d^2}{dz^2} + V(z) \right] f(z) = E f(z)
\]

Solutions \( f(z) \rightarrow \cos [k|z| + \delta(k)] \) \( E = k^2/m \)

Quantization condition when \(-L/2 < z < L/2\)

\( kL + 2\delta(k) = 0 \mod 2\pi \)

Same physics in 4 dim version, but messier
Provable in a QFT
Finite volume scattering

Lüscher method
-scattering in a periodic cubic box (length $L$)
-finite volume energy levels $E(L) \rightarrow \delta(E)$

E.g. just a single elastic resonance

At some $L$, have discrete excited energies

\[ \pi \pi \rightarrow \rho \rightarrow \pi \pi \]
\[ \pi N \rightarrow \Delta \rightarrow \pi N \]
I=1 $\pi\pi$ : the "$\rho$"

Extract $\delta_1(E)$ at discrete $E$

Extracted coupling: stable in pion mass

Stability a generic feature of couplings??

Feng, Jansen, Renner, 1011.5288
What is a form-factor off of a resonance?

What is a resonance? Spectrum first!

Extension of scattering techniques:
- Finite volume matrix element modified

\[
\langle N|J_\mu|N^*\rangle_\infty(Q^2,E) \leftarrow [\delta'(E) + \Phi'(E)] \langle N|J_\mu|N^*\rangle_{\text{volume}}
\]

Requires excited level transition FF’s: some experience
- Charmonium E&M transition FF’s (1004.4930)
- Nucleon 1st attempt: “Roper”->N (0803.3020)

Range: few GeV^2
Limitation: spatial lattice spacing
Some candidates: determine phase shift
Somewhat elastic

$m_\pi \sim 400$ MeV
Isoscalars: flavor mixing determined

Will need to build PWA within mesons

Exotics

$m_{\pi} = 396$ MeV
-isoscalar
-isovector

YM glueball

1102.4299
Prospects

• Strong effort in excited state spectroscopy
  - New operator & correlator constructions → high lying states
• Results for baryon excited state spectrum:
  - No “freezing” of degrees of freedom nor parity doubling
  - Broadly consistent with non-relativistic quark model
  - Add multi-particles → baryon spectrum becomes denser
• Short-term plans: resonance determination!
  - Lighter pion masses (230MeV available)
  - Extract couplings in multi-channel systems
  - This includes $\pi, \eta, K$ in final states
• Form-factors:
  - Use previous resonance parameters: initially, $Q^2 \sim \text{few GeV}^2$
Backup slides

• The end
Baryon Spectrum

“Missing resonance problem”
• What are collective modes?
• What is the structure of the states?

Nucleon Mass Spectrum (Exp): 4*, 3*, 2*

PDG uncertainty on B-W mass

Nucleon spectrum
Finite volume scattering: Lüscher method

Excited state spectrum at a single volume

Do more volumes, get more points

Discrete points on the phase shift curve

energy levels
L ~ 2.9 fm

L ~ 2.9 fm
The interpretation

**DOTS:** Finite volume QCD energy eigenvalues

**LINES:** Non-interacting two-particle states have known energies

\[
E(p) = 2 \sqrt{m_\pi^2 + n \left(\frac{2\pi}{L}\right)^2}
\]

“non-interacting basis states”

\[
\begin{align*}
|q\bar{q}\rangle & \quad |\pi\pi_{100}\rangle \\
& \quad |\pi\pi_{110}\rangle \\
& \quad |\pi\pi_{111}\rangle
\end{align*}
\]

Level repulsion - just like quantum mechanical pert. theory
The interpretation

\[ \sim \sqrt{1} |q\bar{q}\rangle - \sqrt{0.9} |\pi\pi_{100}\rangle \]

\[ \sim \sqrt{0.9} |q\bar{q}\rangle + \sqrt{1} |\pi\pi_{100}\rangle \]
Phase Shifts demonstration: $I=2$ $\pi\pi$

$\pi\pi$ isospin=2

Extract $\delta_0(E)$ at discrete $E$

No discernible pion mass dependence

1011.6352 (PRD)
Phase Shifts: demonstration

$\pi\pi$ isospin=2

$\delta_2(E)$

Graph showing $\delta_2(\epsilon)$ against $k^2 / \text{GeV}^2$. The graph includes data points for different energies with error bars, indicating measurements at 524 MeV, 444 MeV, and 396 MeV. Different energy levels are represented by symbols and colors: blue squares for $16^3$, green triangles for $16^3$, blue circles for $20^3$, green diamonds for $20^3$, red triangles for $24^3$, and red circles for $24^3$. The graph also includes a legend for data sets: Hoogland, Losty, Cohen, and Durusoy.
Extract $\delta_0(E)$ and $\delta_2(E)$ at discrete $E$.