Reciprocal consistency constraints among photoprocesses

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Goal

- Derive a detailed microscopic description of the nucleon current $J^\mu$
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  - Full implementation of *gauge invariance* in terms of *Generalized Ward–Takahashi identities*
  - Assure *reciprocal consistency* of reaction dynamics among all affected photoprocesses
Introduction

\[ \gamma N \rightarrow \pi N \]

\[ \gamma N \rightarrow \pi \pi N \]

\[ NN \rightarrow NN\gamma \]

e.m. nucleon current

\[ \gamma N \rightarrow \gamma N \]

H. Haberzettl, PWA 2011, GWU, 23 May 2011
Electromagnetic Current $J^\mu$ of the Nucleon

How does one describe the current in a Lorentz-covariant Bethe-Salpeter-type approach?
Electromagnetic Current $J^\mu$ of the Nucleon

How does one describe the current in a Lorentz-covariant Bethe-Salpeter-type approach?

- The most general Lorentz-covariant structure of $J^\mu$ requires 12 form factors.  
- Applying gauge invariance, this reduces to 8 form factors.
- Applying time-reversal invariance, this reduces further to 6 form factors.

Bincer, PR118,855(1960)
Electromagnetic Current $J^\mu$ of the Nucleon

How does one describe the current in a Lorentz-covariant Bethe-Salpeter-type approach?

- The most general Lorentz-covariant structure of $J^\mu$ requires **12 form factors**.  
  \[ J^\mu(p', p) = e \left[ \delta_N \gamma^\mu + \delta_N \gamma^\mu_T(F_1 - 1) + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N F_2 \right. \]
  \[ \left. + \frac{S^{-1}(p')}{2m} \left( \gamma^\mu_T f_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N f_2 \right) + \left( \gamma^\mu_T f_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N f_2 \right) \frac{S^{-1}(p)}{2m} \right. \]
  \[ \left. + \frac{S^{-1}(p')}{2m} \left( \gamma^\mu_T g_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N g_2 \right) \frac{S^{-1}(p)}{2m} \right] \]  
  \[ \gamma^\mu_T = \gamma^\mu - k^\mu \frac{k_\mu}{k^2} \]

- Applying gauge invariance, this reduces to **8 form factors**.
- Applying time-reversal invariance, this reduces further to **6 form factors**:
  \[ F_1, F_2, f_1, f_2, g_1, g_2 \]
Electromagnetic Current $J^\mu$ of the Nucleon

How does one describe the current in a Lorentz-covariant Bethe-Salpeter-type approach?

- The most general Lorentz-covariant structure of $J^\mu$ requires **12 form factors**.
- Applying gauge invariance, this reduces to **8 form factors**.
- Applying time-reversal invariance, this reduces further to **6 form factors**:

$$J^\mu(p', p) = e \left[ \right.$$

$$\delta_N \gamma^\mu + \delta_N \gamma_T^\mu (F_1 - 1) + \frac{i \sigma^{\mu \nu} k^\nu}{2m} \kappa_N F_2$$

$$+ \frac{S^{-1}(p')}{2m} \left( \gamma_T^\mu f_1 + \frac{i \sigma^{\mu \nu} k^\nu}{2m} \kappa_N f_2 \right) + \left( \gamma_T^\mu f_1 + \frac{i \sigma^{\mu \nu} k^\nu}{2m} \kappa_N f_2 \right) \frac{S^{-1}(p)}{2m}$$

$$+ \frac{S^{-1}(p')}{2m} \left( \gamma_T^\mu g_1 + \frac{i \sigma^{\mu \nu} k^\nu}{2m} \kappa_N g_2 \right) \frac{S^{-1}(p)}{2m} $$

$$\left. \right]$$

**(Approximation)**

$$\gamma_T^\mu = \gamma^\mu - k^\mu \frac{k}{k^2}$$

**Constraints:**

- no kinematic singularity:

  $$f_1(k^2) \xrightarrow{k^2=0} 0 \quad \text{and} \quad g_1(k^2) \xrightarrow{k^2=0} 0$$

- chiral-symmetry limit:

  $$f_1 \rightarrow \frac{g_A - G_A(k^2)}{g_A} \quad \text{and} \quad f_2 \rightarrow 1$$
Implications of off-shell structure: Pion photoproduction

\[ F_s S(p + k) J_i^\mu (p + k, p) = F_s S(p + k) \left( e \delta_i \gamma^\mu + \frac{i \sigma^{\mu \nu} k_\nu}{2m} e \kappa_i \right) + F_s \frac{i \sigma^{\mu \nu} k_\nu e \kappa_i}{2m} f_{2i} \]

**s-channel:**

**u-channel:**

\[ J_f^\mu (p', p' - k) S(p' - k) F_u = \left( e \delta_f \gamma^\mu + \frac{i \sigma^{\mu \nu} k_\nu}{2m} e \kappa_f \right) S(p' - k) F_u + \frac{i \sigma^{\mu \nu} k_\nu e \kappa_f}{2m} \frac{2m}{2m} f_{2f} F_u \]
Implications of off-shell structure: Pion photoproduction

\[ s\text{-channel:} \]
\[ F_s S(p + k) J_i^\mu (p + k, p) = F_s S(p + k) \left( e \delta_i \gamma^\mu + \frac{i \sigma^{\mu\nu} k_\nu}{2m} e \kappa_i \right) + F_s \frac{i \sigma^{\mu\nu} k_\nu e \kappa_i}{2m} f_{2i} \]

\[ u\text{-channel:} \]
\[ J_f^\mu (p', p' - k) S(p' - k) F_u = \left( e \delta_f \gamma^\mu + \frac{i \sigma^{\mu\nu} k_\nu}{2m} e \kappa_f \right) S(p' - k) F_u + \frac{i \sigma^{\mu\nu} k_\nu e \kappa_f}{2m} \frac{2m}{f_{2f} F_u} \]

One cannot make the connection to low-energy \( \chi \)PT results without such contact terms.
A Word about “Off-shell Effects”

It is often stated that “off-shell effects are not measurable” and that, therefore, any such effects should be summarily banished from any theory.
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Franz Gross on off-shell effects

It is commonly stated that “off-shell effects” are unobservable. This is of course true, but so are wave functions, potentials, and most of the theoretical tools we use to describe physics. A better point is that off-shell effects are meaningless without a theory or model to define them. Almost all models provide such a definition, and off-shell effects should be discussed only in the context of a particular model that defines these effects uniquely.
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Within the Bethe-Salpeter-type equations that originate from effective Lagrangian formulations, the off-shell structure of the nucleon current arises naturally as an integral part of the description of the reaction dynamics.
Electromagnetic Current $J^\mu$ of the Nucleon

For photoprocesses... 

- the generic structural description of the nucleon current, in general, is not good enough
- more details of the current's internal explicit reaction dynamics are required
Electromagnetic Current $J^\mu$ of the Nucleon

For photoprocesses... 

- the generic structural description of the nucleon current is, in general, not good enough; 
- more details of the current's internal explicit reaction dynamics are required.

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Require reciprocal consistency among the various photoprocesses to determine the dynamical structures of the current $J^\mu$. 

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H. Haberzettl, PWA 2011, GWU, 23 May 2011
Introduction

\[ \gamma N \rightarrow \pi N \]

\[ \gamma N \rightarrow \pi \pi N \]

e.m. nucleon current

\[ NN \rightarrow NN\gamma \]

\[ \gamma N \rightarrow \gamma N \]
Dynamical Links between Photoprocesses

\[ \gamma N \rightarrow \pi N \]

\[ \gamma N \rightarrow \pi\pi N \]

\[ \gamma N \rightarrow \gamma N \]

\[ NN \rightarrow NN \gamma \]

e.m. nucleon current
Pions, Nucleons, and Photons

\[ T(a) = 0 + X \]
\[ T(b) = \bar{V} + V T \]
\[ T(d) = U + U X \]
\[ V(c) = U + U \]
\[ \pi N T \text{ matrix} \]

\[ \pi N N \text{ vertex} \]

Tower of \textit{nonlinear} Dyson-Schwinger-type equations

\[ \text{dressed nucleon propagator} \]
\[ \text{propagator determines current} \]

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Nucleon Current $J^\mu$

(a) $=$ $+$

(b) $=$ $+$

Couple photon to dressed propagator:

(a) $=$ $+$ $+$ $+$

(b) $=$ $+$ $+$ $+$ $+$

Tower of nonlinear Dyson-Schwinger-type equations
Pion Photoproduction

- **Pion-production current** $M^\mu$:

  \[ M = b + X b \]  

  \[ b = U \]  

  The internal structures of the dressed nucleon current can be understood by the dynamics of the pion production current.

- **Nucleon current** $J^\mu$:

  \[ J = \text{Diagram} \]  

  Tower of *nonlinear* Dyson-Schwinger-type equations
Rewriting the Production Current

Pion-production current $M^\mu$:

(a) $M = \text{equivalent} + B + X B_T$

(b) $M = \text{equivalent} + B + T B_T$

(c) $B = \text{equivalent} + \text{equivalent} + \text{equivalent}$

Contact-type current $M^\mu_c$:

$M^\mu_c = \text{equivalent} + U L + \text{equivalent} + \text{equivalent}$

Tower of *nonlinear* Dyson-Schwinger-type equations
Rewriting the Production Current

- **Pion-production current $M^\mu$:**

  
  $$M = B + X B_T$$  \hspace{1cm} (a)

  
  $$M = B + T B_T$$  \hspace{1cm} (b)

  
  $$B = U + U_L + U_L$$  \hspace{1cm} (c)

- **Contact-type current $M_c^\mu$:**

  
  $$B = U + U_L + U_L + U_L$$

- **Tower of nonlinear Dyson-Schwinger-type equations**

  partial integral equation

  longitudinal

  (irrelevant for gauge invariance)
### Rewriting the Production Current

**Pion-production current** \( M^\mu \):

\[ M = B + X_B \]

\[ M = B + T_B \]

\[ B = \text{not the full nucleon current} \]

**Contact-type current** \( M_c^\mu \):

\[ \text{not the full nucleon current} \]

**Tower of nonlinear Dyson-Schwinger-type equations**
Nucleon Current $J^\mu$

(a) $N = \varepsilon + T + T + T$

(b) $N = \varepsilon + L + L + L + L$

- Tower of *nonlinear* Dyson-Schwinger-type equations
Nucleon Current $J^\mu$

\[ J^\mu = J^\mu + J^\mu_{(a)} \quad \text{(a) transverse} \]

\[ J^\mu_s = J^\mu_s + J^\mu_s_{(b)} \quad \text{(b) longitudinal} \]

- Tower of \textit{nonlinear} Dyson-Schwinger-type equations

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Nucleon Current $J_\mu$

\[
J_\mu = J_{\mu}^{HH} + J_{\mu}^{T} + J_{\mu}^{T}(a)
\]

\[
J_\mu^s = J_{\mu}^{T} + J_{\mu}^{L} + J_{\mu}^{L} + J_{\mu}^{L}(b)
\]

Gauge Invariance: Ward-Takahashi Identity (WTI)

\[
k_\mu J_\mu(p', p) = k_\mu J_\mu^s(p', p) = S^{-1}(p')Q_N - Q_NS^{-1}(p)
\]

$S$: dressed nucleon propagator
Problems?

- Everything is exact!
- Everything is nonlinear!
- Everything is hideously complicated!
Everything is exact!

Everything is nonlinear!

Everything is hideously complicated!

But... 😊
Let’s cut the Gordian knot!

Do not use $X$. Work with full $T$.

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Cutting the Gordian Knot

\[ M = B + X B_T \]  

\[ J_\mu^S \]  

not the full nucleon current

\[ J_\mu \]

determine approximation by WTI for the nucleon current \( J_\mu \)
Cutting the Gordian Knot

\[ M_{\mu}^C \]

determine approximation of \( M_{\mu}^C \) by generalized WTI for the photoproduction current \( M^\mu \)

\[ (a) \]

\[ (b) \]

\[ (c) \]
Reminder: Generalized Ward–Takahashi Identity

\[ M^\mu = M^\mu_s + M^\mu_u + M^\mu_t + M^\mu_{int} \]

- **Generalized WTI for the full current \( M^\mu \):**

\[
k_\mu M^\mu = -F_s S(p + k) Q_i S^{-1}(p) + S^{-1}(p') Q_f S(p' - k) F_u + \Delta_\pi^{-1}(q) Q_\pi \Delta_\pi(q - k) F_t
\]

- **Equivalent Generalized WTI for the interaction current \( M^\mu_{int} \):**

\[
k_\mu M^\mu_{int} = -F_s Q_i + Q_f F_u + Q_\pi F_t
\]
Reminder: Generalized Ward–Takahashi Identity

\[ M^{\mu} = M_{S}^{\mu} + M_{u}^{\mu} + M_{t}^{\mu} + M_{\text{int}}^{\mu} \]

- **Generalized WTI for the full current \( M^{\mu} \):**

\[
k_{\mu} M^{\mu} = -F_{S} S(p + k) Q_{i} S^{-1}(p) + S^{-1}(p') Q_{f} S(p' - k) F_{u} + \Delta_{\pi}^{-1}(q) Q_{\pi} \Delta_{\pi}(q - k) F_{t}
\]

- **Equivalent Generalized WTI for the interaction current \( M_{\text{int}}^{\mu} \):**

\[
k_{\mu} M_{\text{int}}^{\mu} = -F_{S} Q_{i} + Q_{f} F_{u} + Q_{\pi} F_{t}
\]

Here: \( k_{\mu} M_{\text{int}}^{\mu} = k_{\mu} M_{c}^{\mu} \)

Off-shell constraints!
Approximating $M^\mu_c$

Lowest-order approximation in terms of phenomenological form factors:

$$M^\mu_c = ge\gamma_5 \frac{i\sigma^{\mu\nu}k_\nu}{4m^2} \tilde{\kappa}_N - (1 - \lambda)g \frac{\gamma_5 \gamma^\mu}{2m} \tilde{F}_\pi e - G_X \left[ e_i \frac{(2p + k)^\mu}{s - p^2} \left( \tilde{F}_s - \hat{F} \right) 
+ e_f \frac{(2p' - k)^\mu}{u - p'^2} \left( \tilde{F}_u - \hat{F} \right) 
+ e_\pi \frac{(2q - k)^\mu}{t - q^2} \left( \tilde{F}_t - \hat{F} \right) \right]$$

Don’t try to read the details. What is important is that this is a simple expression, easy to evaluate, and that it helps preserve gauge invariance of the entire production current.
Approximating $J^\mu_s$

\[ J^\mu_s(p', p) = (p' + p)^\mu \frac{S^{-1}(p') Q_N - Q_N S^{-1}(p)}{p'^2 - p^2} + \left[ \gamma^\mu - \frac{(p' + p)^\mu}{p'^2 - p^2} k \right] \frac{Q_N A(p'^2) + A(p^2)}{2} \]

- **Approximate $J^\mu_s$** by the minimal current that reproduces the WTI:

- **Half on-shell:**

\[ S J^\mu_s u = \left( \frac{1}{\not{p} + \not{k} - m} j_1^\mu + \frac{2m}{s - m^2} j_2^\mu \right) Q_N u(p) , \quad \text{with} \quad s = (p + k)^2 \]

- **Auxiliary currents:**

\[ j_1^\mu = \gamma^\mu (1 - \kappa_1) + \frac{i\sigma^{\mu\nu} k_\nu}{2m} \kappa_1 \quad j_2^\mu = \frac{(2p + k)^\mu}{2m} \kappa_1 + \frac{i\sigma^{\mu\nu} k_\nu}{2m} \kappa_2 \]

**Two parameters!**

Ball–Chiu:
- Satisfies WTI
- Nonsingular
- Minimal
- Unique!
On the importance of maintaining gauge invariance

- Preliminary results for $\gamma N \rightarrow \pi N$:

Dashed green curves: w/o $M^\mu_c$

Dynamical Links between Photoprocesses — Bremsstrahlung

\[ \gamma N \rightarrow \pi N \]

\[ \gamma N \rightarrow \pi \pi N \]

\[ \gamma N \rightarrow \gamma N \]

Bremsstrahlung

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Bremsstrahlung $NN \rightarrow NN\gamma$

Bremsstrahlung Current:

$$J_B^\mu = (TG_0 + 1)J_T^\mu (1 + G_0T)$$

$T$: $NN$ $T$-matrix

Compare the photon processes along the top nucleon line above to the meson production diagrams below.

Essential parts of the process can be described as a meson capture process — i.e., as an inverse photoproduction process — in the presence of a spectator nucleon.
Bremsstrahlung $NN \rightarrow NN\gamma$

- Application to KVI data. — Or: Resolving a longstanding problem:

- Inclusion of the four-point interaction current from meson photoproduction brings about a dramatic improvement.
Dynamical Links between Photoprocesses — Two-Pion Production

\[ \gamma N \rightarrow \pi N \]

\[ \gamma N \rightarrow \pi \pi N \]

\[ N N \rightarrow N N \gamma \]

\[ \gamma N \rightarrow \gamma N \]

Two-Pion Production
Basic Two-pion Production Mechanisms

(a)

(b)

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Dynamical Links between Photoprocesses — Compton Scattering

\[ \gamma N \rightarrow \pi N \]

\[ \gamma N \rightarrow \pi \pi N \]

\[ NN \rightarrow NN\gamma \]

\[ \gamma N \rightarrow \gamma N \]

Compton Scattering
Compton Scattering $\gamma N \rightarrow \gamma N$

$s$- and $u$-channel terms employ dressed current just described.

Contact term constrained by gauge invariance.
Conclusions

- There exists a very close relationship between the dressed nucleon current and the pion photoproduction current.

- Exploiting this relationship suggests physically meaningful approximations that work, despite the enormous complexity of the exact formalism.

- Maintaining full gauge invariance (as opposed to mere current conservation) is not a luxury but a necessity for the correct microscopic description of the reaction dynamics.

- Requiring gauge invariance in the form of off-shell (generalized) Ward-Takahashi identities for each subprocess provides a powerful tool for constraining the contributing mechanisms and ensuring overall gauge invariance as a matter of course.

- **Note:** Gauge invariance (as an off-shell condition) cannot be maintained in a non-covariant phenomenological Lagrange-type formalism. At best, one can have non-unique non-relativistic types of current conservation.
Goal

- Derive a detailed microscopic description of the nucleon current $J^{\mu}$:
  - Full implementation of gauge invariance in terms of Generalized Ward–Takahashi identities
  - Assure reciprocal consistency of reaction dynamics among all affected photoprocesses
Goal ✓

Derive a detailed microscopic description of the nucleon current $J^\mu$:

- Full implementation of gauge invariance in terms of Generalized Ward–Takahashi identities
- Assure reciprocal consistency of reaction dynamics among all affected photoprocesses
- As a bonus, this provides a novel* description of the pion photoproduction process that has many features that make it particularly well suited for practical applications

Thank you!

*) In the spirit of HH, Nakayama, Krewald, PRC 74, 045202 (2006), but decisively different in detail.