Spin observables in photon- and pion-nucleon interactions

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Motivation

- Strict $\chi$PT (a systematic expansion in terms of pion mass and momentum) has a limited range of convergence (for $\pi(\gamma)N$ scattering: threshold (or subthreshold (P.Büttiker, U.-G. Meißner)) region).

- Higher energies–Phenomenological models:(Jülich model, Giessen model, ...)

  Explicit treatment of $u$ and $t$-channel analyticity is important.
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- Higher energies–Phenomenological models: (Jülich model, Giessen model, ...)
  Explicit treatment of $u$ and $t$-channel analyticity is important. (A. Gasparyan and M. F. M. Lutz, Nucl. Phys. A 848 (2010) 126)
The scheme

- 2-channel approximation ($\pi N$ and $\gamma N$) $\Rightarrow$ one is limited by energies $\sqrt{s} \simeq 1300$ MeV
- Low energy: tree level amplitude ($u$ and $t$-channel cuts are taken into account) + one loop to chiral order $Q^3$ (in HBChPT)
- Analyticity and unitarity are used to extrapolate the amplitude beyond threshold region.
- Fit free parameters to data.
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Partial Wave Dispersion Relation with subtraction at
\( \sqrt{s} = \mu_M = m_N \)

Unitarity and Analyticity:

\[
T_{ab}(\sqrt{s}) = U_{ab}(\sqrt{s}) + \sum_{c,d} \int_{\text{wthrs}}^{\infty} \frac{dw}{\pi} \frac{\sqrt{s} - \mu_M}{w - \mu_M} T_{ac}(w) \rho_{cd}(w) \frac{T_{db}^*(w)}{w - \sqrt{s} - i\epsilon}.
\]

\( U(\sqrt{s}) \) contains only left hand cuts

\( \rightarrow U(\sqrt{s}) \) can be analitycally continued beyond threshold region (conformal mapping)
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\[\implies U(\sqrt{s}) \text{ can be analytically continued beyond threshold region (conformal mapping)}\]
CDD poles and resonances

\[ T_{ab}(\sqrt{s}) = U_{ab}(\sqrt{s}) + \sum_{c,d} \int_{w_{thrs}}^{\infty} dw \frac{\sqrt{s} - \mu_M}{\pi} \frac{T_{ac}(w) \rho_{cd}(w) T_{db}^*(w)}{w - \sqrt{s} - i\epsilon}. \]

Non-linear integral equation may have multiple solutions!

CDD poles $\iff$ resonances (\(\Delta\), Ropper)
CDD poles and resonances

\[ T_{ab}(\sqrt{s}) = U_{ab}(\sqrt{s}) + \sum_{c,d} \int_{w_{\text{thr}}}^{\infty} dw \frac{\sqrt{s} - \mu_M}{\pi} w - \mu_M \frac{T_{ac}(w) \rho_{cd}(w) T_{db}^*(w)}{w - \sqrt{s} - i\epsilon}. \]

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CDD poles ⇔ resonances (Δ, Ropper)
\( \pi N \) phase shifts (\( S \) and \( P \) waves)

- **\( S_{11} \)**
  - \( \delta \) [degree] vs. \( s^{1/2} \) [MeV]
  - Plot shows the phase shift \( \delta \) for \( S_{11} \) channel.

- **\( S_{31} \)**
  - \( \delta \) [degree] vs. \( s^{1/2} \) [MeV]
  - Plot shows the phase shift \( \delta \) for \( S_{31} \) channel.

- **\( P_{11} \)**
  - \( \delta \) [degree] vs. \( s^{1/2} \) [MeV]
  - Plot shows the phase shift \( \delta \) for \( P_{11} \) channel.

- **\( P_{13} \)**
  - \( \delta \) [degree] vs. \( s^{1/2} \) [MeV]
  - Plot shows the phase shift \( \delta \) for \( P_{13} \) channel.

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  - \( \delta \) [degree] vs. \( s^{1/2} \) [MeV]
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\( \pi N \) phase shifts (\( S \) and \( P \) waves)

\[ \begin{align*}
\delta & \quad \text{[degree]} \\
S_{11} & \quad \text{[degree]} \\
S_{31} & \quad \text{[degree]} \\
P_{11} & \quad \text{[degree]} \\
P_{31} & \quad \text{[degree]}
\end{align*} \]

\[ \begin{align*}
& \text{[MeV]} \\
\text{\( s^{1/2} \)} & \quad [\text{MeV}] \\
\text{\( s^{1/2} \)} & \quad [\text{MeV}]
\end{align*} \]

$\pi N$ phase shifts ($S$ and $P$ waves)

\[ \begin{align*}
\delta_{S_{11}} &\quad \text{for } S_{11} \\
\delta_{P_{11}} &\quad \text{for } P_{11} \\
\delta_{P_{31}} &\quad \text{for } P_{31} \\
\delta_{P_{33}} &\quad \text{for } P_{33}
\end{align*} \]
Pion photoproduction

$s$- and $p$-waves multipoles and differential observables are well described up to $\sqrt{s} = 1300$ MeV (at order $Q^3$).

Threshold data are not included in the fit! (Isospin symmetric case)
Differential cross section for Compton scattering off the proton

\[
d\sigma/d\Omega_{\text{C.M.}} \quad \text{[nb/sr]}
\]

\[ s^{1/2} = 995 \text{ MeV} \]

\[ s^{1/2} = 1049 \text{ MeV} \]

\[ s^{1/2} = 1014 \text{ MeV} \]

\[ s^{1/2} = 1065 \text{ MeV} \]

\[ s^{1/2} = 1078 \text{ MeV} \]

\[ s^{1/2} = 1084 \text{ MeV} \]

\[ s^{1/2} = 1105 \text{ MeV} \]

\[ s^{1/2} = 1145 \text{ MeV} \]

\[ J \leq \frac{3}{2} \]

\[ \text{full} \]
Differential cross section for Compton scattering off the proton

No additional free parameters!
Differential cross section for Compton scattering off the proton

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\[ J \leq \frac{3}{2} \text{ full} \]
Differential cross section for Compton scattering off the proton

\[ \frac{d\sigma}{d\Omega} \text{ C.M.} \] [nb/sr]

- $s^{1/2} = 1157$ MeV
- $s^{1/2} = 1174$ MeV
- $s^{1/2} = 1190$ MeV
- $s^{1/2} = 1220$ MeV
- $s^{1/2} = 1228$ MeV
- $s^{1/2} = 1243$ MeV
- $s^{1/2} = 1259$ MeV
- $s^{1/2} = 1274$ MeV
- $s^{1/2} = 1295$ MeV

$J \leq \frac{3}{2}$

- ▲-LEGS 2001
- □-SAS 1993
- ○-MAMI 2001
### Threshold $p$-wave multipoles

<table>
<thead>
<tr>
<th>Multipole</th>
<th>Our Values</th>
<th>HBχPT ($Q^3$)</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1 (\pi^0 p)$ [$10^{-3}/m^2_{\pi^+}$]</td>
<td>10.2</td>
<td>9.4</td>
<td>$9.46 \pm 0.05 \pm 0.28$</td>
</tr>
<tr>
<td>$P_2 (\pi^0 p)$ [$10^{-3}/m^2_{\pi^+}$]</td>
<td>$-10.7$</td>
<td>$-10.0$</td>
<td>$-9.5 \pm 0.09 \pm 0.28$</td>
</tr>
<tr>
<td>$P_3 (\pi^0 p)$ [$10^{-3}/m^2_{\pi^+}$]</td>
<td>10.3</td>
<td>10.6</td>
<td>$11.32 \pm 0.11 \pm 0.34$</td>
</tr>
</tbody>
</table>
Energy dependence of the beam asymmetry.

![Graph showing the energy dependence of the beam asymmetry.](image-url)
Energy dependence of the beam asymmetry.

Sign change at threshold!
Energy dependence of the beam asymmetry.

Sign change at threshold!

D-waves are important (C. Fernandez-Ramirez, et al. 2009)
Energy dependence of the beam asymmetry

\[ \Sigma (\theta = 90^\circ) \]

- Gasparyan, Lutz 2010
- MAID
- O. Hanstein et al. 1997
- S. Kamalov et al. 1999

- MAMI 2001
- MAMI 2010 (M. Ostrick, private communication)
Energy dependence of the double polarization observable $F$. 

![Graph showing energy dependence of $F$.](image)
Energy dependence of the double polarization observable $F$. 

D-waves do not contribute at threshold.
Energy dependence of the target asymmetry.
Energy dependence of the target asymmetry.

Access to P-waves at $\pi^+ n$ threshold
No additional parameters need to be adjusted for Compton scattering!
Proton spin polarizabilities in units of $10^{-4}\,\text{fm}^4$

<table>
<thead>
<tr>
<th></th>
<th>$\chi_{\text{PT}}, Q^3$</th>
<th>$\chi_{\text{PT}}, Q^4$</th>
<th>DR</th>
<th>our values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{E1E1}$</td>
<td>$-5.93$</td>
<td>$-1.41$</td>
<td>$-4.3$</td>
<td>$-3.68$</td>
</tr>
<tr>
<td>$\gamma_{M1M1}$</td>
<td>$-1.19$</td>
<td>$3.38$</td>
<td>$2.9$</td>
<td>$2.47$</td>
</tr>
<tr>
<td>$\gamma_{E1M2}$</td>
<td>$1.19$</td>
<td>$0.23$</td>
<td>$0.0$</td>
<td>$1.19$</td>
</tr>
<tr>
<td>$\gamma_{M1E2}$</td>
<td>$1.19$</td>
<td>$1.82$</td>
<td>$2.1$</td>
<td>$1.19$</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>$4.74$</td>
<td>$-4.02$</td>
<td>$-0.7$</td>
<td>$-1.16$</td>
</tr>
<tr>
<td>$\gamma_\pi$</td>
<td>$4.74$</td>
<td>$6.39$</td>
<td>$9.3$</td>
<td>$6.14$</td>
</tr>
</tbody>
</table>

Empirical values:

\[
\gamma_0 = -\gamma_{E1E1} - \gamma_{M1M1} - \gamma_{E1M2} - \gamma_{M1E2} \\
\quad = (-1.01 \pm 0.08 \pm 0.13) \times 10^{-4}\,\text{fm}^4 ,
\]

\[
\gamma_\pi = -\gamma_{E1E1} + \gamma_{M1M1} - \gamma_{E1M2} + \gamma_{M1E2} \\
\quad = (8.0 \pm 1.8) \times 10^{-4}\,\text{fm}^4 .
\]
Energy dependence of the beam asymmetry.
Energy dependence of the double polarization asymmetry $\Sigma_{2x}$.
Proton Compton scattering

Energy dependence of the double polarization asymmetry $\Sigma_{2z}$.

\[ \theta_{\text{lab}} = 90^\circ \]

\[ \theta_{\text{lab}} = 150^\circ \]

\[ \theta_{\text{lab}} = 30^\circ \]

\[ E_\gamma \text{ [GeV]} \]

Gasparyan, Lutz 2010
Pasquini et al. 2007
Proton Compton scattering

Energy dependence of the double polarization asymmetry $\Sigma_{2z}$.

Importance of unitarity and analyticity constraints
Summary

- A method to extrapolate chiral amplitudes beyond threshold region is reviewed.
- Causality and unitarity constraints are utilized to stabilize the extrapolation.
- The processes $\pi N \rightarrow \pi N$, $\gamma N \rightarrow \pi N$ and $\gamma N \rightarrow \gamma N$ are well described up to $\sqrt{s} = 1300$ MeV.
- Predictions for various polarization observables for neutral pion photoproduction and proton Compton scattering as well as for proton spin polarizabilities are presented.